HOW DIVISIVE IS AN ALTERNATIVE IN A PROFILE OF RANKINGS?

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OUTLINE

1. The **context**: measuring consensus, diversity, polarisation, cohesiveness in profiles of rankings

2. Definition of **divisiveness metric**, analysis of bounds, algorithmic questions: robustness and control

3. **Empirical** analysis of divisiveness measures, platforms for building a collective government program

DETECTING WINNERS

Standard input in social choice and rank aggregation:
a **set of strict rankings** over alternatives (a profile)

4 voters	3 voters	2 voters
A	B	C
E	C	D
D	E	E
C	D	B
B	A	A

Table 0.1. Five candidates, five winners

Social choice theory proposed
**countless methods to define
the winning/most agreed upon
alternative**

What else can we detect/measure in profiles of strict rankings?

NOTATION AND RUNNING EXAMPLE

In the running example we are **prioritising over projects** that a city hall will invest on

The "generating question" is: *Rank the following projects in order of priority*

There are n users and 5 **issues** (not alternative, possibly interdependent)
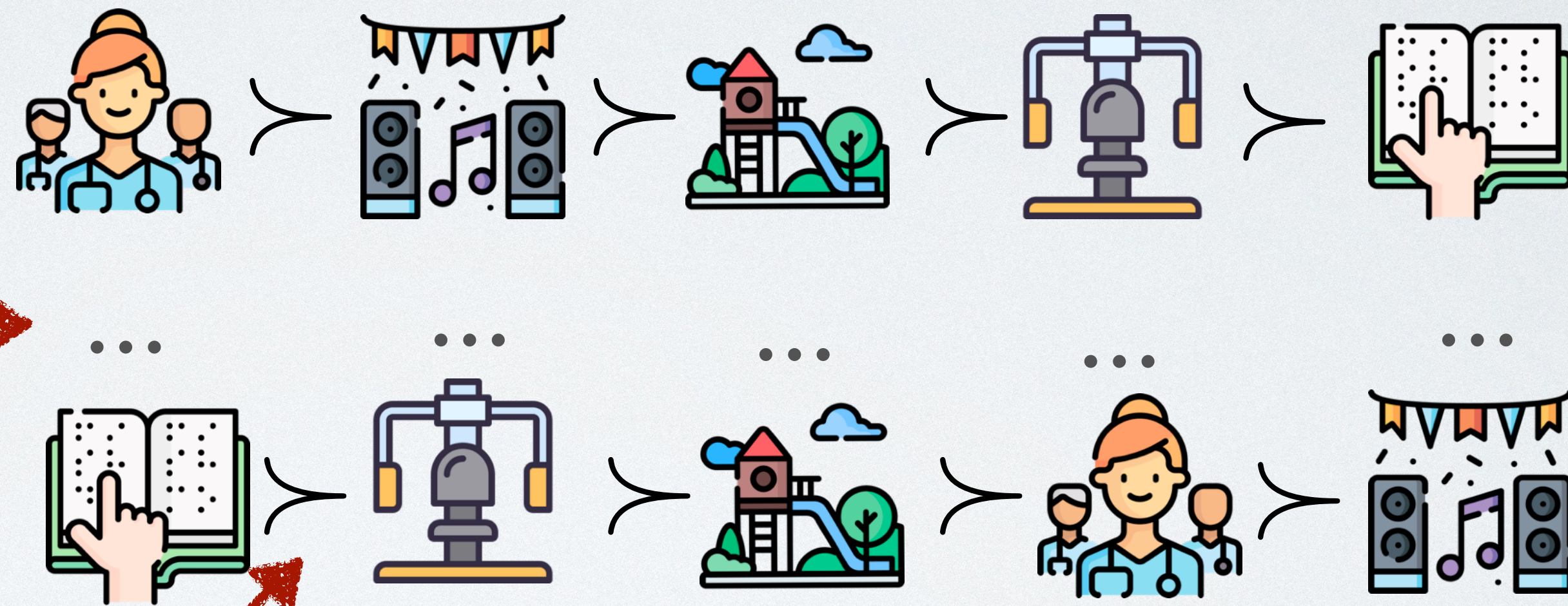
Read "📖 is preferred to ⚙ by this individual"

Icons: freepik from flaticons.com

We assume that the users know how their preferences are aggregated (eg Borda, Copeland)

MEASURING AGREEMENT: CONSENSUS, COHESIVENESS

Define a numerical function that measure how consensual or cohesive is a profile of rankings
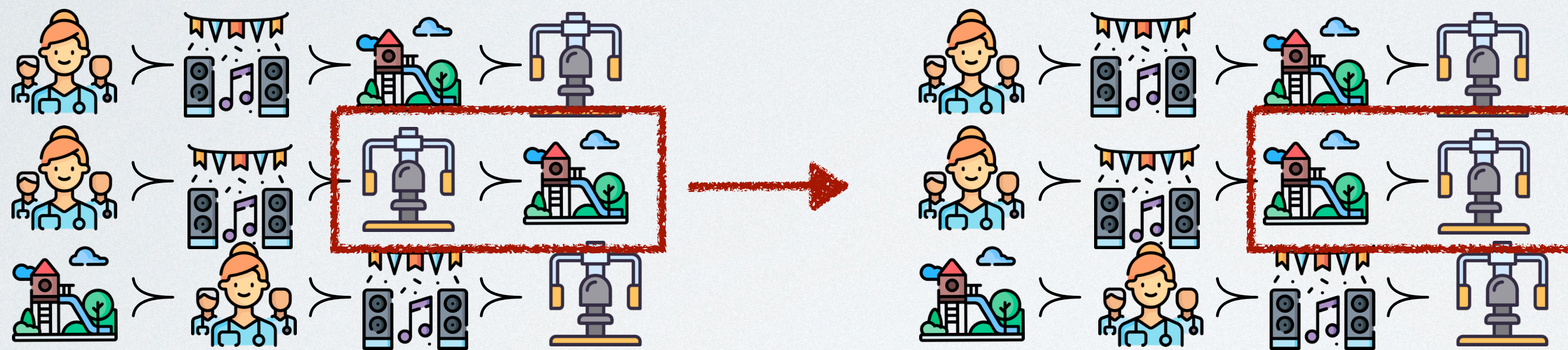
Independence axiom: a swap of a contiguous pair towards the majority strictly improves cohesiveness

A large number of axiomatic characterisations (mostly based on pairwise comparisons with some exceptions)
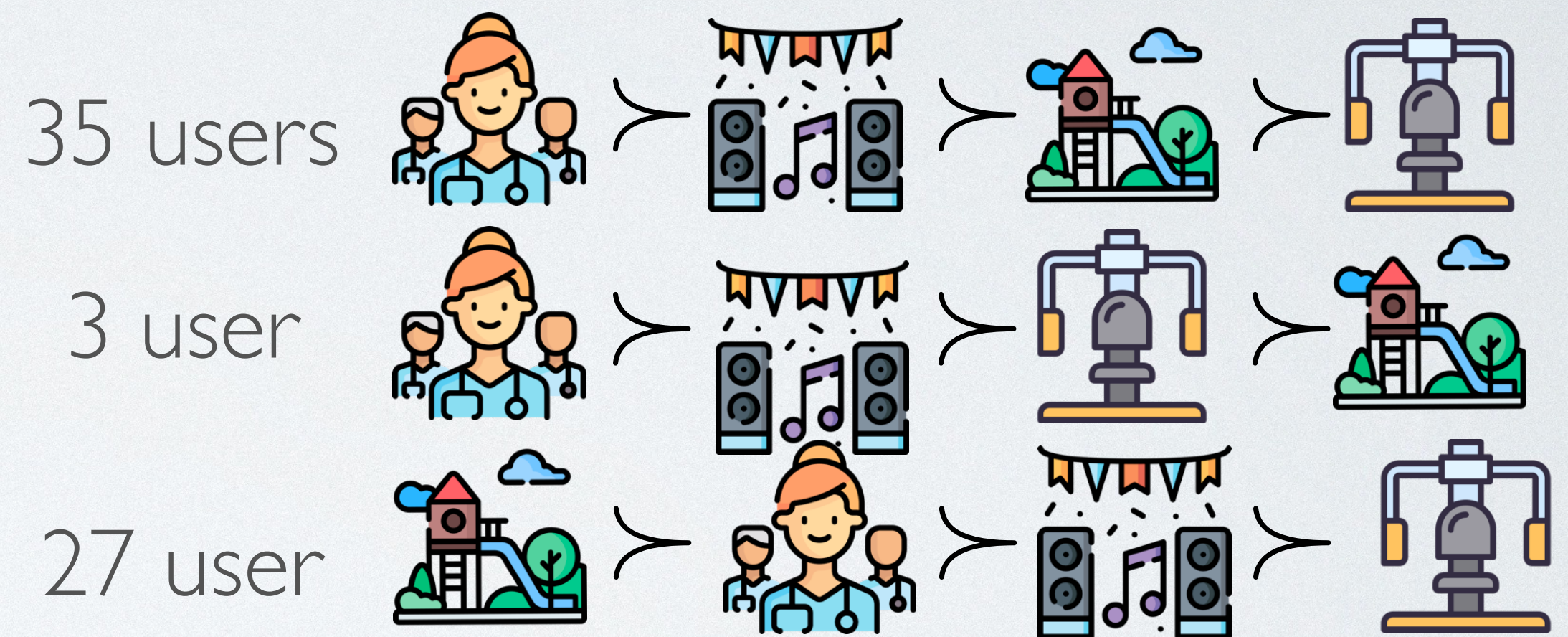
Various papers by Alcalde-Unzu and Vorsatz. General setting by Bosch (2006).

MEASURING (DIS?)AGREEMENT: DIVERSITY

How to decide which of two preference profiles is more diverse?

Three possible approaches:

1. Counting the different rankings
2. Averaging the disagreements among rankings
3. Measuring distance to a compromise ranking

35 users

3 user

27 user

Research question: does diversity influence classical social choice problems?

Hashemi and Endriss. ECAI 2014. Karpov, Group Dec Negot. 2017.

