Topics in Modal Logic (Fall 2024)

Sixth tutorial (5 December 2024): exercise sheet

Exercise 1 (Scott continuity for modal automata) Let $\mathbb{A} = (A, \Theta, \Omega, a_I)$ be a modal automaton relative to a set P of proposition letters, and let q be a proposition letter in P. As usual we write $C := \wp(\mathsf{P})$.

Assume that the state space A of $\mathbb{A}$ is partitioned as $A = A_0 \uplus A_1$ and that

- $a_I \in A_0$;
- $\Omega(a)$ is odd, for every $a \in A_0$;
- $\Theta(a, c) \in \mathtt{1ML}(A_1)$, for all $a \in A_1, c \in C$ (i.e., no $b \in A_0$ appears in any formula $\Theta(a, c)$ with $a \in A_1$);
- each occurrence of $b \in A_0$ in a formula $\Theta(a, c)$ is in the scope of a diamond (and hence, not in the scope of a box);
- $\Theta(a, c) = \bot$ whenever $a \in A_1$ and $q \in c$.

Let $\mathbb{S} = (S, R, V)$ be a tree model with root r, and assume that $\mathbb{A}$ accepts $(\mathbb{S}, r)$. Then there is some *finite* subset $Q \subseteq V(q)$ such that $\mathbb{A}$ accepts $(\mathbb{S}[q \mapsto Q], r)$. (Hint: work towards an application of König's Lemma.)

Exercise 2 (disjunctive formulas) Prove Theorem 10.38.

Exercise 3 (closure properties of logical automata) Prove Theorem 11.11.