Bridges and Arches

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A practical investigation task for pupils at upper secondary school level who have no experience with COACH. Estimated working hours: 5. In practice, video measurement with COACH (the first 3 activities) takes up 2 hours.

Preface
COACH is a computer learning environment that has been developed originally for natural sciences (physics, chemistry, and biology). In this program you can easily collect data from different sources, display data in graphical or tabular format, process data mathematically, and so on.
The most recent version of COACH offers the possibility of collecting data from videoclips. You can record yourself a videoclip or you can use an existing videoclip for the purpose of carrying out practical investigation tasks. You may think of studying the orbit, velocity, and acceleration of a thrown basketball, investigating the motion of a car on a roller coaster, exploring the collision of two cars, and so on.
Actually, we are going to use the data video tool for doing measurements in digital images. In version 5 of COACH it is still required to convert the digital image into a videoclip or to duplicate the digital image into a sequence of pictures, even though you do not see much difference, as all frames are the same. In an upcoming version of the software this is not the case anymore.
The program COACH works with projects, each consisting of several activities. This practical investigation task relates to the project Bridges and Arches.
Looking at bridges and arches you often see curved shapes that resemble a parabola. Looking at hanging ropes, chords and chains you see similar shapes. Of course, not every curved line is a parabola. But for bridges and arches the shape is quite often a parabola. There are good reasons for this. In this practical investigation task you are going to explore this. First you will verify the parabolic form by collecting data from digital photographs, and next you will try to find a theoretical explanation for this.

The Learning Material
The explanation and the tasks can be read almost completely at the computer screen. We have also provided them in printed form, so that you can choose the format that you find most convenient.

Report
Part of this practical investigation task is writing a report. Herein we expect the answers to the exercises that ask for a reply, sometimes short, sometimes long, but in any case clear and to the point. We also would like to know which parts of the work you found easy or difficult. When you close an activity, then COACH asks "Save changed result?". Click on 'Yes'. In this way, you also get a digital part of your report!
Furthermore, we expect a summary of the time that you spent on various parts of the investigation task (a sort of lab journal).

The Project Bridges and Arches

This project consists of four activities: *Bixby Creek Bridge*, *Zeeburger Bridge*, *Weights*, and *Necklace*. The topic is the mathematical shape of bridge spans and hanging chains.
In this first two activities you will use the data video tool of COACH to explore the mathematical shape of two bridges. In the other activities you will approach the subject from a theoretical point of view.

Activity 1: Bixby Creek Bridge

In this activity you will get acquainted with some facilities that COACH offers. Part of the work has done, as you will see. In the next activity you will have to apply what you have learnt here.

Start COACH, (login as student) and choose the project *Bridges and Arches*.
Click the 'Open activity' button (the second button from the left in the toolbar) and choose activity *1. Bixby Creek Bridge*.

Introduction

Now you see the activity menu bar, the activity toolbar, and four
windows: one is provided with a video measurement, next to it is
some text, and the bottom part of the screen contains two empty
windows.

Alongside, in the data video window, you see a photograph of the
Bixby Creek Bridge in California, highway 1, 13.3 miles south of
Carmel. The central span is 320 feet long and 265 feet high. This
span is supported by an arch underneath.
Furthermore, you see a coordinate system, a red indication of a
scale of 100 m, and twelve measured points (white crosses) on the
supporting beam of the bridge. One data point on the image is
wrongly placed.
So, twelve frames of the videoclip of this bridge have been used. By
the way, the videoclip consists of 30 frames. The non-used frames
are indicated in the video control bar, at the bottom of the data
video window, by grey strokes.

In this activity you will use the tools that COACH offers to find
out that the span has a parabolic shape.

This introduction is in a text window. In addition there is a text
with exercises and tasks. In order to see these, left-click on the
yellow 'Display Text' button in the activity toolbar. Choose
'Tasks'. The shape of the mouse cursor changes into a text symbol.
Move the cursor to an empty window and click in this window.

1. COACH automatically places the coordinates of the measured points
   into a table and it prepares a diagram (a data plot). To make the
   table visible, press the 'Display Table' button in the activity
   toolbar. Choose 'Bixby Creek Bridge' and click in the empty
   window.
   If you right-click in the table window, you get a menu. Choose
   'Display as a Diagram'. Because we do not need the introductory
   text anymore, you can click in this text window.