MEASURING DISAGREEMENT: POLARISATION

Classical work in Economics distinguished measures of polarisation from measures of inequality
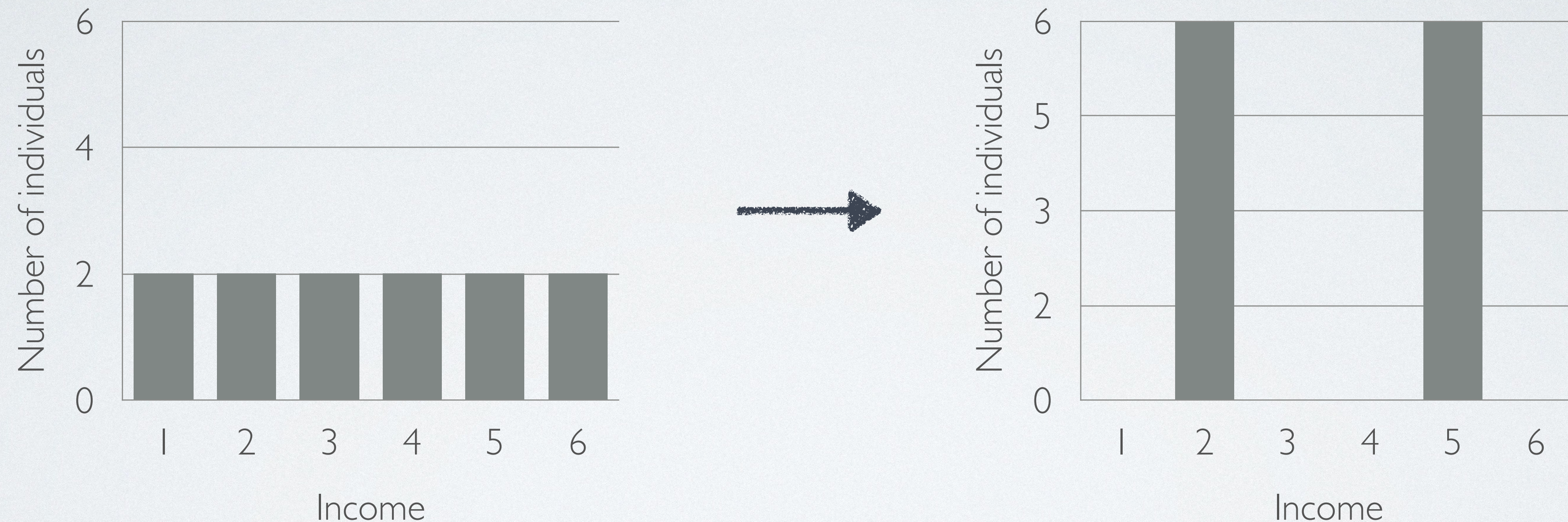
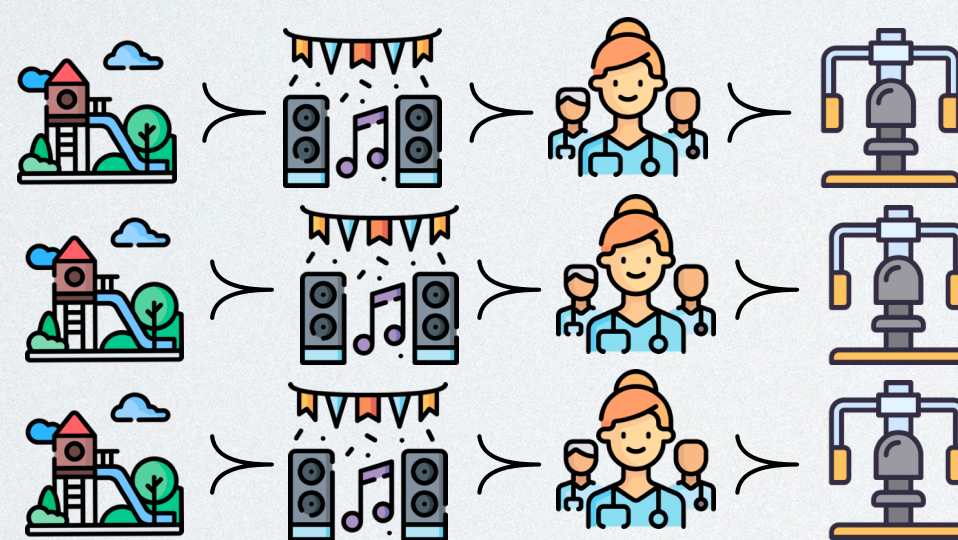From left to right: **less inequality, more polarisation**

Esteban and Ray. On the measurement of polarization. Econometrica. 1994

POLARISED PROFILES OF RANKINGS

Compare profiles of rankings based on average disagreement of pairwise comparisons:

Formal definition (then normalised over pairs and users): $\sum\limits_{(a,b)\in A^2} n - d(a,b)$

Minimum polarisation

Maximum polarisation

Two completely opposed camps

...

n! users each submitting a different ordering

Can, Ozkes, Storcken. Measuring polarization in preferences. MSS 2015
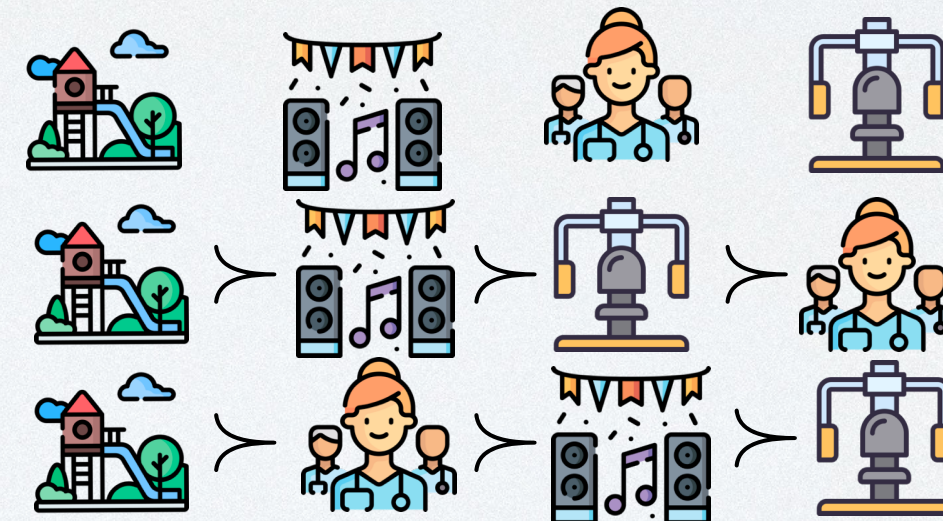
KEMENY-BASED MEASURES

k-Kemeny distance as the minimal swap distance of a set of k rankings to the profile (1-Kemeny is the standard Kemeny distance)

Diversity index basically averaging the k-Kemeny distances, **polarisation** index as the difference between 2-Kemeny and 1-Kemeny

Two completely opposed camps

Polarisation index 1
Diversity index 1/2

...

Polarisation index ~0
Diversity index high (depends on m)

Faliszewski et al. Diversity, agreement, and polarization in elections. IJCAI 2023

OBSERVATIONS

Averaging pairwise agreement/disagreement is a popular
notion studied under several different names

All proposed measures are applied on entire
preference profiles, global measures

Measures are hard to compute and require complete data
(apart from the simplest average agreement/disagreement)

Our research questions:
Can we explain what makes one particular profile polarised/diverse?
Can we identify "divisive" issues or proposals in a given profile of rankings?
Can we do with large numbers of alternatives?

OUTLINE

2. Definition of divisiveness metric, analysis of bounds, algorithmic questions: robustness and control

Measuring and Controlling Divisiveness in Rank Aggregation

Rachael Colley[1] , **Umberto Grandi**[1] , **César Hidalgo**[2,3,4] , **Mariana Macedo**[2] , **Carlos Navarrete**[2]

[1]IRIT, Université Toulouse Capitole, France
[2]Center for Collective Learning, ANITI, TSE, IAST, IRIT, Université de Toulouse, France
[3]Alliance Manchester Business School, University of Manchester, UK
[4]Center for Collective Learning, CIAS, Corvinus University, Hungary
{rachael.colley,umberto.grandi}@irit.fr, {cesar.hidalgo, mariana.macedo, carlos.navarrete}@univ-toulouse.fr

UNPACKING POLARISATION

3 users

I user

I user
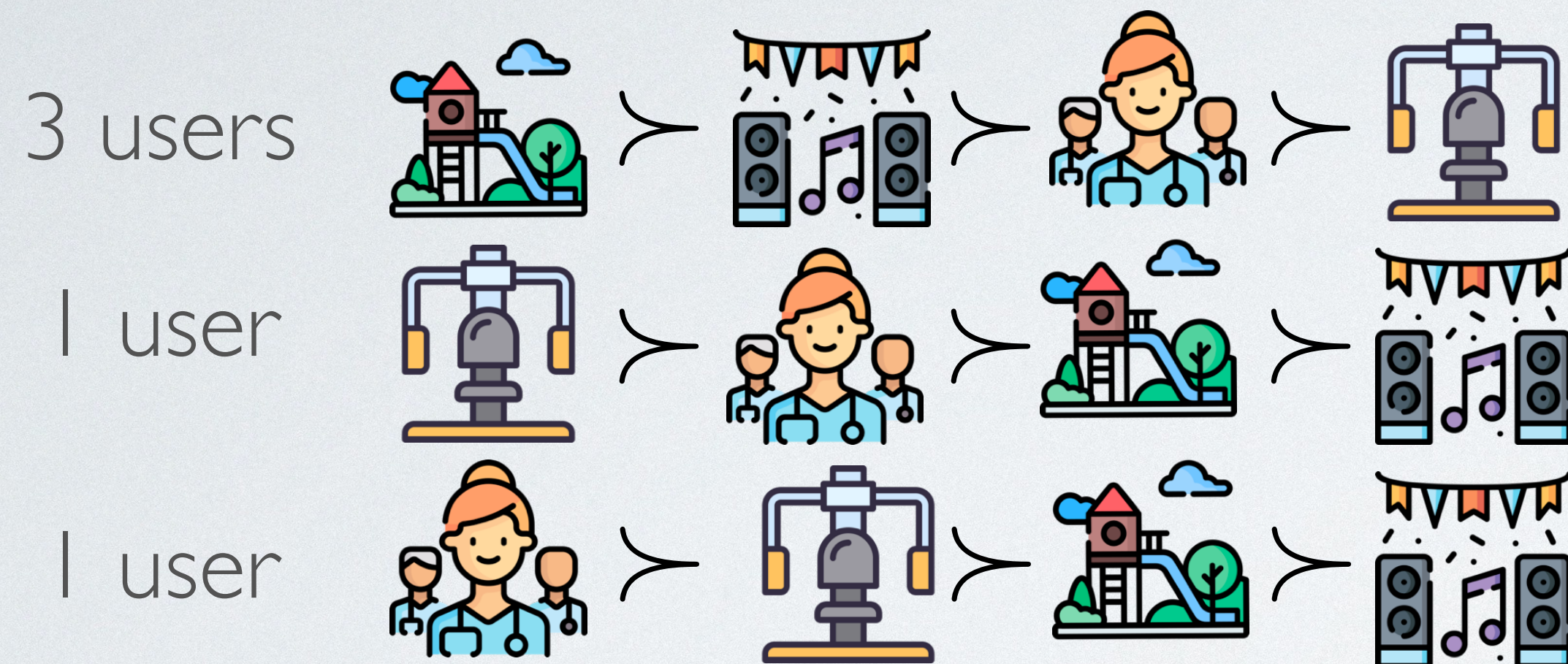
This profile has high polarisation. If we wanted to decrease polarisation, or to take advantage of it, where should we start? Can we explain why is it polarised?
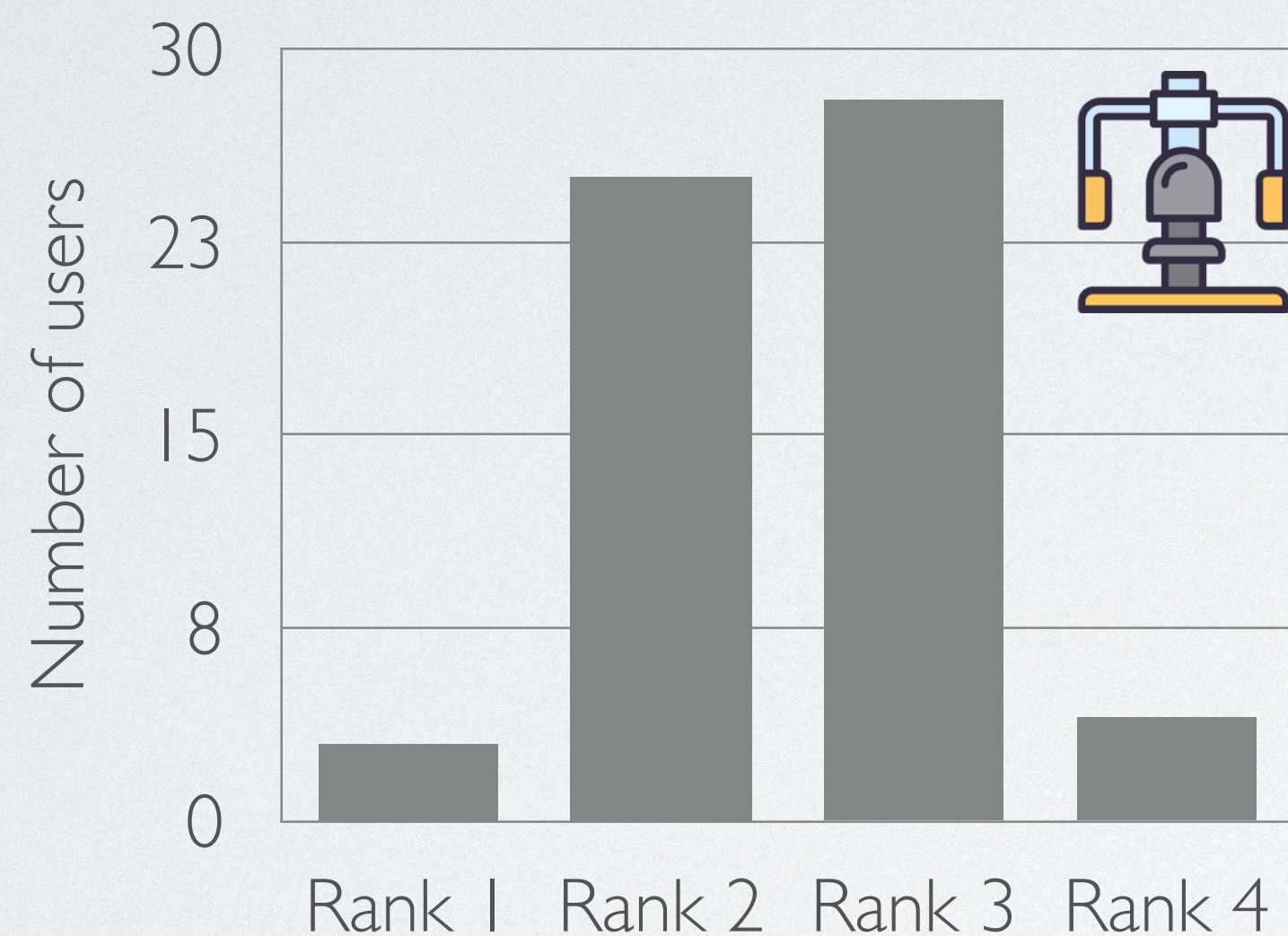
We aim at moving from comparing profiles (in terms of polarisation), to **comparing issues inside a single profile**

