Mathematics and Physics of Walking

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A practical investigation task for pupils at upper secondary school level who have no experience with COACH. Estimated working hours: 8.

Preface

Coach is a computer learning environment that has been developed originally for natural sciences (physics, chemistry, and biology). In this program you can easily collect data from different sources, display data in graphical or tabular format, process data mathematically, and so on.
Coach offers the possibility of collecting data from video clips. You can record yourself a video clip or you can use an existing video clip for the purpose of carrying out practical investigation tasks. You may think of studying the orbit, velocity, and acceleration of a thrown basketball, investigating the motion of a car on a roller coaster, exploring the collision of two cars, and so on.
Actually, we are going to use the data video tool for measuring human movements. You will use existing video clips of walking, jogging, race walking, and running. Maybe you can record your own video clips with a webcam, e.g., for the purpose of studying your own locomotion or the movement of a fellow pupil. In short, you will play the role of a movement scientist.
The program Coach works with projects, each consisting of several activities. This practical investigation task relates to the project `Walking`.

Reasons for Human Movement Research

Humans walk almost all in the same manner: gait is a periodic movement of each foot from one position of support to the next position of support in the direction of progression. The process of walking involves moving the center of gravity up and down and swinging of arms. Yet, one can often recognize an acquaintance from a distance by his or her walk. Apparently some personal characteristics are present in gait. This offers crime fighters the possibility to identify thieves and hooligans running away from a crime scene by means of video recordings of security cameras. Gait recognition could be a useful addition to individual identification methods such as finger printing and iris recognition.
Researchers in biomechanics try to develop techniques for measuring individual gait characteristics for authentication and recognition of persons from the way they walk One of the issues is whether the movements of the human limbs can be described well by trigonometric functions. Not every periodic movement can be described well by a sine graph, but for (certain phases of) walking and running it often works fine. There are reasons for this. Gait recognition is based on the mathematical model of curves that are built up from various sinusoids. Here, the more scientific term is gait signature, which is based on the Fourier representation of periodic motion. In this practical investigation task you will apply simple mathematical and physical models and compare theoretical results with measurements.
Quite another motive for movement science comes from motion-picture industry. One can often recognize a person's state of mind from his or her gait and posture. The gait of a cheerful person differs from the one of a depressed person. The emotional aspects of walking can also be described experimentally in mathematical terms. Creators of animated cartoons and

science fiction movies have an interest in this: they often do not want to have robot-like figures in their movies, but instead creatures that have emotion-like motions. Mathematics gives them a helping hand.
Movement scientists also support athletes with their gait analyses. By measuring and analyzing human motion and the forces involved they can give good advice to sportsmen in order to improve the sporting achievements. For example, a small improvement in the technique of a hurdler can make the difference between championship and loss.
Finally, gait analysis is popular because of medical importance of results: orthopedic surgeons, kinesiologists, therapists, rehabilitation engineers, prosthetists and many others are all involved in the treatment of movement disorders and are all interested in the modeling of the gait cycle. The models can be used to assist diagnosis in disorders, to determine the effect of prosthesis, to determine the success of particular rehabilitation techniques, and to determine the effect of certain shoes and specific surfaces. Mathematical models are useful in the design of such products.

The Learning Material
The explanations and the tasks can be read almost completely at the computer screen. We have also provided them in printed form, so that you can choose the format that you find most convenient. The computer tasks in the first activities are written down in so much detail that it may give you the impression of drill and practice work. We have done this with the purpose that you can familiarize yourself with video measurement as quickly and as independently as possible. The mathematics and physics is mainly present in the biomechanical topic itself. By the way, after this project you have learnt so much about Coach that you can continue to use it in other work without much difficulty.

Report
Part of this practical investigation task is writing a report with a text processor about the last three Coach activities. Herein we expect the answers to the exercises that ask for a reply, sometimes short, sometimes a bit longer, but in any case clear and to the point. We also would like to know which parts of the work you found easy or difficult. When you close an activity, then COACH asks “Save changed result?”. Click on 'Yes'. In this way, you also get a digital part of your report!
You can copy and paste tables, diagrams, and text from Coach into a Word document. You are advised to start Word, in conjunction with Coach. Then you can copy intermediate results like graphs, tables, and whatsoever into a Word document. You can decide later what you will use of this in the final report.
Furthermore, we expect a summary of the time that you spent on various parts of the investigation task (a sort of lab journal).

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The ‘Walking’ Project

This project consists of five activities:
1. Video measurement of walking.
2. Gait analysis.
3. Arm movements during normal walking.
4. Swing phase of the leg in low speed walking.
5. The complete gait cycle of low speed walking.

The main topic of this project is the (mathematical) shape of human movements during walking. We have included some extra practical work for enthusiasts at the end of this project. In the first three activities the main purpose is to get acquainted with the possibilities of Coach for investigating human movements in walking by measurements of video clips. You will also use simple models to look at the collected data from a more theoretical perspective. In the last few activities the self-reliance of investigating human gait is increased.

Activity 1: Video measurement of walking

In this activity you will get acquainted with some facilities of Coach. Part of the work has already been done, as you will see. In the next activity you will have to apply what you have learnt here.

Start Coach, (log in as student; no password needed) and choose the activity *1. Video measurement of walking*.

Introduction

You see an activity menu bar, an activity toolbar, and four windows:

Upper-left: a video measurement of a magnified and sharpened video clip.
Upper-right: a window with the original video clip.
Down-left: an empty window.
Down-right: a text window.

In the upper-right window you see a short movie in which Mr. Holleman walks along a building. This video clip has been created outside with a webcam, which was attached to a laptop computer. Every second 25 frames were recorded.
Play the movie to get an impression of the movement.

The movie clip has been magnified and sharpened with video editing software to make measuring on the computer easier. In the upper-left data video window you see a coordinate system, a light-blue indication of the scale labeled 1 meter, and data points (red and lilac dots) tracing the locations of the right shoulder and hand over time. By the way, one of the data points on the hand has been deliberately misplaced.
The video clip consists of 32 frames, which are represented on the video control bar, at the bottom of the data video window, by small strokes.

In this activity you will use the tools of Coach to collect data from digital video clips. Place results of your work only in your report if there are good reasons for doing so.
We shall mostly pay attention to
- the graphical display of collected data;
- the linking between graphs and the video clip.

This introduction is in a text window. In addition there are other texts with exercises and tasks. To see them, left-click on the yellow 'Display Text' button in the activity toolbar. First choose '2. Linking tables and graphs with a video measurement'. The shape of the mouse cursor changes into a text symbol. Move the cursor to an empty window and click in this window. In the same way you can obtain the other texts windows. Actually, you don't need these windows because all texts are on paper as well. Choose what you find most convenient.