2. In the table you see the x-coordinate and the y-coordinate of the measured video points P1. In the diagram you see the corresponding data plot.
   Click in the top row of the table. Notice that the data video window displays the first-used frame.
   Click in the second row of the table. Now the second frame has been selected and you can drag the wrongly placed white cross to a better location. Do this and look at the result in the diagram and the table.
   In this way you could move more measured points. (By the way, there are more ways of doing this as you will see or find out yourself)
3. Right-click in the diagram (or press the tool button, second button from the left in the button bar of the diagram window).
   Choose 'Analyse' and the sub-item 'Function-fit'.
   At 'Function Type' you see that the graph of f(x)=ax is drawn. The value of a(=0,38) is next to it.
   Choose the parabolic function type.
   Press the 'Auto fit' button.
   Now you see that the video points fit well to a parabola.
   If you press 'OK', then the parabola is added to the diagram window. If you choose 'Stop', then the diagram window remains the same.
   Write down the coefficients of the parabola and press OK.
4. Right-click in the diagram.
   Choose 'Zoom to fit'. Notice the red cross through the looking glass on the leftmost button in the button bar of the diagram window. Press this button. Two ways to zoom in or out.
5. Right-click again in the diagram.
   Choose 'Create/edit diagram…'. In the dialog window you see among other things the name of the window, the option to display a grid, eight columns to choose from, and at the bottom facilities to change the display of the graph.
   Click on C1. We have nothing to do with it, because time is connected with the videoclip, and we do not using images changing in time. That is why it is marked 'Invisible'.
   Look at columns C2, C3, and C4. Do you recognise in the last column the formula of the quadratic fit?
   Make the function fit invisible and experiment with the facilities to display data points with various markers and colours.
6. By all odds, the distance between the consecutive piers that support the span is constant. This you could use to check the location of the video points. COACH can compute these distances for you.
   Choose 'Create/edit diagram…'.
   Click on the fifth column.
   Choose 'Connection: Formula'.
   Click in the formula text region
   Press the 'Wizard' button.
   Click twice to create the formula Delta(P1X)
   Take as quantity: distance piers
   Unit: m
   Decimals: 2

Leave the activity.
Answer 'Yes' to the question 'Save changed result?'.

Activity 2: Zeeburger Bridge

In this activity you analyse the shape of the Zeeburger Bridge in Amsterdam across the Amsterdam-Rhine Canal. You record the coordinates of points on the arch bridge and you derive from these data the height as a function of the horizontal distance.
In this example you learn how to carry out measurements with the data video tool of COACH when dealing with still images. The instructions are very detailed and they guide you through the example.

If necessary, start COACH and choose activity *2. Zeeburgerbrug.*

Open the videoclip
1. Right-click in the upper-left window called 'Data Video' and choose 'Open Video'.
2. Select the movie 'Zeeburger Bridge'.

Play the videoclip
- The videoclip shows the Zeeburger Bridge in Amsterdam across the Amsterdam-Rhine Canal.
- Playing the videoclip by pressing the play button to the left of the frame controller below the movie does not make much sense because all frames are the same. The frame controller becomes useful only after the data collection.

Calibrate the videoclip
Before you can collect meaningful data from the videoclip you must calibrate the distances in the videoclip. Data about the bridge or the width of the Amsterdam-Rhine Canal on this location can be searched elsewhere or can be roughly estimated. Quite often the creators of the videoclip have added to the first frame an indication of length scale. In this clip you see a bar of length 80 m.
1. Right-click in the data video window and choose 'Coordinate settings…'.
   - Scale: because the vertical and horizontal scale are the same, select 'Same scale in all directions'.
   - Origin: Because all frames are the same, select the origin 'Same at all frames'.
   - Time Calibration: this is irrelevant for image measurement. But in case you choose 1 frame per second, the frame number corresponds with the time in seconds.
     Press 'OK'. Note that a new window, viz. The Scale Settings window, pops up.
2. Now you can start with the actual setting of the scale. Drag and resize the end points of the scale-ruler (red by default) to match the horizontal white ruler in the first frame. Specify the scale length of 80 m in the window 'Scale Settings'.
3. If desired, you can move the coordinate system by dragging its origin (drag the small circle). Locations for which the mathematical formulas will be simplest for symmetry reasons are the top of the arch or the points on the road straight under the top of the arch.