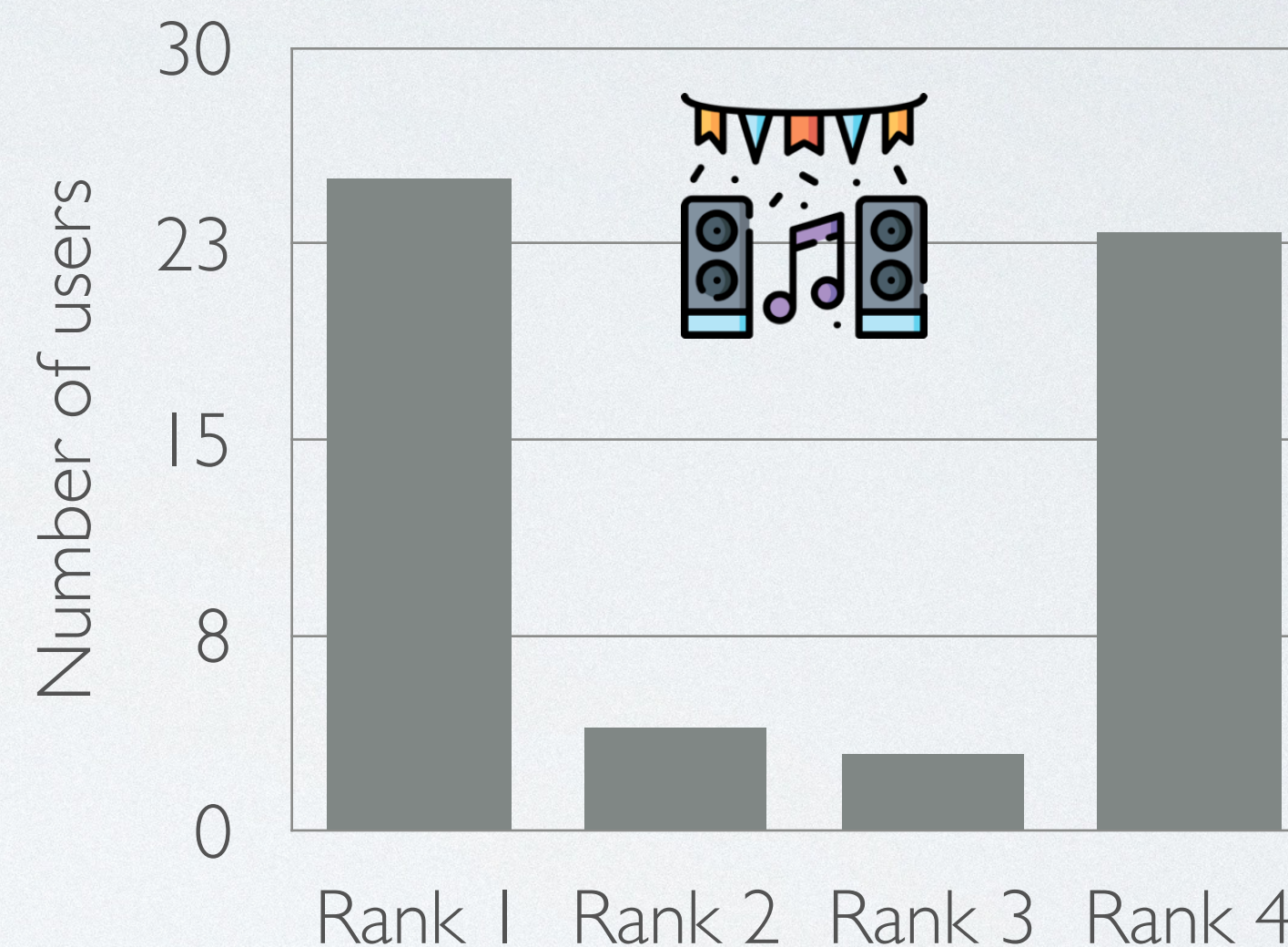$$\frac{1}{n}\sum_i (rank(a, \succ_i) - \text{avg-rank}(a))^2$$

RANK VARIANCE

The variance of a distribution is **widely used in social sciences** to measure polarisation

Less polarising issue

23 users rank "reduce noise" at the bottom of their ranking (rank 4)

More polarising issue

Two issues: the variance is not related to the aggregation function used (eg Borda)
It is hard to compute on incomplete data without imputing missing preferences

DIVISIVENESS IN CHILE

During the Chilean protests in 2019-20, César Hidalgo and Carlos Navarrete (now also affiliated to IAST!) run a successful application which **extracted pairwise comparions over hundreds of proposals** to be included in the new constitution
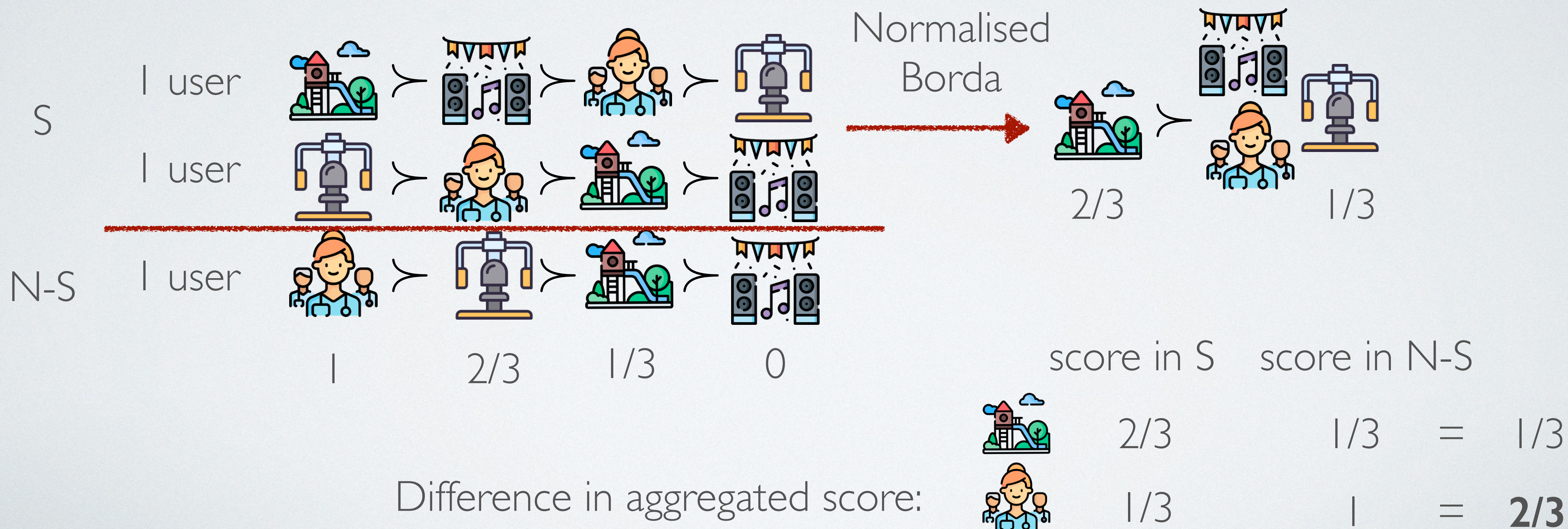

They collected 7.4M responses (pairwise comparisons). See last part of the talk for similar experiment in France and Brazil.

Users can see collective measures:
A ranking of agreement (Borda score)
A ranking of how **divisive** an issue is

DIVISIVENESS, FIXED SUB-POPULATION

The divisiveness of issue b for subpopulation S is the difference of the score (Borda, Copeland) of issue b in S and its score in the complement subpopulation N-S

Normalised Borda

S

1 user

1 user

N-S

1 user

1   2/3   1/3   0

2/3   1/3

score in S   score in N-S

2/3   1/3   =   1/3

Difference in aggregated score: 1/3   1   =   **2/3**

DIVISIVENESS

Let N(b>c) be the set of all users that prefer issue b to c

The divisiveness of issue b in profile P is the **average divisiveness of b wrt subpopulation N(b>c) for all other issues c**, discounted by the size of the two subpopulations (alpha between 0 and 1)
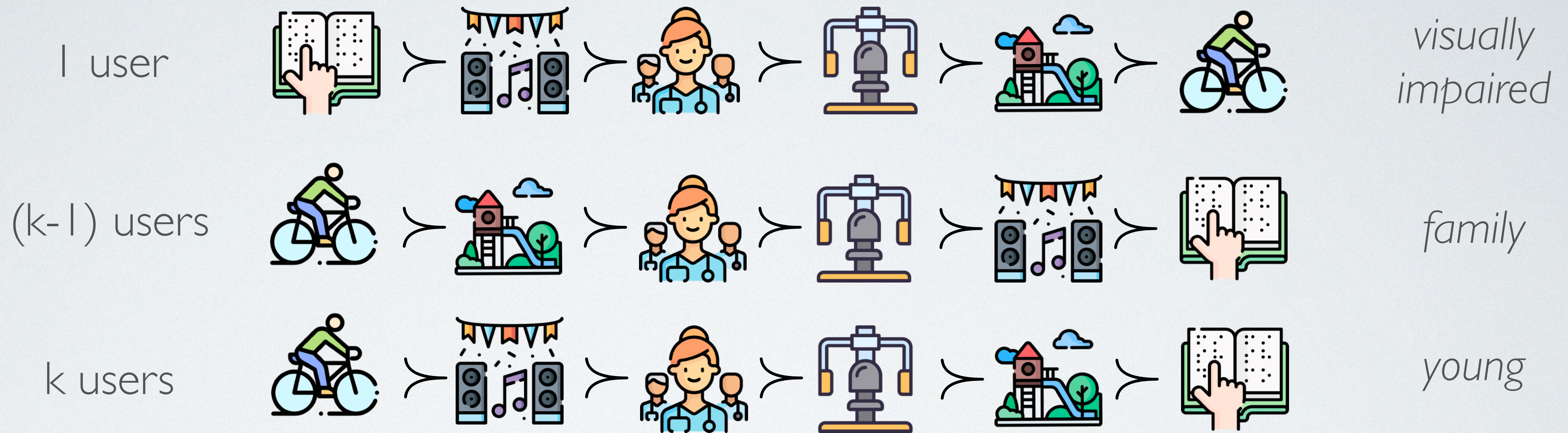
normalising factor