Linking tables and graphs with a video measurement

1. Coach automatically places the coordinates of the measured points into a table and it prepares a diagram (a data plot). To make the table visible, press the 'Display Table' button in the activity toolbar. Choose 'Holleman walks' and click in the upper-right window (we do not use the original movie anymore).
   If you right-click in the table window, you get a menu. Choose 'Display as a Diagram'. Because we do not need the introductory text anymore, you can click in this text window.

2. You see four graphs in the diagram. Explain which graph belongs to what quantity. Why is one red graph almost constant and the other one increasing? Of the two increasing graphs, the red graph is in turns above and below the lilac graph. Express what this means for the gait.

3. Right-click in the diagram window or press the tool button of this window (iconified by a hammer symbol). Select 'Scan', place the cursor somewhere near the time-axis. Notice that the corresponding frame appears in the data video window, that the corresponding row is highlighted in the table window, and that a legend appears in the diagram window from which it is easy to read off values of displayed quantities. The legend can be dragged to any location in the diagram window, for example to a location where it does not hide parts of the graphs.
   Drag the cursor in the diagram window and notice that the frame displayed in the video clip en and the highlighted row in the table change, too. By pressing ← and → on the keyboard (whilst the data video window or the diagram window has the focus) you can step through the frames of the video clip and the corresponding points of the graph. In this way you can easily make the connection between interesting points of the graphs and the frames in the video clip.

4. Drag the cursor in the diagram window until it is located at the time for which the data collection went wrong and the wrong location of the right hand was measured. Move the video point to the correct location and watch the result in the diagram and table window.
   In this way you can relocate more video points. (There are more ways of accomplishing this as you will see later or find out yourself.)
   Stop scanning in the diagram window.

Collecting data from the video clip

1. In this activity we stopped measuring prematurely: on the last four frames of the video clip, the location of shoulder and hand have not been measured. You have to finish this yourself.
   Start measuring by clicking the green 'Start' button in the activity toolbar. The video clip jumps automatically to the first

frame that has not been measured yet. Move the cursor across the data video window towards the middle of the sticker attached to the right shoulder and left-click this point. Next, move the cursor to the middle of the piece of paper that Mr. Holleman holds in his hand and click this point. Now you have collected the coordinates of these two locations and the video clip moves all by itself to the next frame for measurement. Repeat this until you are done (the 'Start' button changes color from red to green).

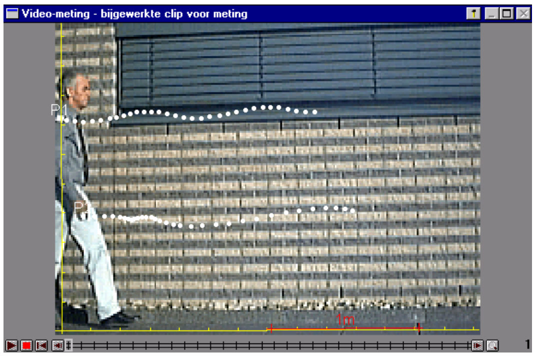
Zooming

1. The x- and y-coordinates of the right shoulder and the right hand during walking have been plotted against time in the diagram window. Time t=0 has been chosen to correspond with the first frame of the video clip.
   Right-click in the diagram window. Choose 'Zoom to fit'. Notice the red cross through the magnifying glass on the leftmost button in the button bar of the diagram window. Press this button, which we will refer to as the zoom-button. So there are at least two ways to zoom in and out.

2. Another way of zooming is the following: drag an area to zoom in. This area will be enlarged to fill the whole diagram. Zoom in a couple of times on the graph of the vertical position of the shoulder. By clicking the zoom-button in the diagram window you return to the previous zoom state.

Leave the activity.
Answer 'Yes' to the question 'Save changed result?'.

Activity 2: Gait analysis



In this activity you will get acquainted with some facilities that Coach offers. The central theme is the determination of a curve that fits the data.

If necessary, start Coach and choose the activity *2. Gait analysis*.

Introduction

In the movie clip, Mr. Holleman walks along a building. You see a coordinate system, a light-blue indication of the scale labeled 1 meter, and data points (red and lilac dots) tracing the locations of the right shoulder and the right hand over time. The video clip consists of 32 frames, which are represented on the video control bar, at the bottom of the data video window, by small strokes.

The graphs of the coordinates of the right shoulder and the right hand have already been prepared.

In this activity and others you will learn that normal gait can be described well with trigonometric functions. Here we concentrate mostly on the movements of shoulder and hand during normal walking.

Some observations about normal walking

1. By watching the video clip and the corresponding graphs closely you can already find out a lot of things about normal walking. Some questions as a result of measuring the video clip:
   What periodic movements do the legs make during walking and what phases can be distinguished in the gait cycle?

What can you say about the coordination of arm and leg movements? How are shoulder height and phase of leg movement correlated? How many meters per second does Mr. Holleman walk in this movie? What is your estimation of the duration of the gait cycle, i.e., the time that it takes to go through a complete period of walking? Use this to compute the step frequency, i.e. the number of steps per minute. (Do not forget to explain what you mean by one step)

2. You can easily measure in the data video window distances and angles with a computerized ruler and protractor. Right-click in the data video window or use its tool button. Select 'Ruler'. Drag the endpoints of the ruler so that they lay on the points between which you want to know the distance. Measure the length of a single step in this way.

3. From the measured stride length and the step frequency you can compute the average walking speed. How many meters per second does Mr. Holleman walk in this video clip? Is this in agreement with the answer that you found before? What is this walking speed expressed in kilometers per hour?

Practicing the creation of new diagrams

1. Right-click again in the diagram. Choose 'Create/edit diagram…'. In the dialog window you see among other things the name of the window, the option to display a grid, eight columns to choose from, and at the bottom facilities to change the display of the graph. Click C1. The 'Clock' is connected with the time of the video clip. Look at columns C2, C3, C4, and C5. You will probably recognize the coordinates of the video points. Make the vertical coordinates of the video points invisible.

2. Click the yellow button 'Display Diagram' in the activity toolbar. Instead of choosing an existing diagram, you will now create a new diagram that will only display the coordinates of the right hand: Press the 'New diagram' and give this window a sensible name (e.g. 'right hand'). Choose as first column the connection 'Clock' and introduce as 2^{nd} and 3^{rd} columns the x- and y-coordinate of the right hand, respectively. Click OK and select the diagram just made: you have now two graphs in one diagram. Find out how you can plot the vertical position of the hand against the horizontal position. Do you recognize this graph?