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4. Press 'OK' in the window 'Scale Settings'. Calibration is finished now.

Optional Settings

Selection of the number of points per frame, markers, and colours
1. Right-click in the data video window and choose 'Points…'.
2. We will measure 1 point per frame. So, confirm by pressing 'OK'.
3. Go to 'Markers and Colours…' and choose your favourite colours and markers for the video points (a white big dot works fine).

Selection of frames
You fix which frames of the video clip you want to use for data collection. There are four possible ways of selecting frames.
1. Right-click in the data video window and choose 'Frames'
2. If you want to collect as much data as possible, choose 'Use All frames'. This is the default choice of COACH.
3. In this example however we want to collect 12 data points. Therefore choose 'Select Automatically' and enter the number of frames you want to select. All selected frames are black in the frame bar; all deselected frames are grey.

At first sight it may look strange to specify first which frames you want to select for data collection, because all frames are the same. But keep in mind that COACH uses the frame number to keep the location in the table where the measurement will be stored up to date. You cannot sort afterwards the data in an ascending order with respect to the horizontal coordinate. If you want to trace a curve during the measurement and at the same time leave the option of additional measurements open, then you must make gaps between selected frames, e.g. by choosing 'Select Automatically' and entering the requested number of frames.

Collect Data
1. Check if the calibration and the coordinate settings are to your liking. If necessary, make the coordinate system visible by right-clicking in the data video window and by selecting 'Axes'. If desired, adjust the calibration.
2. Start measuring by pressing the green 'Start' button.
3. Move the cursor across the data video window. Choose a suitable point on the front arch of the Zeeburger Bridge. Click on this video point.
4. Click in the next frame on the arch to the right of the previous video point. Repeat this for all (selected) frames.
5. If you want to stop earlier because you are of opinion that you have collected sufficient data, press the red 'Stop' button.
6. If you want to keep track of the measured points in the videoclip, select 'Trace' under the tool button (or by right-clicking in the data video window). The marker of the video points will appear.

Display Collected Data
1. Right-click in the video-window and select 'Display as a Diagram…' to plot the data points. The shape of the cursor changes into the form of a small diagram. Click in the window where you want to place the diagram.
2. In the diagram, the horizontal (P1X) and vertical (P1Y) Cartesian coordinates of the video points are plotted against time (after our calibration equal to the frame number). Change the diagram: Make the time invisible and plot P1Y against P1X. Hint: this goes easiest by connecting the variable P1X to the x-axis; time will automatically be set invisible. If desired, you can change the names P1X and P1Y into X and Y, respectively.

3. If you also want to display the data in tabular form, select in the data video window 'Display as a Table'.
4. The program offers the possibility to 'replay' the videoclip, the diagram, and the table, simultaneously, i.e., to show in what order which points have been measured. Press the 'Replay' button (the small green button next to the 'Start' button). Determine the duration of the replay (e.g. 5 sec.) and press the green button in the dialog window. Now you can see the replay of the data collection and at the same time watch the diagram and the table evolve.

Correct Video Points

If you are not satisfied with a data point, you can go back to the frame in the video clip that corresponds with it and correct the measurement. For better results you are advised to zoom out the video screen.

1 First enlarge the data video window by pressing the 'maximize' button in the window (the second button to the right). Hereafter, press once or twice the ↑ arrow.
2 Via ← and → you can step through the selected frames
3 Update the video point by dragging it to the desired location.
4 When you are finished, minimise the data video window again.

Adding Extra Video Points

It is possible to add extra video points after the measurement is finished and in case not all frames have been selected yet.

1. Select a frame with a grey marker in the frame controller (hint: you can go through all frames in the frame controlled by the 'Page Up' and 'Page Down' keys). Press the insert key (INS). The selected image becomes black on the frame controller.
2. Press the green 'Start' button to collect data points for the additionally selected frames. The data are inserted at the right spot in the diagram and the table.
3. To delete a video point, select the corresponding frame in the frame controller and press the delete key (Del). Try this.