If 0 this term disappear

$$DIV(a) = \frac{1}{m-1} \sum_{b \neq a} \left(\ell \frac{\#(N_{a>b}) \cdot \#(N_{b>a})}{n^2} \right)^{\alpha} DIV(a, N_{a \succ b})$$

Number of users prefer a to b

Divisiveness of issue a wrt population that prefers a to b
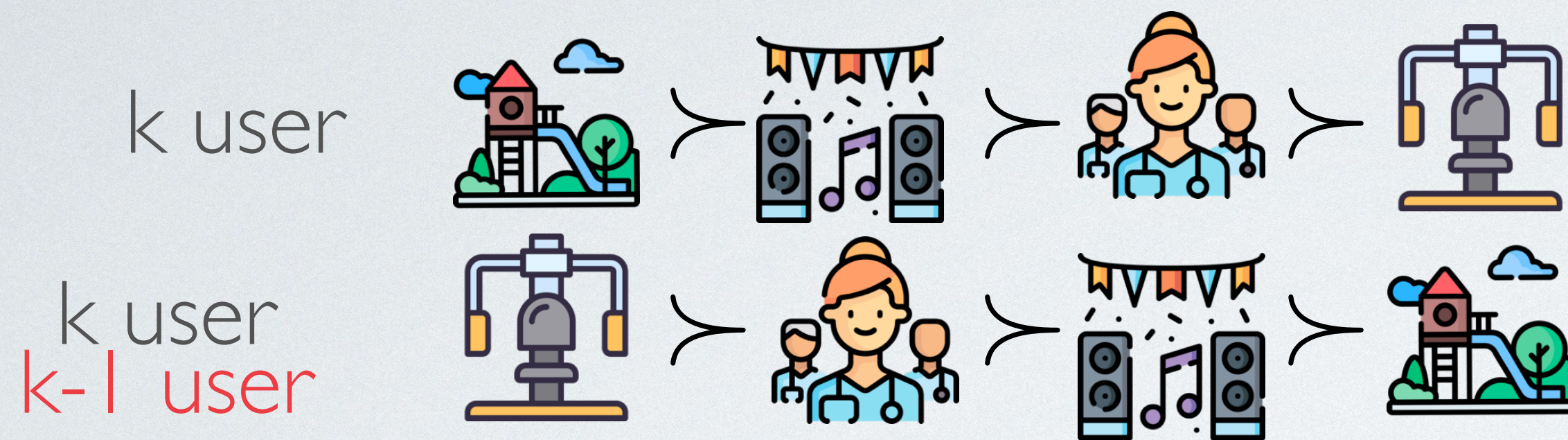
POLARISATION AND MINORITY OPINIONS

1 user   → 📖 ≻ 🔊 ≻ 👩 ≻ 🎏 ≻ 🏰 ≻ 🚴   visually impaired

(k-1) users → 🚴 ≻ 🏰 ≻ 👩 ≻ 🎏 ≻ 🔊 ≻ 📖   family

k users   → 🚴 ≻ 🔊 ≻ 👩 ≻ 🎏 ≻ 🏰 ≻ 📖   young

Assuming k>=5, l=4 (normalisation)

If *alpha=0* then 📖 and 🚴 are the most divisive issues

If *alpha=1* then 🔊 and 🏰 are the more divisive issues

BOUNDS EXAMPLES

k user

k user
k-1 user

Fully polarised profile:
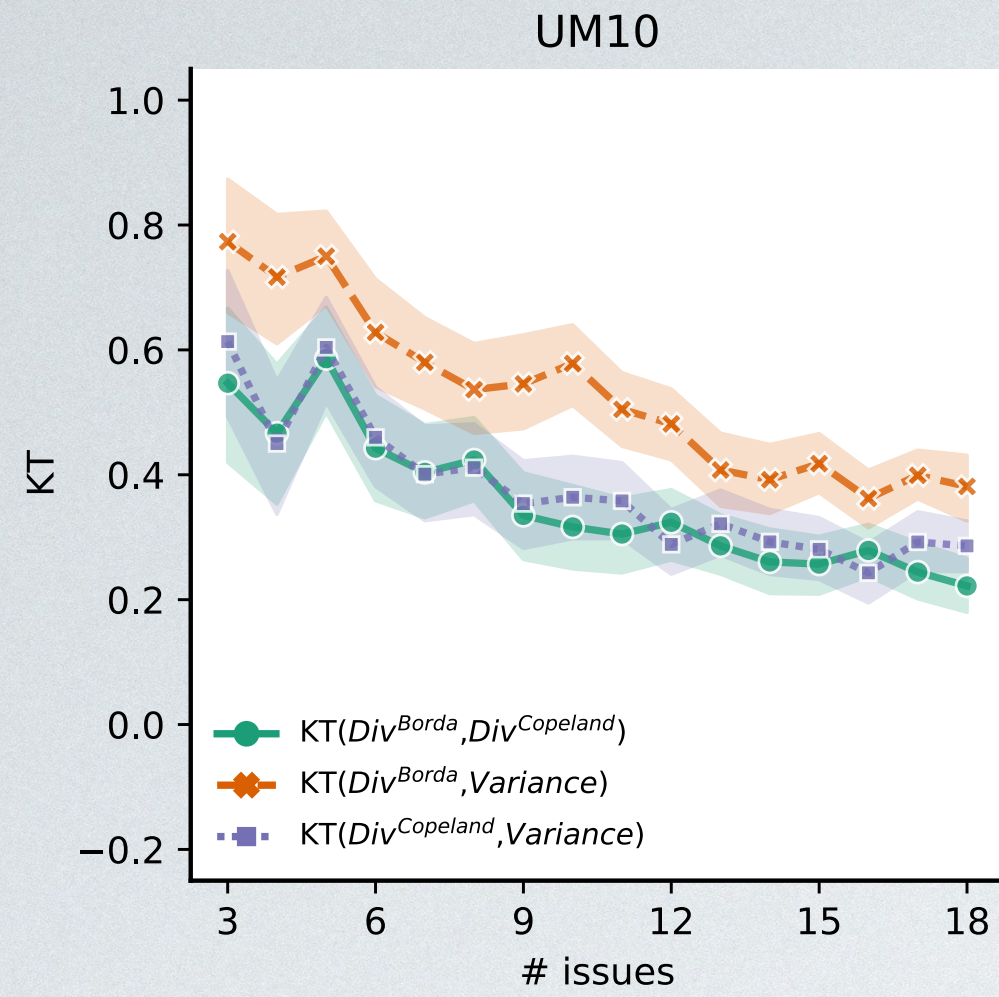divisiveness of 🔧 🏰 = 1 (Borda, Copeland)
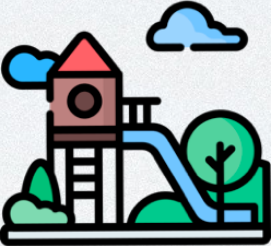
…
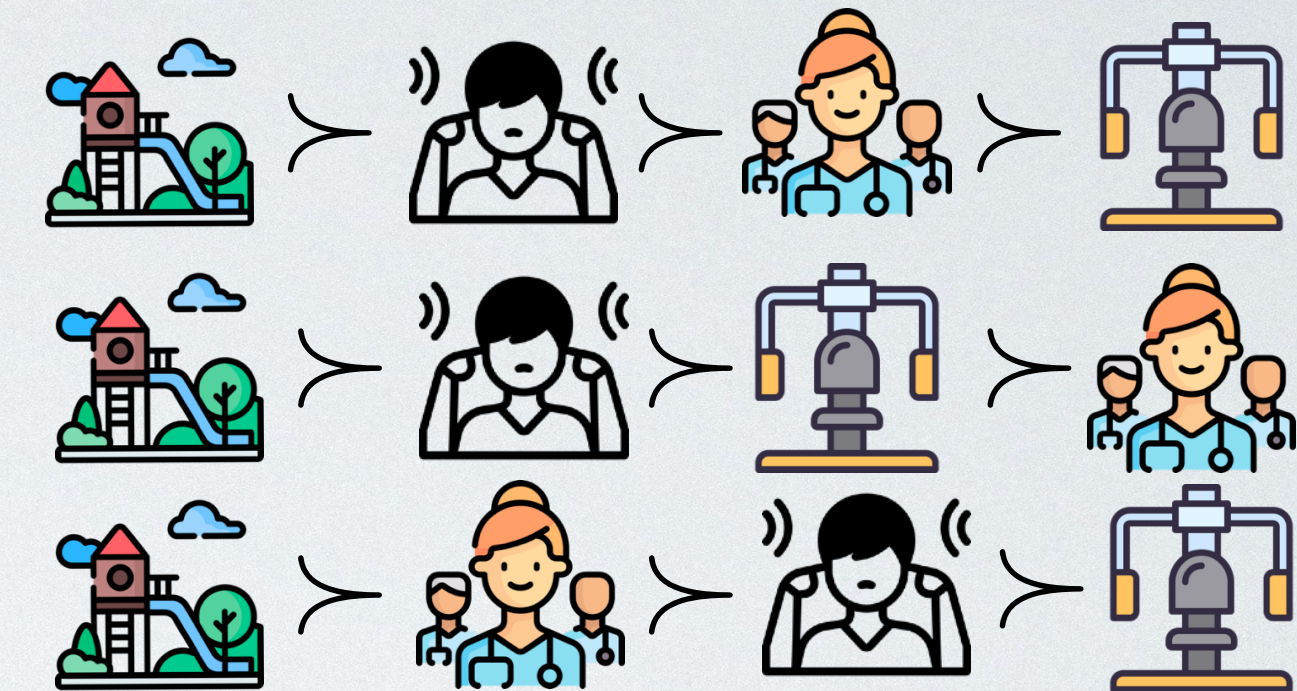
m! users each submitting a
different ordering