3. Experiment in the diagram just made with the Coach facilities to display data points with various markers and colors.

Determination of a curve that fits the data

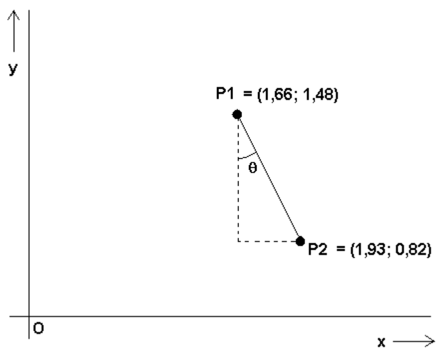
1. Plot the horizontal position of the right shoulder in a new diagram against time. Only draw the measured points and do not connect them with straight lines.
   Right-click in the diagram (or press the tool button, second button from the left in the button bar of the diagram window).
   Choose 'Analyse' and the sub-item 'Function-fit'. At 'Function Type' you see that the graph of f(x)=a.x+b is drawn. The value of a(=1,38) and b(=-9E-3) of the straight line are next to it. Press the 'Auto fit' button. The straight line that fits the data best is computed. What does a change of the value of 'a' mean in terms of the line? What effect has a change of the value of 'b' for the line?
   If you press 'OK', then the straight line is added to the diagram window. If you choose 'Stop', then the diagram window remains the same. Write down the formula of the line and press 'OK'.

2. Plot the horizontal position of the right shoulder in a new diagram against time. Only draw the measured points and do not connect them with lines. Zoom in so that the graph is displayed as large as possible. It is not a sine graph because the graph has a tendency to increase in the right direction. What could be the reason for this phenomenon? (Hint: measure in the video clip with the 'Ruler' the difference in height between window and pavement on the left- and right-hand side of the window).

3. We are going to do our best to find a reasonable formula for the vertical position of the shoulder as a function of time.
   Let us first determine the trend in the graph: do a straight-line function fit of the data points. Add this line to the diagram.
   Plot the graph of the difference of the measured vertical position of the shoulder and the straight line, giving it its own vertical axis. You can do this as follows:
   - Right-click in the diagram window and select the menu item 'Create/edit diagram…'
   - Choose an empty column and change the connection into 'formula'
   - Choose 'Axis: second vertical'
   - Click in the formula text region after 'Formula:'
   - Press the 'Wizard' button (the button with the magic hat)
   - Click on known quantities and mathematical operations to make the correct formula.
   - Fill out the name of the quantity introduced and (optionally) specify the unit.
   - Press 'OK' and zoom to fit by pressing repeatedly the zoom-button until you have reached your goal of having the graph of the difference displayed as large as possible.

   If everything went right you now have a set of points that lie pretty much on a sine graph. Use function-fit to find the best sinusoid. The sum of this sinusoid and the previously found straight line describes the original data pretty well. Let Coach plot the graph of this sum to check this.

4. If you know the position of the right shoulder and right hand at a certain time, then you can compute the angle that a stretched arm at that time makes with the imaginary vertical axes passing through the shoulder. Let us compute a concrete example before the general case: in the diagram you can read off that at time t=1.2s the position of the shoulder (P1) equals (1.66, 1.48) and that the position of the right hand (P2) equals (1.93, 0.82). Positions are in meters with respect to the coordinate system.
   Use the opposite sketch to compute the indicated angle θ.
   Hint: First, compute the tangent of this angle. Use this result to find out which angle (in radians) corresponds with this. Check your answer in the video clip by measuring the angle in the correct frame with the computerized protractor (if desired, maximize the data video window and the clip for better reading).



5. Let us now try the general case: we denote the positions of P1 and P2 as (x_1, y_1) and (x_2, y_2), respectively. Verify that the following formula holds:

$$\tan(\theta) = \frac{x_2 - x_1}{y_1 - y_2}$$

In other words: you can compute the angle with the formula

$$\theta = \tan^{-1}\left(\frac{x_2 - x_1}{y_1 - y_2}\right).$$

In the Coach activity is already a diagram window present, entitled 'arm angle', which uses this formula (with arctan as synonym of $\tan^{-1}$). Make this diagram visible. Measure the arm angle in the first frame of the video clip with the computerized protractor and check your answer in the graph of the arm angle.

Leave the activity.
Answer 'Yes' to the question 'Save changed result?'.

Activity 3: Arm movements during normal walking

In this activity you analyze the movements of arms during walking at normal speed. You measure in the data video window the angle of the line from the right shoulder to the right hand with the imaginary horizontal line passing through the shoulder. From this you derive the arm angle, by us defined as the angle of the stretched arm from the shoulder to the hand with the imaginary vertical axis passing through the shoulder. The arm angle is a function of time.

In this example you learn how to carry out measurements on digital video clips with Coach. The instructions are very detailed and guide you through the example.

If necessary, start Coach and choose the activity *3. Arm movements during normal walking*.

Open and play the video clip

1. Right-click in the upper-left window called 'Data Video' and choose 'Open Video'. Select the video clip 'Walking'.

2. The video clip, which has been recorded outside with a webcam and a laptop computer, shows a person who is walking at normal speed along a building. Play the video by pressing the Play button in the video control bar, at the bottom of the data video window.

3. The video clip consists of 42 frames, which are represented on the video control bar by small strokes. By pressing ← and → on the keyboard (whilst the data video window has the focus) you can step through the frames of the video clip. Do this and notice that there is something strange about frame 22: it is identical to the previous frame. This means that, while recording the video clip, at this point a frame has been missed ("frame dropped"). You must take this into account, later when you collect data from the video clip.

Calibrate the video clip

Before you can collect meaningful data from the video clip you must calibrate the distances in the video clip. Quite often the creators of a video clip have added to the first frame an indication of dimensions. In this clip you must manage with the single remark that the square paving stones all have a width of 50 cm.

1. Right-click in the data video window and choose 'Coordinate settings…'.
   - Scale: because the vertical and horizontal scales are the same, select 'Same scale in all directions'.
   - Origin: Because we want to study the arm movements with respect to the shoulder select the origin of the coordinate system for all frames separately, via 'First point clicked at each frame'.
   - Time Calibration: the video clip has been recorded with a frame rate of 30 frames per second. This information is used to connect the frame number with the time t (in seconds), as soon as you have decided which frame corresponds with t=0. Leave the current choice at 't=0 at first frame'. Press 'OK'. Note that a new window, viz., The Scale Settings window, pops up.

2. Now you can start with the actual setting of the length scale. Drag and resize the end points of the scale-ruler (red by default) to match two neighboring paving stones along the imaginary horizontal line where the footsteps are placed (why should you actually do this?). Specify the scale length of 100 cm in the window 'Scale Settings'.

3. Press 'OK' in 'Scale Settings'. Calibration is finished now.

Optional settings

Selection of the number of points per frame and their display

1. Right-click in the data video window and choose 'Points…'.

2. We want to measure 1 point per frame, viz., the middle of the right hand. So, confirm the current setting by pressing 'OK'.

3. Go to 'Markers and Colours…' and choose your favorite colors and markers for the video points (a blue big dot works fine).

Selection of frames

You fix which frames of the video clip you want to use for data collection. There are four possible ways of selecting frames.