Tasks

Analyse the mathematical shape of one of the arches of the Zeeburger Bridge. In mathematical terms: what function has a graph that fits well with the arch and stays closest to the measured video points.

1. Make a diagram in which P1Y is plotted against P1X, if you have not done this yet, and change the names P1X and P1Y. If you replace in the 'Create/Edit diagram…" dialog window the quantity P1X, connected with column C2, by x, and the quantity P1Y of column C3 by y, then you have "normal" mathematical symbols.
2. Also make a table of x and y: Right-click in the diagram window or press the tool button in this window. Select 'Display as a Table'. The cursor gets a small tabular shape. Click in the window where you want to place the table.
3. Right-click in the diagram window or press the tool button of this window. Select 'Scan', place the cursor at a point in which you are interested and read off its values. You can also read off the values in the table.
4. Verify the answer found in (3) in the data video window by measuring the position with a computerised ruler: Right-click in the data video window or use its tool button. Select 'Ruler'. Drag the endpoints of the ruler so that they lay on the points between which you want to know the distance.

5. In the diagram, the data points seem to lie on a parabola. Right-
   click in the diagram window or press its tool button. Select
   'Analyse' ▸ 'Function-fit' and try to find the parabola that fits
   best.

In other COACH activities about bridges and architectural structures you will encounter the
parabola again and again. There are good reasons for this and with some knowledge of
mathematics and physics you can find them.

Activitity 3: Five Weights

Introduction
The parabolic shape of the Zeeburger Bridge and of the Bixby Creek Bridge is no coincidence.
In this activity, the tasks and your own ingenuity will lead you to a mathematical and physical
explanation.

Physical Model
The arch of the Zeeburger Bridge can be seen as a parabolic shape, on which hang equal
weights at equal distances (in horizontal direction). Each beam "bears" a same portion of the
road surface.
The arch of the Creek Bridge is a parabolic shape which support equal weight placed at equal
distances form each other (again, equal distances in horizontal direction).
We prefer to study the "inverted" model: a weightless chain on which hang equal weights at
equal horizontal distances (See the picture below)

Choose the activity 3. Weights
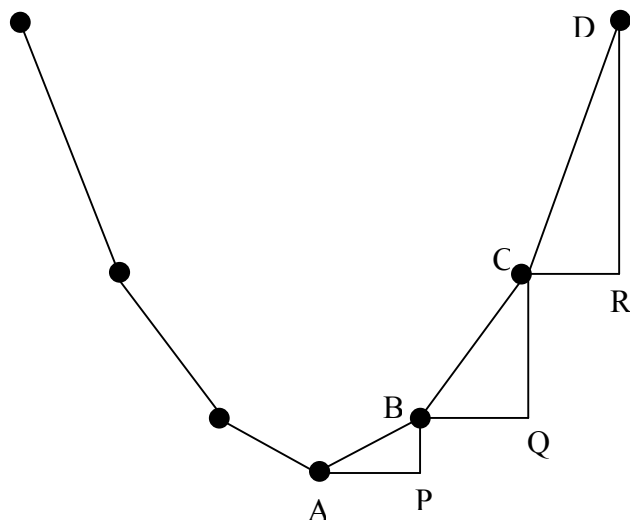
The videoclip shows a chain
on which hang five equal
weights. These weights have
placed symmetrically such
that the horizontal distance
is always 1 dm.
Coordinate system and
scaling have already been
chosen.

Tasks
1   Select   seven
video points: the two
suspension points and the
five points on the chain to
which the five weights have
been attached.

Text windows in COACH do not offer (yet) all possibilities of a real text editor. This is why
the next exercises are on paper only. The above drawing is a simplified display of the video
image.

2   Measure in the data video screen with the protractor the angles
     of inclination ∠A, ∠B, and ∠C.
     Measure in the diagram window the slope of the segment AB, of
   segment BC, and of segment CD.

Maybe you notice some regularity in the angles and/or slopes or in their ratios?

3   Draw on paper the graph of the standard parabola $y = x^2$ with A(0,0), B(1,1), C(2,4), and D(3,9). Compute $\angle A$, $\angle B$ en $\angle C$. Compute the slope of AB, of BC, and of CD.