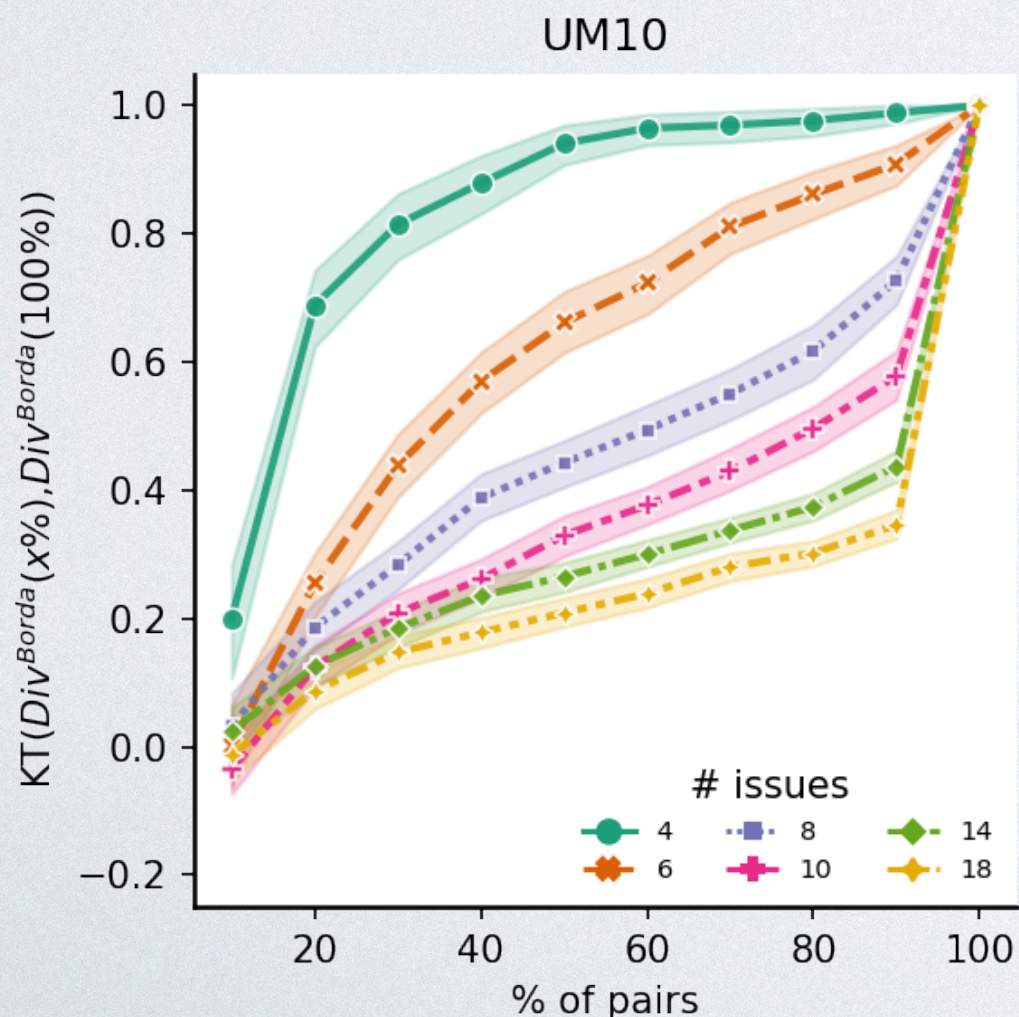Uniform profile:
all issues have the same divisiveness

RESULTS

UM10

When the number of issues >10 the Kendall-Tau correlation between the rankings obtained from Rank-Variance and Divisiveness (with Borda, Copeland) is **lower than 0.4**
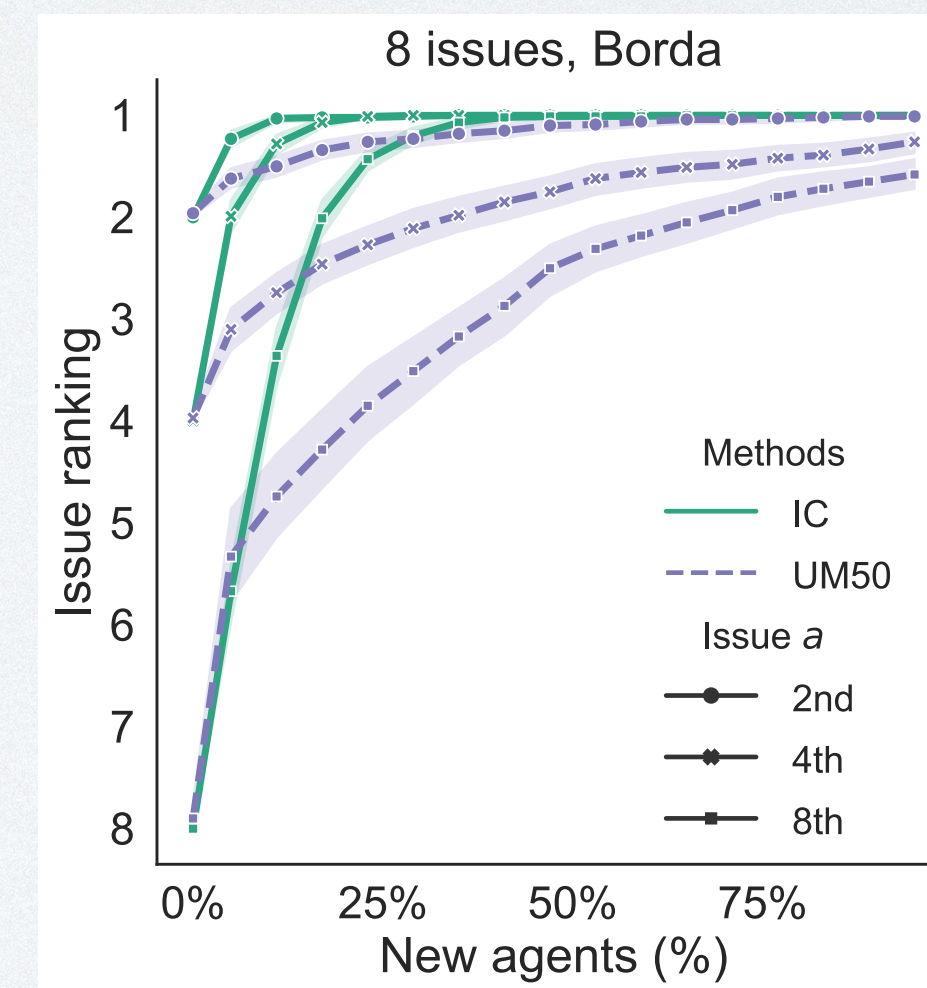
Given issue 🏰 we can find in poly time the subpopulation S that is most divided on 🏰

UM10

Divisiveness is **not robust (=easy to disrupt)**: deleting between 10/20% of pairs is sufficient to drop KT correlation below 0.5

8 issues, Borda

Need to add **20-30% fake profiles** to manipulate an issue on top of divisiveness

OUTLINE

Understanding political divisiveness using online participation data from the 2022 French and Brazilian presidential elections
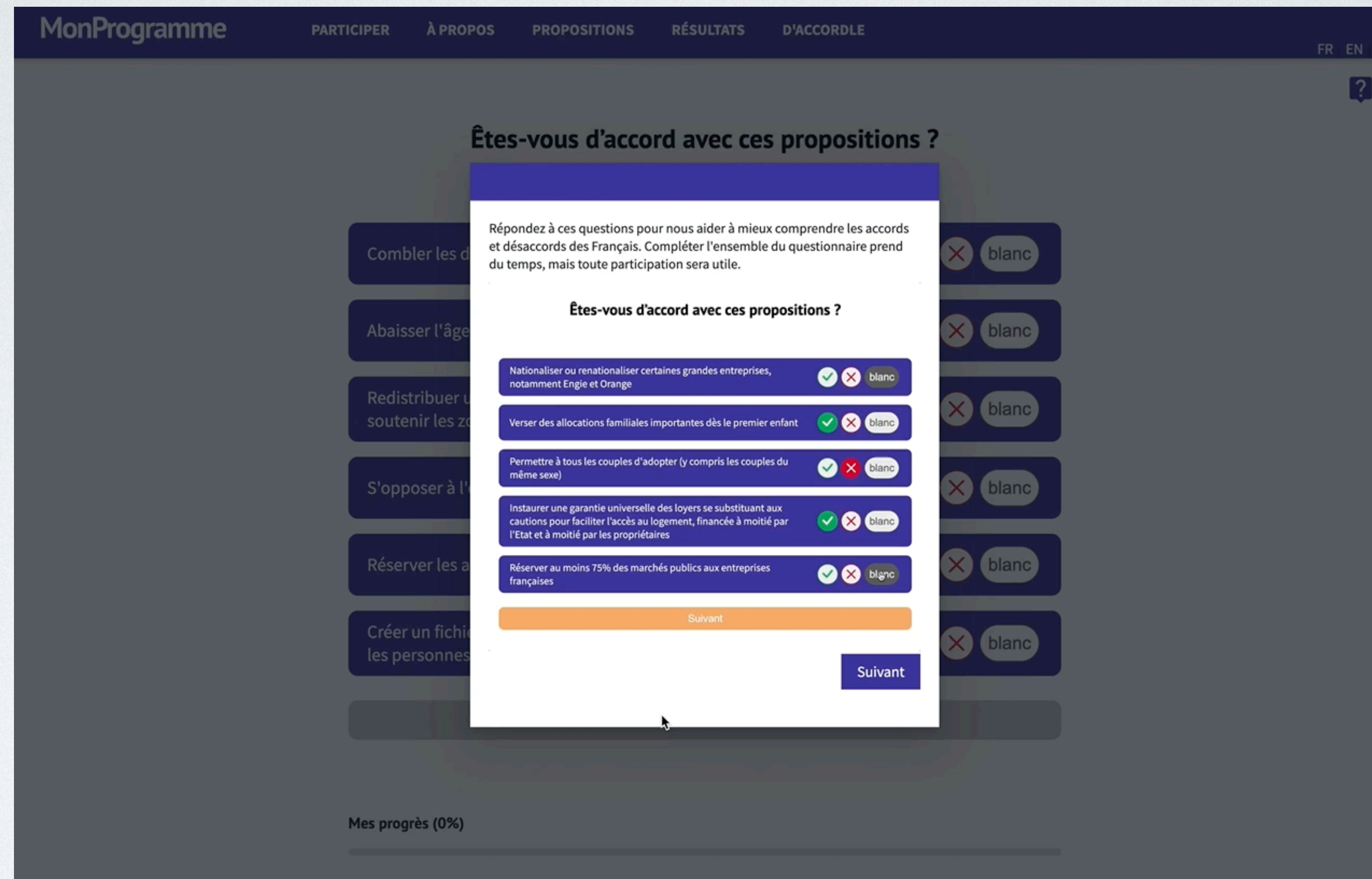
Carlos Navarrete [1], Mariana Macedo [1], Rachael Colley [2], Jingling Zhang [1], Nicole Ferrada[1], Maria Eduarda Mello [3], Rodrigo Lira [4], Carmelo Bastos-Filho [4], Umberto Grandi [2], Jérôme Lang[5] & César A. Hidalgo [1,6,7] ✉

3. Empirical analysis of divisiveness measures, building a collective government program

PREFERENCE ELICITATION PLATFORMS

monprogramme2022.org
Participants: 1 175
Pairwise comparisons: 1 705 104

brazucracia.org
Participants: 740
Pairwise comparisons: 157 280

COLLECTIVE GOVERNMENT PROGRAM

~100 political proposals extracted from the programs of the candidates

Drag and sort the proposals according to your preferences

| 1 | Stop the construction of wind turbines |
2	End the 35-hour working week
3	Refrain from any military intervention without the mandate of the United Nations
4	Submit foreign investments to the approval of a High Council for Economic and Digital Sovereignty
5	Prohibit the burkini at municipal swimming pools