1. Right-click in the data video window and choose 'Frames'

2. If you want to collect as much data as possible, choose 'Use All frames'. This is the default choice of Coach. But as you have already noticed, frame 22 cannot be used. Also is the automatic selection of a number of evenly spaced frames less suitable because the right hand is out of sight after the first 37 frames (you better check this beforehand).

3. In this example we want to collect data points on equidistant frames from 1 till 36, omitting frame 22. For example, we can choose frames from 1 till 36 with intermediate steps 2, i.e. omitting each time a single frame. In other words, we choose frame 1, 3, 5, .., 33, 35. To do this, choose the option 'Select these…' and enter the formula 1-36$2. With this formula you express your choice of ‘frames from 1 till 36 with step size 2’. All selected frames are black in the frame bar; all deselected frames are gray.

By the way, after your choice of frames to collect data from, you can always use the Insert key to add more frames. With the Delete key, you can deselect frames for measurement.

Collect data

1. Start measuring by pressing the green 'Start' button. The color of this button changes into red to indicate that you can stop measuring at any time.

2. Move the cursor across the data video window and click on the right shoulder. Recall that the coordinate setting have been chosen such that the first point clicked in each frame is the origin of the coordinate system for that frame.

3. Next, click on the right hand.

4. The video clip jumps to the next frame for measuring. Repeat step 2 and 3 for all selected frames.

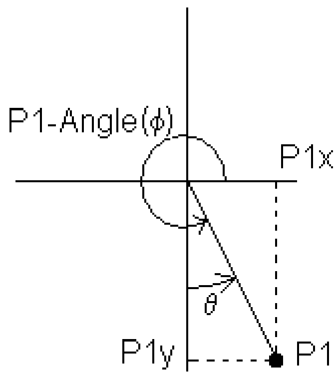
5. If you want to stop earlier because you are of opinion that you have collected sufficient data or because you want to make changes to the settings, press the red 'Stop' button. But for a good result of your analysis of the arm movements you better leave the settings as they are now and collect data on all selected frames.

6. If you want to keep track of the measured points in the video clip, select 'Trace' under the tool button (or by right-clicking in the data video window). The marker of the video point will appear.

Display of collected data

1. Right-click in the video-window and select 'Display as a Diagram…' to plot the data points. The shape of the cursor changes into the form of a small diagram. Click in the window where you want to place the diagram.

2. In the diagram, the horizontal (P1X) and vertical (P1Y) Cartesian coordinates of the video points are plotted against time. Recall that that we are more interested in the arm angle, i.e., the angle that the stretched arm makes with the imaginary vertical line that passes through the shoulder. The arm angle is a function of time. Luckily you collect by clicking more than the position of a video point. The distance from the video point P1 to the origin O of the coordinate system and the P1-Angle ϕ of the line OP1 with the horizontal axis are also collected. See the opposite figure. The arm angle is denoted here with θ.



   Change the diagram: plot 'P1-Angle' against 'time'. If desired, you can also change the name 'P1-Angle' into 'phi'.
   Note that the angle is measured in degrees and not in radians. This is an activity option that has been set by the author of this activity. Just have a look at the activity option via the menu item 'activity options'. Trigonometric functions take this choice of degree as unit into account: now, for instance, sin(90)=1.

3. Determine the arm angle θ as a formula in 'P1-Angle', add the graph of the arm angle to the diagram, and make 'P1-Angle' invisible.

Update video points

If you are not satisfied with a data point, you can go back to the frame in the video clip that corresponds with it and update the measurement. For better results you are advised to zoom out the video screen.

1. Click in the data video window to put the focus on this window and press once the arrow key ↑. The video clip is enlarged so that it fits in the data video window. If this is not large enough, press the 'maximize' button in the window (the 2^{nd} button the right).

2. With the arrow keys ← and → you can step through the selected frames.

3. Update a video point by dragging it to the desired location. The coordinate system cannot be relocated: if it is misplaced, go through the following steps:
   - remove a measurement in a frame by pressing the Delete key;
   - insert the frame again for measurement via the Insert key;
   - update the measurement by pressing the green 'Start' button.

4. When you are finished, it is a good idea to minimize the data video window again to its default size.

If desired, add extra data points

It is possible to add extra video points after the measurement has been finished and in case not all frames have been selected yet.

1. Select a frame with a gray marker in the video control bar (hint: you can go through all frames in the video control bar by the 'Page Up' and 'Page Down' keys). Press the Insert key. The selected image becomes black on the video control bar

2. Press the green 'Start' button to collect data points for the additionally selected frames. The data are inserted at the right spot in the diagram and the table.

3. To delete a video point, select the corresponding frame in the video control bar and press the Delete key. Try this.

Analysis of the arm movements

Analyze the arm movement. In mathematical terms: find a formula that has a graph that fits well with the measured arm angles, i.e., that stays close to the measured angles.

1. The graph of the arm angle has the shape of a sine graph. Right-click in the diagram window or click on the tool button of this window. Select 'Analyse' > 'Function-fit' and try to find the best sinusoid, i.e., a function of the form

$$angle(t) = a\sin(b\,t + c) + d\ .$$

2. What is in this diagram the duration of a complete period? Compute the period with the formula from function fit? Use the foot movement to estimate the period of the gait cycle. Are these results in agreement? Actually, this should be the case. If not, check your work.

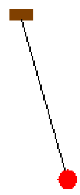
3. The arm rotation is the difference in angle between the arm when it is as far as possible in the forward direction and the arm when it is as far as possible backward. How large is the arm rotation

according to the graph? How large is the arm rotation according to the function fit?

4. In which phase of walking is the person in the video clip when his right arm is directed forward as far as possible? And when the arm is as far as possible backward?

5. What is the state of equilibrium of the right arm for the walker and at which gait phase is this state of equilibrium passed through? What is the amplitude of the arm swing?

6. A standard topic in school physics is the harmonic motion of a pendulum. The pendulum consists of a heavy weight that is attached to a light rope and that swings to and fro. For the period of a pendulum of length l, i.e., the time needed for one complete swing, the following formula holds:



$$\text{period} = 2\pi\sqrt{\frac{l}{g}}\ ,$$

where g is the gravitational constant (g=9.8 m/s^2).
The arm swing around the state of equilibrium resembles the motion of a pendulum. Measure the length of the arm (including the hand), and compute the period with the above formula. Is the answer in agreement with the period of the arm swing that you determined in a previous exercise? If not, can you come up with a reason why this formula cannot be applied in this case?