4   Investigate whether the regularity that you have found in exercise (2) also occurs for the standard parabola. If you did not find a regularity in (2), try if you can find the regularity of exercise (3) back in (2), approximately. (If desired, compute the coordinates of more points of the standard parabola.)

Theoretical Closure

You have seen that the model of weights hanging on a weightless chain indeed leads to a parabolic shape. Hopefully you have also found that there exists a simple relation between the slopes of consecutive straight segments of the chain. This regularity can be expained by equilibrium of forces acting on the chain. The figure below is the basis of our reasoning:
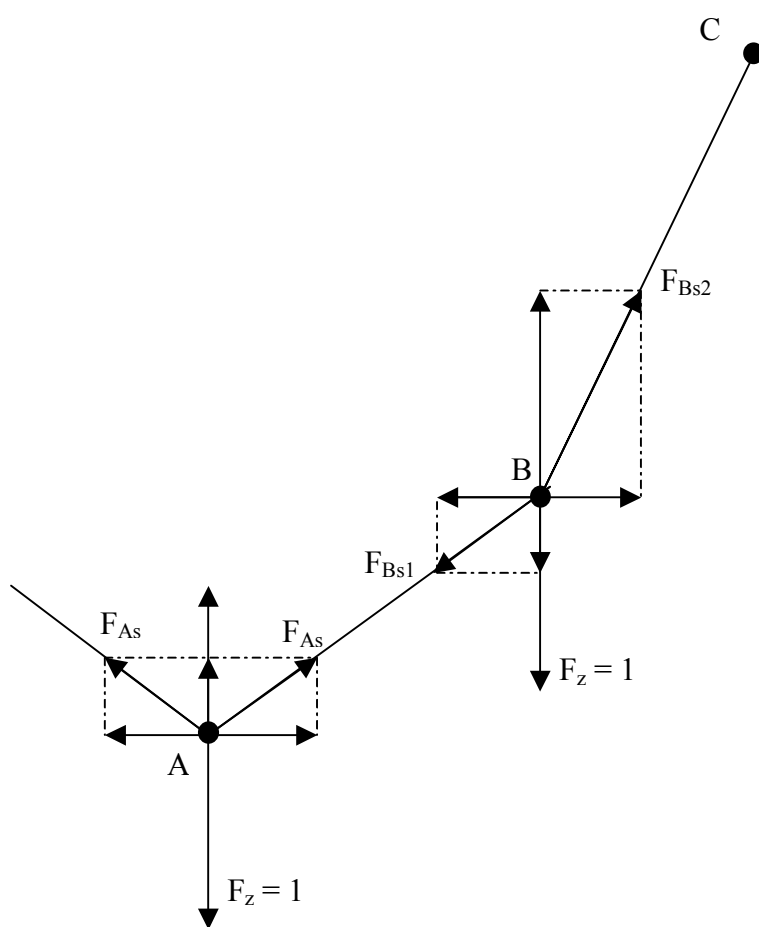
The (equal) weights lead to a gravitational force F_z in each point of application. For simplicity we take $F_z = 1$.

The weight A hangs in equilibrium by the tension forces in F_{As} in left- en right-segment of the chain. Because of symmetry the tension forces on the left and on the right are equal in magnitude.

Easy reasoning gives that the vertical component of F_{As} must be equal to $0,5F_z$.

A gravitational force and two tension forces cause the equilibrium of the point of application B.

Exercise
Reason out that the slopes of AB, BC, and CD (if another weight would be placed) are in ratio 1 : 3 : 5 .

Epilogue
Not every hanging chain has a parabolic shape. More strongly: Christiaan Huygens (1629-1695) proved that an *unloaded* homogeneous chain certainly *not* hangs in a parabolic shape. Such a freely hanging chain you can find in activity *4. Necklace*.
With slopes in consecutive ratio 1 : 3 : 5 : 7 …, do you always get a graph of a parabola? And can you explain this?
What can be said if the weights are not all the same?
What can be said if equal weights hang symmetrically, but are even in number (i.e., no weight in the top)?
These and other questions you could investigate another time.