Next | Go to results

Which proposal do you

Reduce agricultural production costs and marketing price

Co... encouraging physical activity for primary care

⦾ Both | ∅ Neither | Skip ⟳
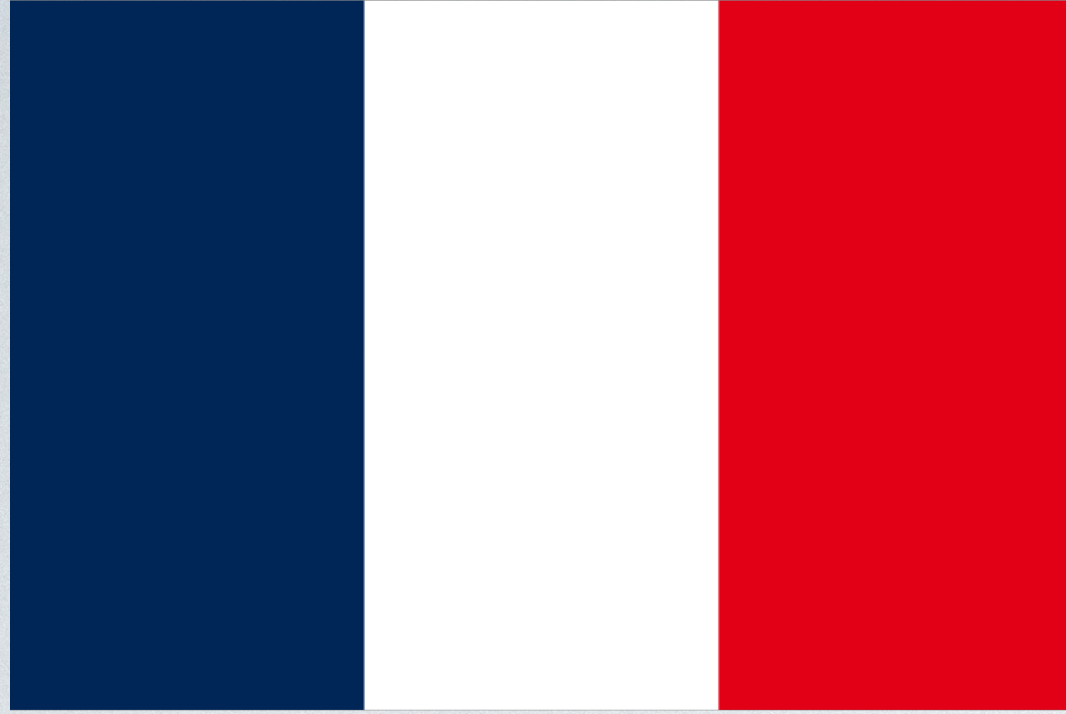
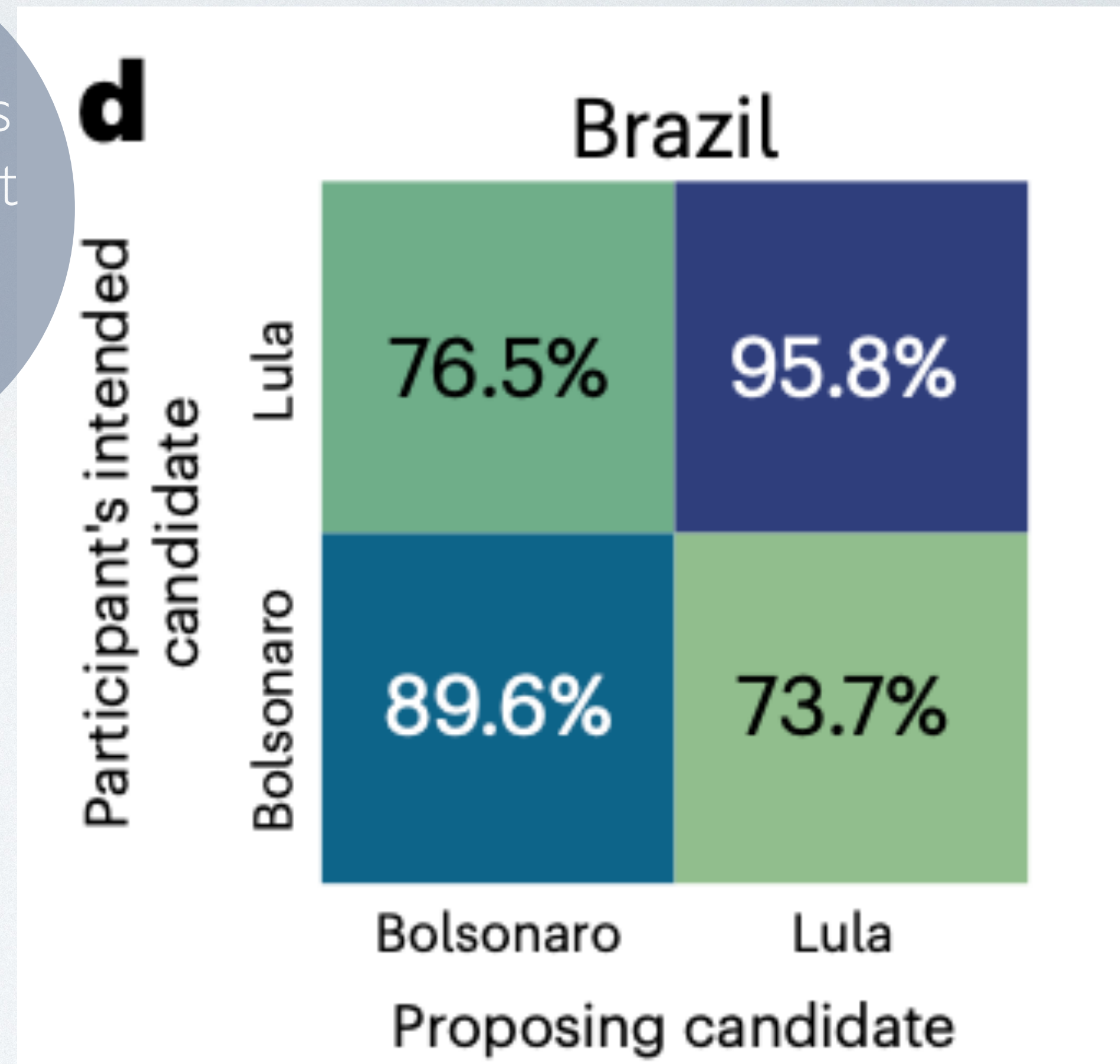| Id	Name	Win percentage
1	Plan to use 100% renewable energies by 2050	74.7%
2	Increase personnel in public hospitals	72.0%
3	Increase the minimum wage	70.6%
4	Further develop the French nuclear park	70.5%
5	Include ecology in the Constitution	68.2%
6	Reduce working hours to 32 hours per week	67.1%
7	Create a citizen income	66.5%
8	Cap prices of essential products: gas, electricity, food	66.0%
9	Prohibit any salary difference of more than 1 to 20 in a company	65.9%
10	Devote 3% of GDP to research and development	65.7%
11	Create a Constituent Assembly to pass to the Sixth Republic	65.7%
12	Lower retirement age to 60	64.8%
13	Increase the industrialization of the country	63.8%
14	Ensure a minimum pension is equivalent to the minimum wage	62.5%
15	Restore the solidarity tax on wealth (ISF)	62.0%
16	Increase number of doctors in rural underserved areas	62.0%
17	Increase teacher salaries	61.8%
18	Nationalize or renationalize some large companies (EG Telecom / Orange)	59.4%
19	Ban dangerous pesticides (eg neonicotinoides)	59.1%
20	Establish a Citizens' Initiative Referendum	58.2%

Borda score on incomplete data

Results are only representative of the opinion of the participants

| 115 | Remove of the TV licence fee | 29.0% |
116	Defend regional languages and cultures	27.6%
117	Restore ENA (the National School of Administration)	25.9%
118	Protect hunting and fishing rights	24.5%
119	Establish full autonomy for Corsica	22.5%
120	Prohibit the burkini at municipal swimming pools	18.1%

DO (RIGHT) LEFT WING VOTERS AGREE MORE WITH (RIGHT) LEFT VOTERS?
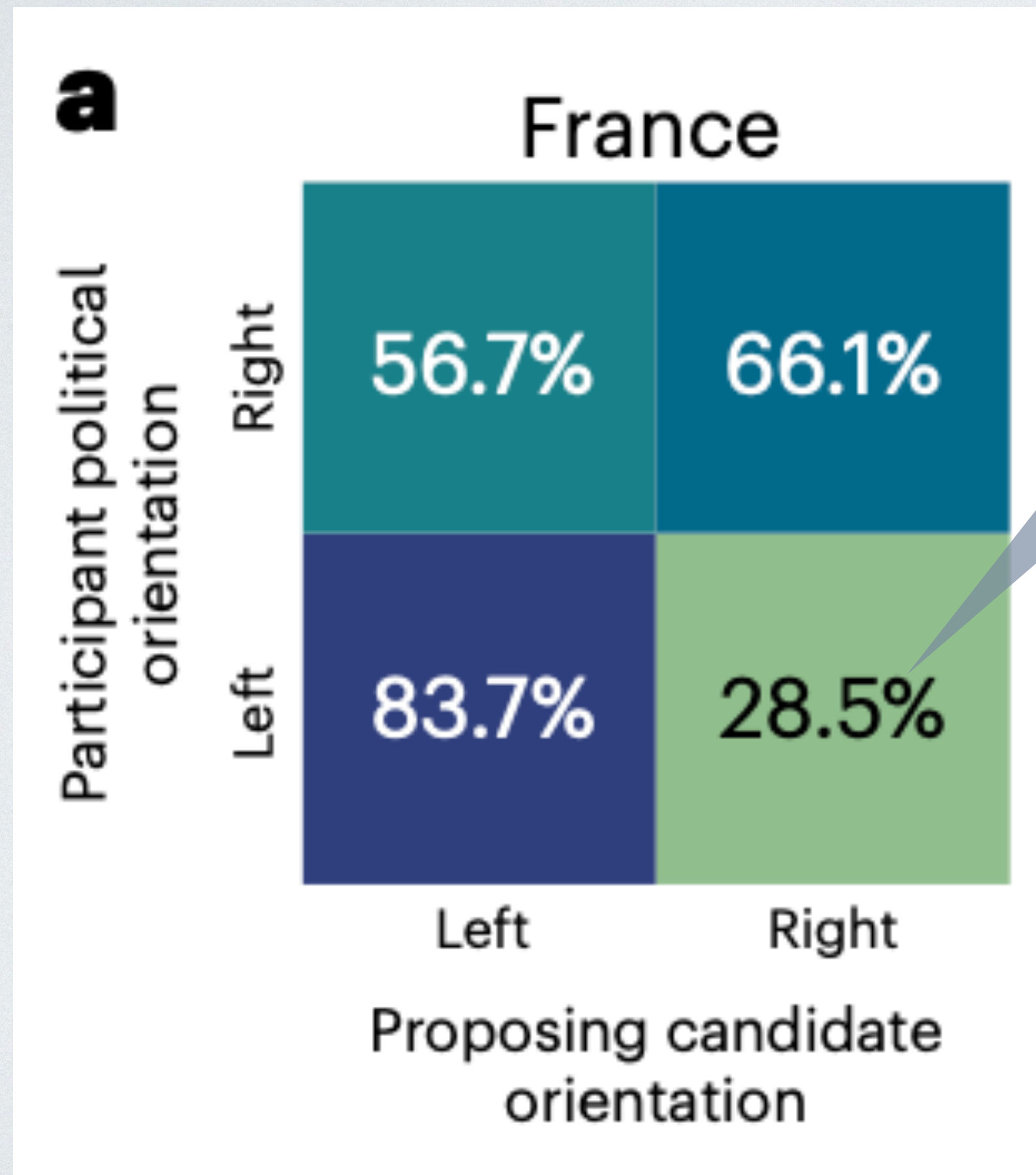
Excluding Macron's proposals