7. In scientific research, the arm movement is sometimes modeled as a harmonic motion that is driven by an external force. For this model, there exists a 'natural period' that is given by the following formula:

$$\text{period} = 2\pi\sqrt{\frac{I}{2m\,g\,d}}\ ,$$

where I is the moment of inertia of the arm (including the hand), m represents the mass of the arm, g is the gravitational constant, and d is the distance of the center of gravity of the arm to the rotational axis, i.e., the shoulder. You can use this formula, even when you do not know what the moment of inertia actually means. In measurements with humans it has been found that the center of gravity is located away from the shoulder at around 53% of the total arm length (hand included). The weight of the arm is for men about 5% of the body weight. The moment of inertia I of the arm with the shoulder as rotation axis can be calculated with the following formula:

$$I = m\,k^2\ ,$$

where k is the so-called radius of gyration of the arm with the shoulder as rotational axis; it is equal to about 64.5% of the arm length. Compute the natural period for the arm according to this model and compare the answer with the period of arm movement that you have measured before.

8. Physicists agree about one thing of the periodic arm movement of normal walking: the formula for the natural period can be written as

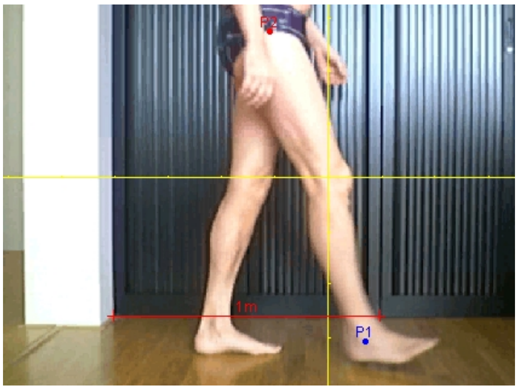
$$\text{period} = 2\pi\sqrt{\frac{c\,l}{g}}\ ,$$

where l is the arm length (including the hand), g is the gravitational constant, and c is a constant that depends on the stature of the walker. Determine the constant c for the walker in the video clip. Compare this with the number that you expect to find according to the model in the previous exercise.

9. In the first part of this analysis you have approximated the data with a sinusoid. Determine the graph of the difference between the graph of the measured data and the sinusoid. Find out whether this graph of the difference has again the shape of a sinusoid. If this is the case, then you can describe the arm movement pretty well by the sum of two sinusoids. This would actually mean that the model of a periodically driven harmonic motion is applicable. Draw the sum of these graphs to see with your own eyes how well the formula describes the arm movement.

Leave the activity.
Answer 'Yes' to the question 'Save changed result?'.

Activity 4: Swing phase of the leg in low speed walking



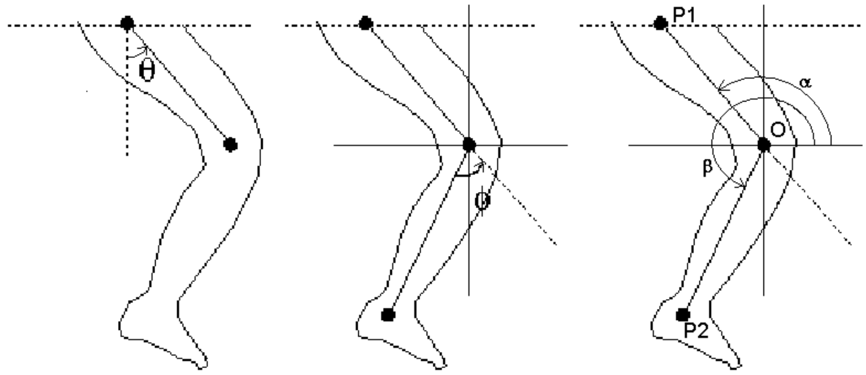
Gait is a periodic movement of each foot from one position of support to the next position of support in the direction of progression. One of the issues is whether these movements can be described well with trigonometric functions. In this activity you focus on the leg movement: you measure the angles of the thigh and lower leg have with respect to the (moving) coordinate system that has the knee as origin. You try to find experimentally mathematical relations between the quantities. We distinguish the swing phase of gait and the complete gait cycle. This activity is about the swing phase. If necessary, start Coach and choose the activity *4. Swing phase of the leg in low speed walking*

Introduction
In the data video window you see a short movie in which a person walks slowly in a study along two cupboards. The width of one cupboard is 1 meter. Take perspective into account when calibrating the video clip: place the scale-ruler of 1 meter on the imaginary horizontal line that connects the places where the right foot is flat on the floor (hint: use the parquet strips for a good estimation). This video clip has been recorded with a webcam and with a frame rate of 30 frames per second.

In this activity we shall see that the movements of the lower limbs can be described well by trigonometric functions. We shall look mostly at the right thigh and lower leg of the walker.

1. Play the movie to get an impression of the motion.

Measurement of angles during the swing phase
The hip angle (θ) and the knee angle (ϕ) are defined in the figure below.



So, the knee angle is 0 degrees if the leg is completely stretched. You can easily measure the angles α and β as functions of time in the video clip. Just take the knee as the origin of the (moving) coordinate system and click in each frame on the hip and right foot (not only the positions but also the angles α and β are obtained).

1. Prove the following formulas for the hip angle and knee angle in terms of the angles α and β: $\theta = \alpha - 90$ and $\phi = \alpha - \beta + 180$.

2. Select the frames in the video clip in which the right leg completes a full swing, i.e., start with the frame in which the toes get off the ground and end with the frame in which the heel strikes the floor again. Measure the angles α and β for this time interval and draw the graphs of the angle β, the hip angle, and the knee angle (as functions of time) in separate diagram windows.

Analysis of the swing phase

1. Verify that the angle β and the knee angle can be described well by a sinusoid. In case of the angle β, also check whether the difference between the measured angle and the sinusoid can be approximated once more with a sinusoid.

2. In the sinusoidal approximation of the angle β, what is the period of the swing phase? Like in the case of the arm movements during walking you can compare this period with the theoretical, `natural period' of a harmonic motion that is periodically driven by an external force. The formula is now

$$\text{period} = 2\pi\sqrt{\frac{k^2}{2\,g\,d}}\ ,$$

   where k is the radius of gyration of the lower leg (including the foot) with the knee as axis of rotation, g is the gravitational constant (=9.8m/s^2), and d is the distance from the center of gravity of the lower leg to the rotational axis, i.e., the knee. In measurements with humans it is found that k is equal to 73.5% of the length of the lower leg (including the foot) and that d is equal to 60.6% of the lower leg length. Use this formula to calculate the natural period of the lower leg of the walker in the movie and compare the answer with the previously measured value.

3. Verify that the hip angle θ during the swing phase can be approximated reasonably well by the sum of a parabola and a sinusoid. For those who wonder why we do not take a sum of two sinusoids: look what happens when you do a sinusoidal function fit for the hip angle.

