

Logic, Agency, and Games, Day Five

Topic flow:

1 These first four days, you have had an introduction to dynamic-epistemic logics of agency, and hopefully you have come to appreciate the natural connection between logic and games.

2 I had to select, and omitted many topics, such as group knowledge, further game equivalences, Forward Induction, or repercussions of theory of play such as agent diversity. You have the course material available, and if you are really desperate to read more, the references are there.

3 Behind these specific topics there are grand methodological issues behind current logics of agency, or even, between the various branches of logic represented at a summer school like this. I mentioned the interface of *logic and probability*: rivals or partners? (can send references on request), and I had also wanted to talk about the interface of logics of agency and the theory of *dynamical systems*, or stated differently, between 'high' and 'low' rationality (a paper attached).

4 I did talk about one such grand theme: *information levels*. In logic there are many levels for representing information that agents have, from coarse (sets of possible worlds) to fine (syntax, yes, I know that one of you incisively questioned this way of putting things). I believe that it is essential to understand how these levels are related, not just for the study of agency, but also to see the cooperation of model theory and proof theory in the right light. I signaled one emerging theme in recent research: 'tracking' (or non-trackability) of information updates between levels.

5 Specific illustration, one level up from plausibility models: *evidence models*. Richer representation of what information agents have received, deconstructs public announcement into new operations of evidence addition and evidence removal. Recursion axioms still work for getting complete logics, and they now even suggest new static operators, such as varieties of conditional belief that had not been considered before, or a new 'instantial neighborhood logic' (INL) where we can also say what objects occur in neighborhoods. Everything connects to everything else: we are now using INL to explore a finer notion of game equivalence, different from the powers that were introduced before, getting a new logic of game operations in the process.

6 Levels are related by *mappings*. Downward: evidence models generate a plausibility order, plausibility models become epistemic models by forgetting the order. Upward: e.g., families of upward plausibility-closed sets induce evidence models. Such mappings, upward or downward, also support translations between the languages appropriate to the levels (cf. our earlier discussion of invariances and languages). So, richer and poorer views are not in competition.

7 *Tracking*: commuting diagrams for updates at two levels (see the attached paper). This works for many nice cases, and this two-level harmony has been exploited already, e.g., to give richer descriptions of moral reasoning in deontic logic. However, there are also interesting non-trackability results, showing that a richer level may have its own updates that do not 'reduce'. We showed how evidence addition is trackable, while evidence deletion is not (with some involved pictures). Tracking and non-trackability results are especially prominent these days at the interface of probability theory and qualitative forms of belief revision such as AGM theory. Currently, I am doing a broad study of tracking using category-theoretic perspectives.

8 Note: both directions are natural: giving more structure, and giving less structure. In cognitive reality we must compress information, otherwise our brains would go wild. There may be a sort of temporal zigzagging dynamics through the World of Levels.

9 We also did a brief Q & A session. I really appreciated all your questions and points (also in email, also on earlier days), and please do not hesitate to write to me with more of these!