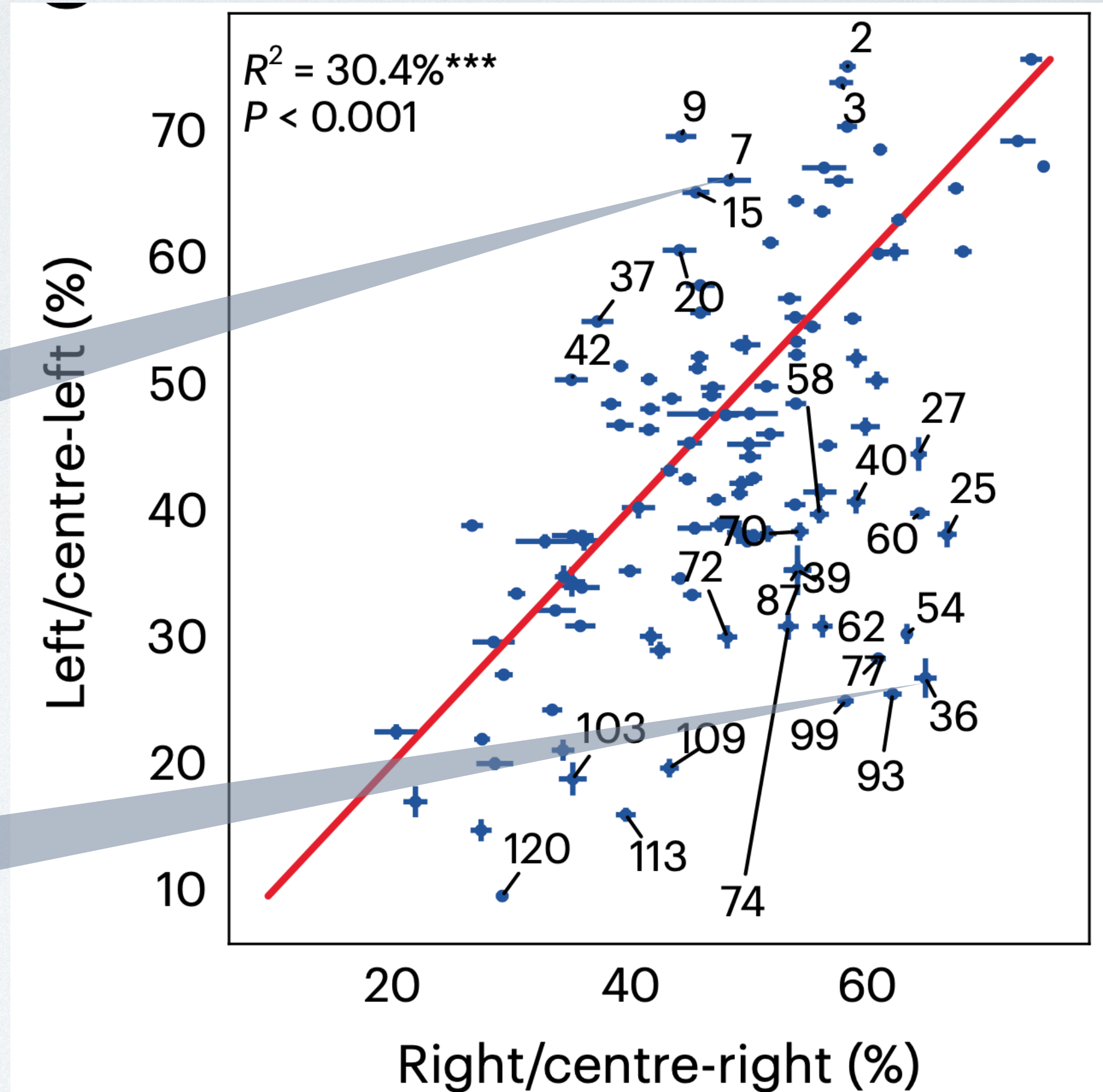# DIVISIVENESS AND POLITICAL ORIENTATION
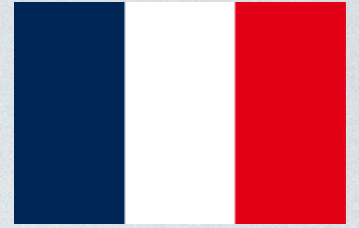
Labeled proposals have a 15% difference between the win percentage (=divisiveness wrt political orientation split of the population)

7.Create a citizen income

0.17 divisive

0.38 divisive: the difference in "win rate" is 38% points between L and R

36.Restoration of border control by France leaving Schenghen

DIVISIVENESS AND AGE

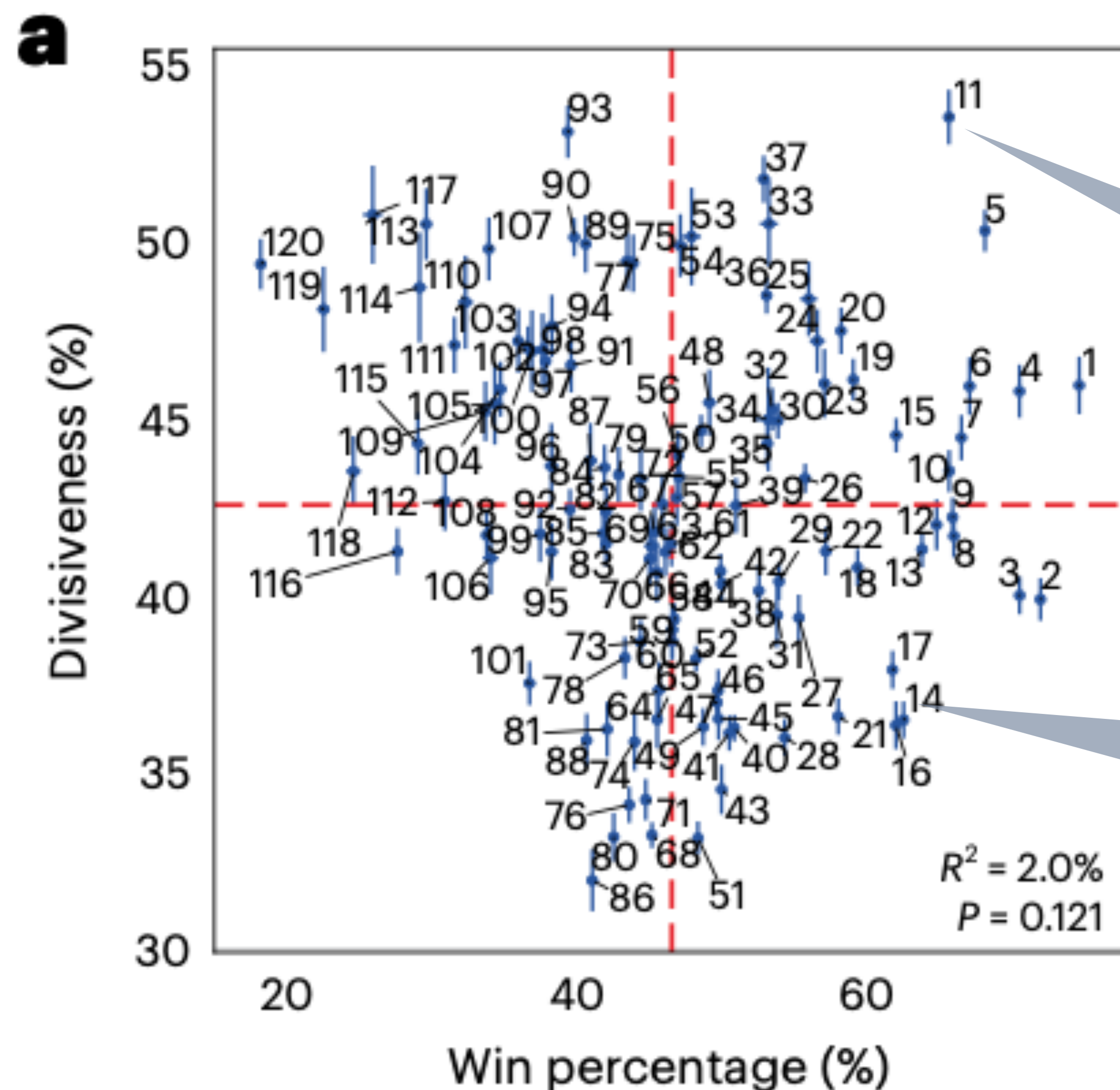40.Reserve social security assistance to people with French nationality

25.Abolition of the law of the soil

110.Reduce the tax on real estate wealth by exempting it from 50% of the main residence

Divisiveness seems to be a multidimensional phenomenon: need to use an "agnostic" measure

$R^2 = 73.2\%***$
$P < 0.001$

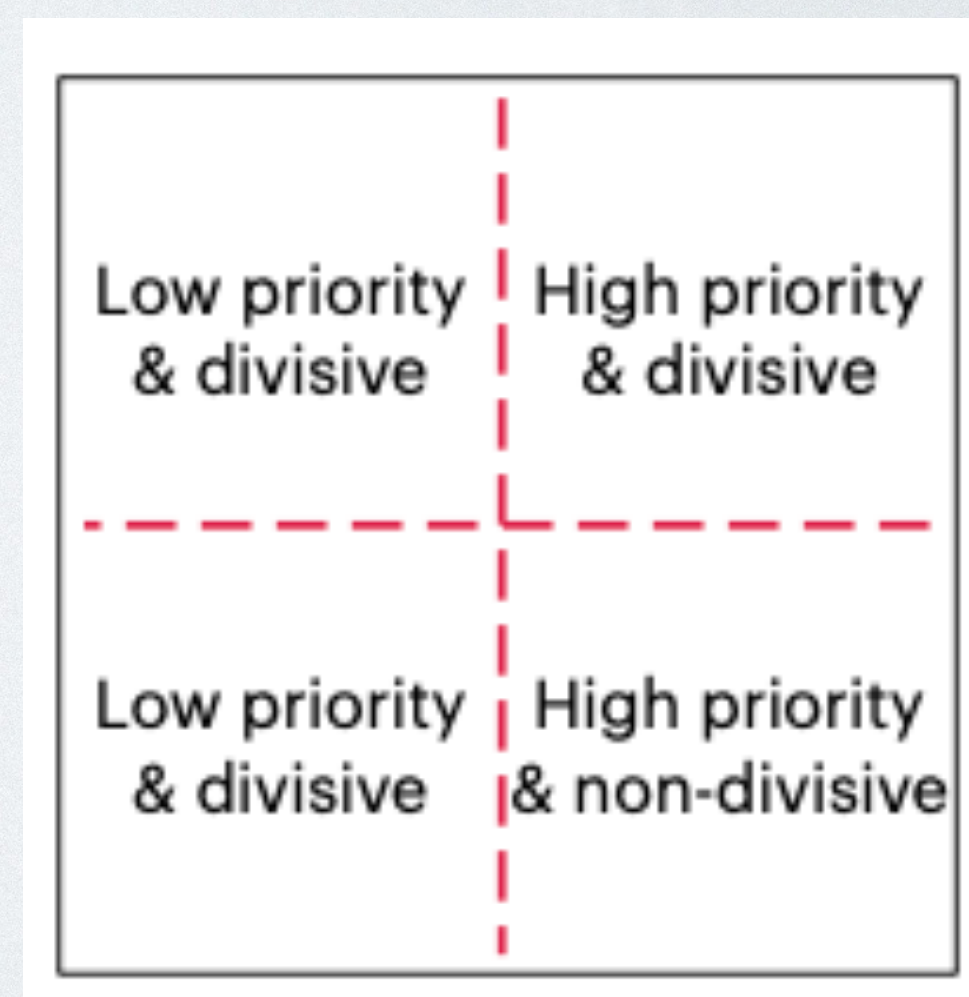Younger participants (%)