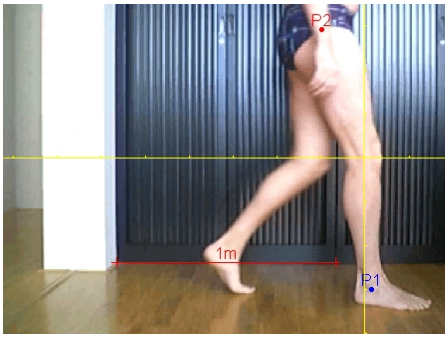
4. Suppose that the thigh and the lower leg of the walker are of equal length, i.e., OP1 = OP2 in the above figure. Hen you can compute the angle of the line P1P2 with the imaginary vertical line through P1. Let us call this angle the leg angle. Prove the following formula: leg angle = $(\alpha+\beta)/2-180$.

5. Let Coach plot the graph of the leg angle against time. Verify that the leg angle can be approximated well by a sinusoid.

Leave the activity.
Answer 'Yes' to the question 'Save changed result?'.

Activity 5: The complete gait cycle of low speed walking

This activity is a continuation of the previous investigation in which you have measured the angles of the thigh and lower leg with respect to the (moving) coordinate system that has the knee as origin and in which you have found experimentally mathematical descriptions of the angles during the swing phase. This activity is not merely about the swing phase, but about the complete gait cycle, including the stance phase when the leg supports the upper body. If necessary, start Coach and choose the activity *5: The complete gait cycle of low speed walking.*



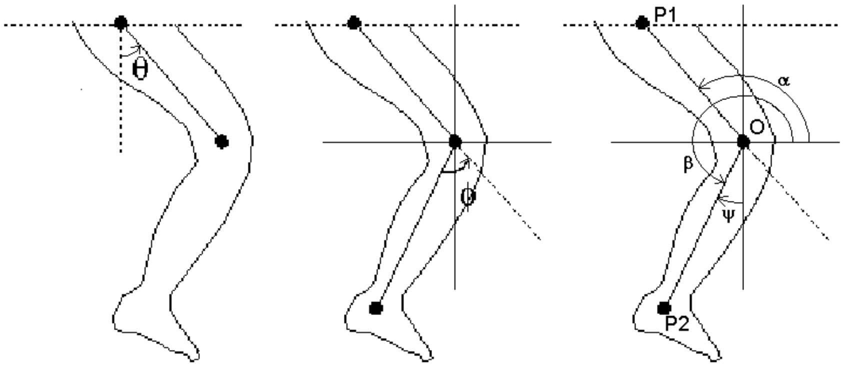
Introduction

In the data video window you see a short movie in which a person walks slowly in a study along two cupboards. The width of one cupboard is 1 meter. The calibration of the video clip takes perspective into account. This video clip has been recorded with a webcam and with a frame rate of 30 frames per second.

In this activity we shall see that the movements of the lower limbs can be described well by trigonometric functions. We shall look mostly at the right thigh and lower leg of the walker.

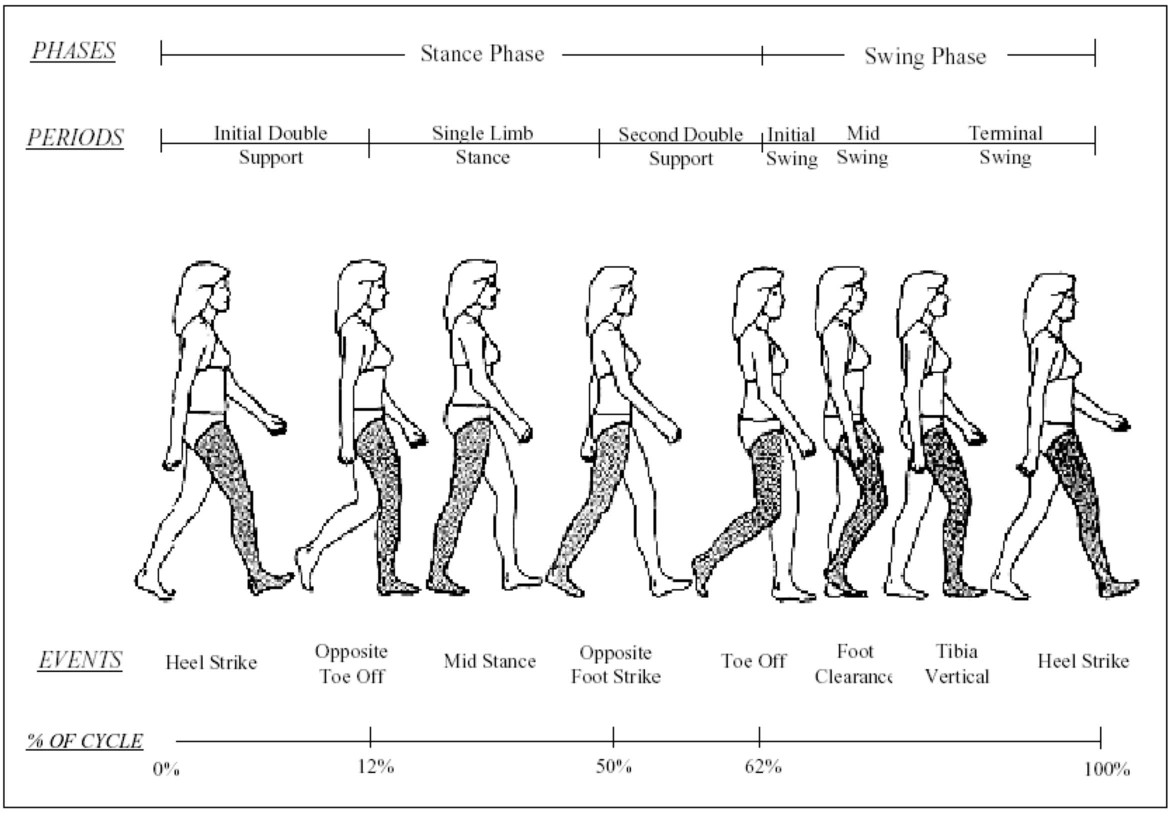
Measurements of angles and distances

Again we define the hip angle (θ) and the knee angle (ϕ) as in the figure below. The angles α (P1-Angle) and β (P2-Angle) have already been measured as a function of time by taking in each frame the knee as origin O of a (moving) coordinate system. We have selected the frames in the video clip for measurements in which the right leg completes at least one gait cycle. We start with the first frame in which the right foot is flat on the floor and the lower leg is perpendicular to the floor, and we end with the frame when this has happened once more. The time calibration has been chosen such that t=0 coincides with the beginning of our segment of frames. The measured angles have already been plotted against time in the diagram window. The lower leg angle (ψ) is defined as $\psi=270-\beta$.



1. Draw the graphs of the lower leg angle, the hip angle, and the negative value of the knee angle (as functions of time) in separate diagram windows. The reason that we consider the lower leg angle and -knee angle is because we want to compare our measurement results with results from the literature in human movement science.

2. The following picture describes the various phases in human walking and their relative duration. This picture has been taken from Rose & Gamble, "Human Walking", Williams and Wilkins, 1994.



Typical normal walk cycle illustrating the events of gait (Rose and Gamble, 1994)

Use the video clip and your graphs to measure the duration of the phases in the walk of the video clip. Figure out the duration of the gait cycle, the duration of double support, and anything else that you find noteworthy to measure.

3. Measure the stride length and the length of the right leg of the walker in the video clip. What is the ratio between stride length and leg size of the walker for the motion shown? If you have not done this before, measure the duration of the gait cycle. Calculate from the measured data the average speed of the walker.

Analysis of the leg movement

1. Compare your graphs of the hip angle and lower leg angle (ψ) with the following graphs that have been taken from Yam et al, Gait Recognition By Walking and Running: A Model-Based Approach. ACVV 2002: the 5th Asian Conference on Computer Vision, Melbourne, Australia.

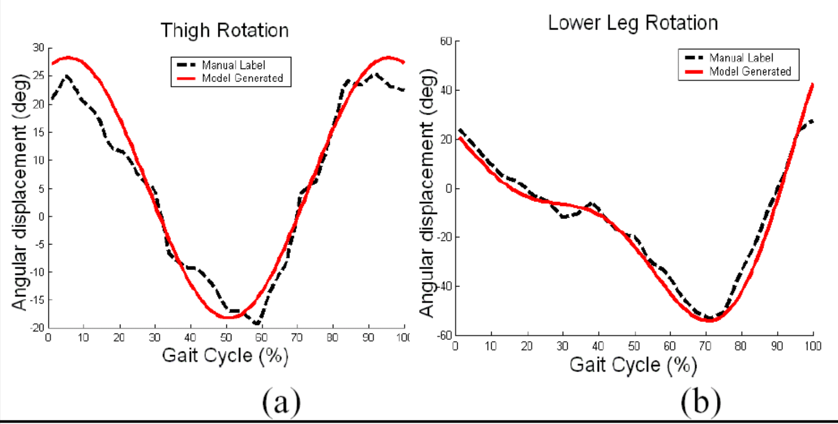
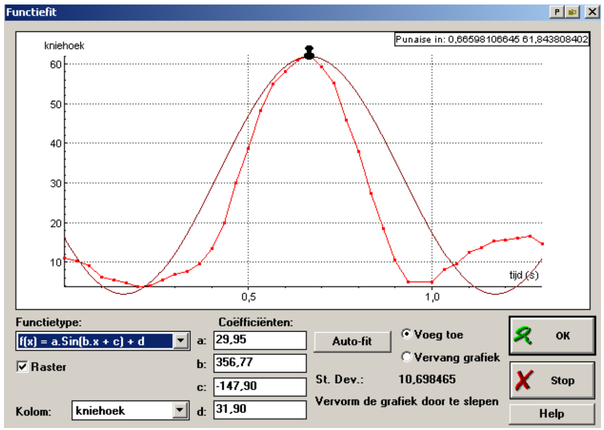


Fig. 4: Sample output of the thigh and lower leg motion model superimposed with the manually labelled data.

What similarities and difference do you notice?

2. The mathematical model that the authors of the afore-mentioned conference paper use is the same as we have applied before: a sum of two sinusoids. Apply this model to the hip angle and the lower leg angle that you have measured.

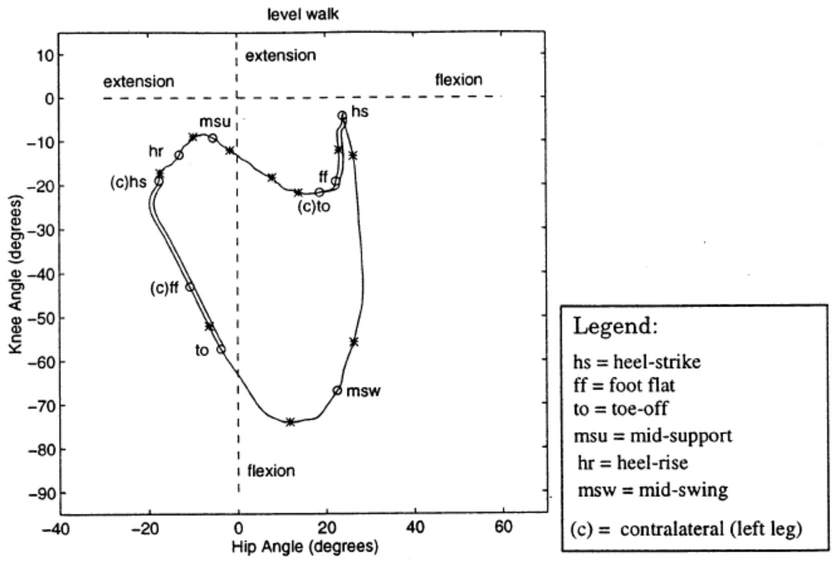
   Function fit in practice: some hints
   If you select in function fit the formula of a sinusoid, then it could happen in this investigation task that you do not find the best fit by pressing 'Auto-fit'. The reason is that Coach takes the currently drawn sinusoid as a starting point in a numerical process to a new fit and it may end up in an unwanted curve. The software is not to blame here, but the mathematical algorithm fails. The only remedy is to choose manually a good starting point.

   The screen dump shows a manual fit in action. The formula is a sinusoid; the measured knee angle has been plotted against time. First we have dragged the original sine graph such that its maximum coincides with the maximum of the measured data of the knee angle. By left-clicking we have fixed the pin. Now the sinusoid can be deformed into another sinusoid with the same maximum by dragging some point of the sinusoid. As soon as the sinusoid is close enough to the measured data the 'Auto-fit' button can be pressed to set Coach to work for finding a good sinusoidal function fit.
   When Coach stops the process initiated by 'Auto-fit' this does not automatically mean that the best fit has been found. Coach only is of opinion that the process of numerical approximation has already leaded to a reasonable result. A measure of the quality of the solution found is the standard deviation (St. Dev.). By pressing 'Auto-fit' once more it may happen that the numerical approximation process continues and that a better approximation, i.e., with smaller standard deviation, is found

3. Another graphical representation of motion that is much used in studies of human walking is the so-called hip-knee cyclogram. In this diagram the knee angle is plotted against the hip angle for a

complete gait cycle. Mathematically, it is nothing more than a parameter curve (time is the parameter in this case).
Create the following cyclogram: plot the negative value of the knee angle against the hip angle. Compare your diagram with the following diagram (taken from A. Goswami, A new gait parameterization technique by means of cyclogram moments: Application to human slope walking. Gait & Posture, 8 (1998), pp. 15-36).



Has your cyclogram the same shape? Are the characteristic points in the movement pattern that are mentioned in the above legend also present in your graph and at the same place?

4. In which phase of walking is the right knee almost stretched? With which points or pieces in the hip-knee cyclogram do these phases correspond?

5. Besides in the swing phase there is another moment during which the right knee is bent. When? With what point or piece of the hip-knee cyclogram does this correspond? Can you think of a physical explanation this bending of the knee?

6. During walking you also rotate your hip along the vertical axis of your body. Think up a reason for this?

Leave the activity.
Answer 'Yes' to the question 'Save changed result?'.

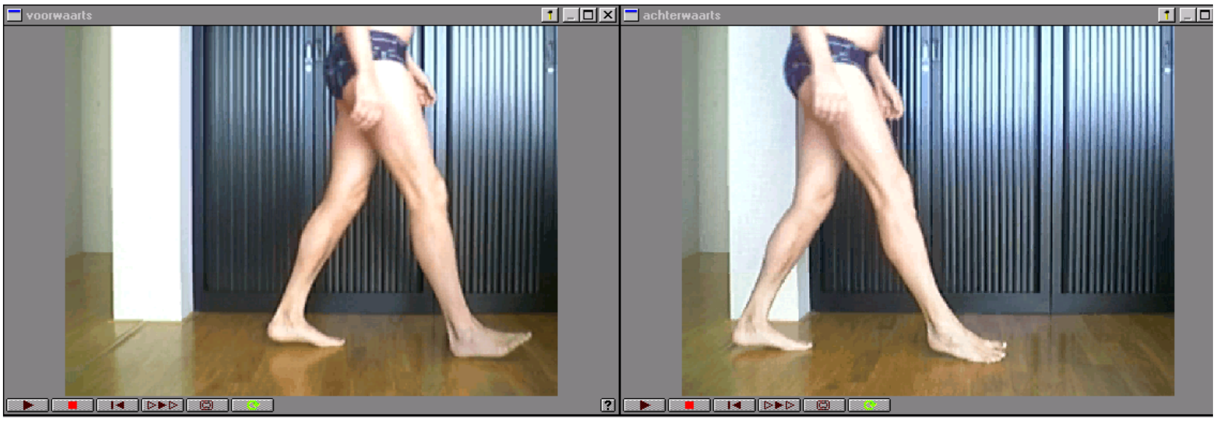
Extra practical work about human locomotion

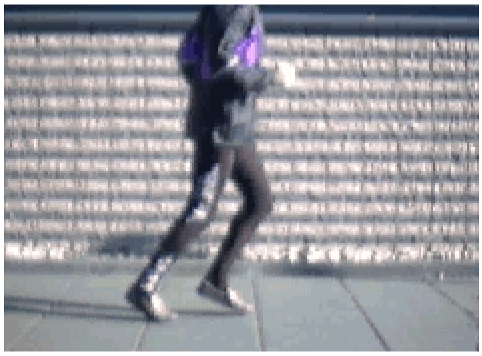
In the project `Walking` more activities have been prepared. They are related investigations that you could do.

Activity A: Backward versus forward human locomotion

In this activity are two video clips of one and the same person walking forward and backward. What are the differences and the common points in these gait patterns?

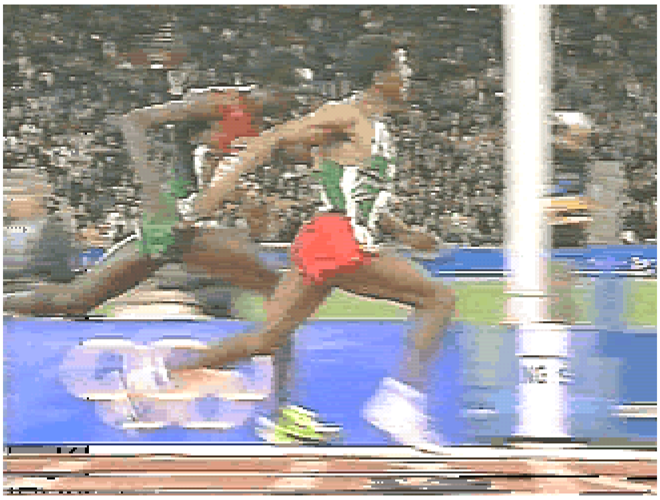
The data video window contains a movie in which a person actually walks backward, but the video clip displays this locomotion in reverse mode.





Activity B: Jogging

This activity contains a video clip of the same person that walks in activity 3. But now he is jogging. Investigate the gait pattern. Two questions that you could ask: What is the stride length and the step frequency for this mode of running? Can the mathematical model of a sum of two sinusoids for the leg angles again be applied?



Activity C: Sprinting

In the video clip, Haile Gebrselassie from Ethiopia and Paul Tergat from Kenia compete for the first price in the men's 10 km during the Sidney Olympic Games of 2000. Like four years before in the Atlanta Olympic Games, the Ethiopian wins. Use this video clip to investigate the sprint movement. Some topics that can be investigated:

- leg movements;
- step frequency and stride length;
- factors that contribute to the victory of Gebrselassie.

Activity D: Race walking

This activity contains the following five video clips about the women's 20 km road walk during the Sidney Olympic Games of 2000:

1. the last few meters of the Olympic champion, Wang Liping (China);
2. the finish of the silver medallist, Kjersti Tysse-Plätzer (Norway);
3. the finish of the bronze medallist, Maria Vasco (Spain);
4. Jane Saville (Australia), who leads the pack of athletes approaching the Olympic stadium;
5. The disqualification of Jane Saville a few hundred meters before the finish line.

The data video window contains the arrival of the gold medallist in the Olympic stadium. In this video clip you have a side view of the gait of a race walker. The fourth movie of Jane Saville leading the pack of athletes give you a front view. These movies give you a good impression of the to typical gait.

Use these video clips to investigate the race walking gait. What are the official rules of race walking and how do they influence the walking technique? Use Internet to find data about race walking that are relevant for a description of this gait.

Activity E: own video clips of human locomotion

Of course you can record yourself video clips with a digital camera or with a webcam: record and analyze the human movements of yourself and fellow pupils. You can think of movements like normal walking, jogging, running, but you can also have a look at hopping, crutch walking, fitness movements, dance movements, and silly walks like goose walking. You can investigate whether the gait patterns change when instead of normal walking you add heavy weight to wrists and ankles and whether walking with high heel shoes differs from walking with regular shoes. In short, you may use your imagination to study an interesting motion. But before you do this, make a work plan and discuss this with your teacher.

Activity F: Investigating the walking speed-frequency relation

In human science it is reported that a person who is constrained to walk at a given speed v on a treadmill chooses a particular step frequency f and step length $d = v / f$. It is commonly observed that humans use a fairly well defined set of v and f combinations that one might term the natural walking gait. The empirical relation is $f = C v^b$, where C is a constant and b=-.58 seems to be a good fit for humans over normal walking speeds. Go to a fitness center and carry out treadmill experiments to investigate the v - f curves of you and fellow pupils. Also record some movements with a digital camera or webcam for gait analysis purposes.