Older participants (%)
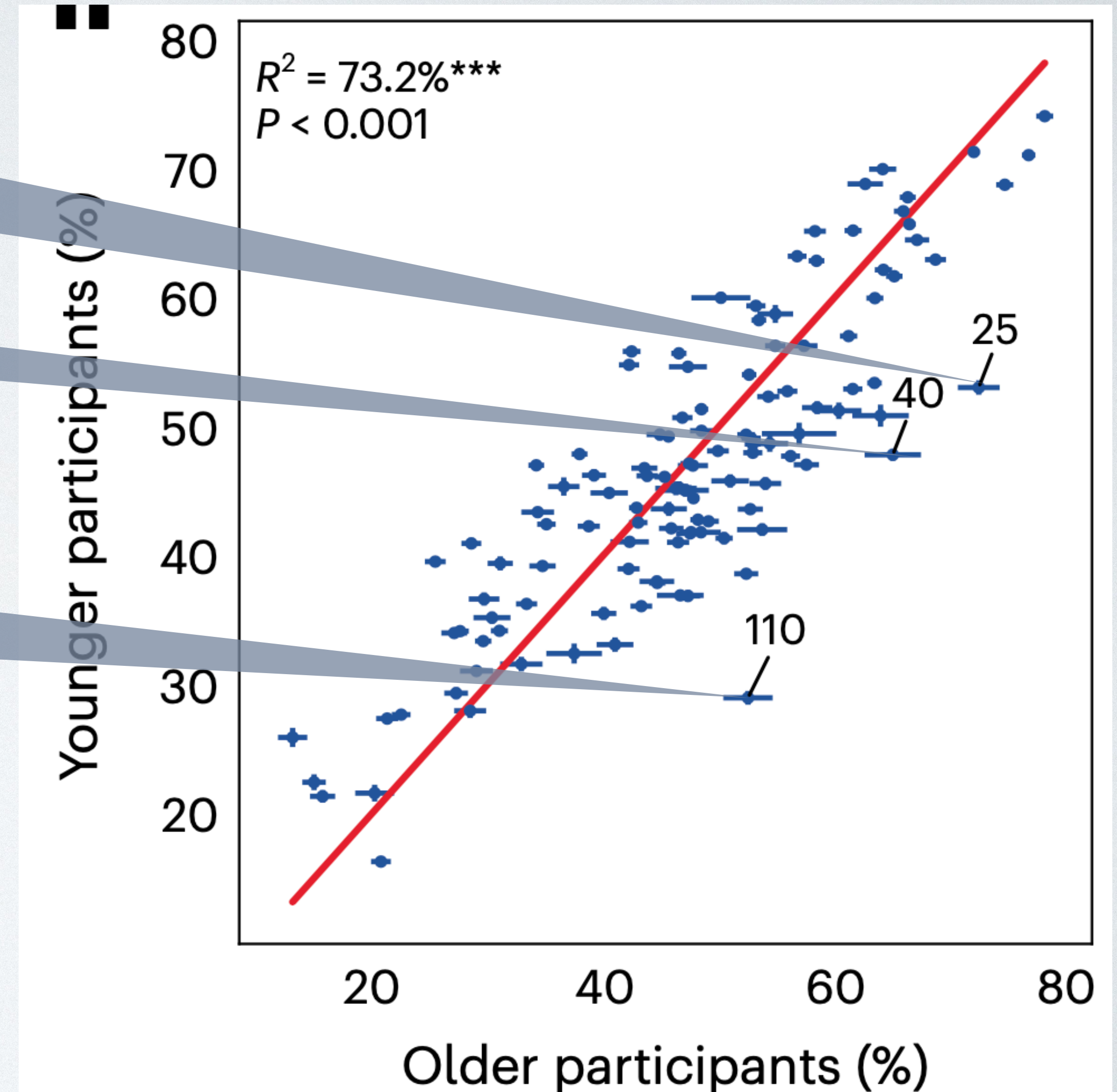
DIVISIVENESS (AGNOSTIC)

Divisiveness provides information that is unavailable to the respective aggregation function (in this case Borda, Copeland)

11. Create a national assembly to move to the VI republic

14. Increase the number of doctors in rural undeserved areas

Divisiveness

- rank issues *within* a profile
- explicit dependence on the score used to aggregate rankings
- It can be used to understand the tensions inside a democracy: asking people what they want, measuring what divides them

Theoretical analysis

- from polarisation to minority detection depending on alpha
- easy to disrupt by deleting pairwise comparisons
- can be controlled by adding (large numbers of) users, size of population matters

Many **open directions** for future work!
- Finding divisiveness measures that are more robust (need less data to be accurate)
- Use divisiveness to compare profiles, relation with latest polarisation measures

MEASURING DISAGREEMENT: POLARISATION

Simple version of polarisation measure: $K \sum_{i,j} \pi_i^{1+\alpha} \pi_j |y_i - y_j|$

population with income $= y_i$

population with income $= y_j$

Dissimilarity between values $y_i \quad y_j$

Among the **postulates** assumed:
- High degree of homogeneity within group
- High degree of heterogeneity across groups
- **Groups of insignificant size carry little weight**
- The size of the overall population has no influence

Number of individuals (PI)

6

3

0

1 2 3 4 5 6

Income (y)

ADAPT POLARISATION ON RANKINGS

Idea: rank pairs of issues by their disagreement d(a,b)

Population I

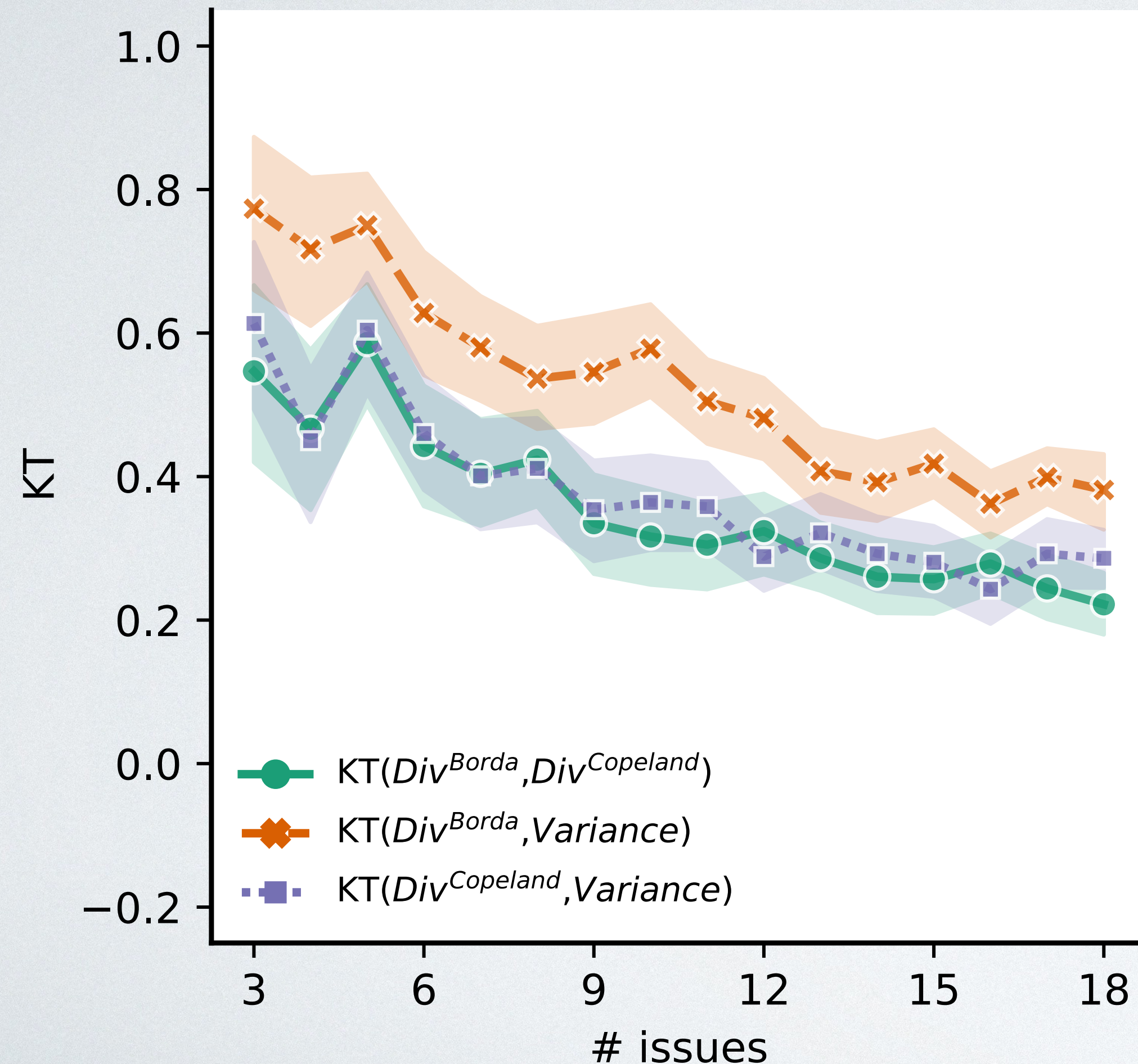Population II
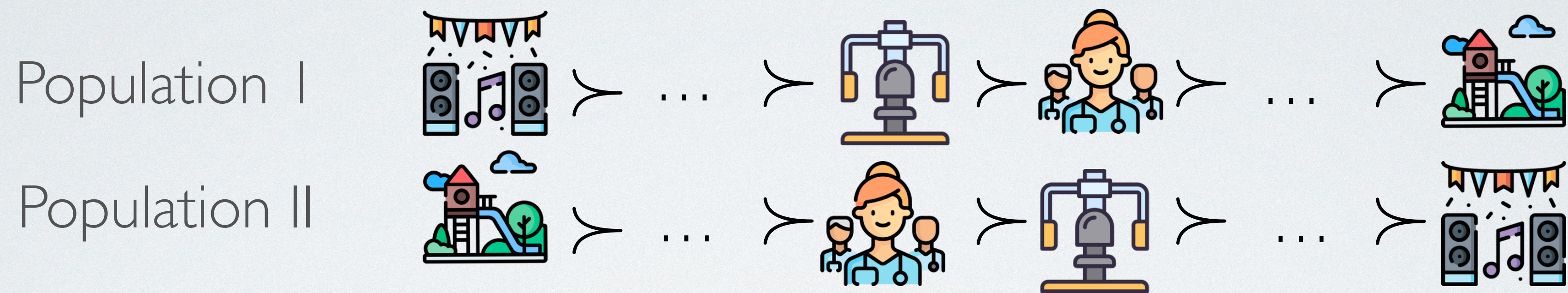
  has the same pairwise disagreement than

Further observation: all four alternatives will have a similar Borda score (agreement)

Possible idea: average the pairwise disagreement of against all other alternatives?
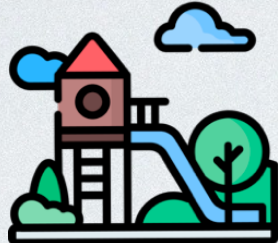
RELATION WITH VARIANCE

UM10

We generated 100 profiles of 100 linear orders using **IC, UM10, UM50** (Urn model, different correlation factors)

When the number of issues >10 the Kendall-Tau correlation between the rankings obtained from Rank-Variance and Divisiveness (with Borda, Copeland) is lower than 0.4
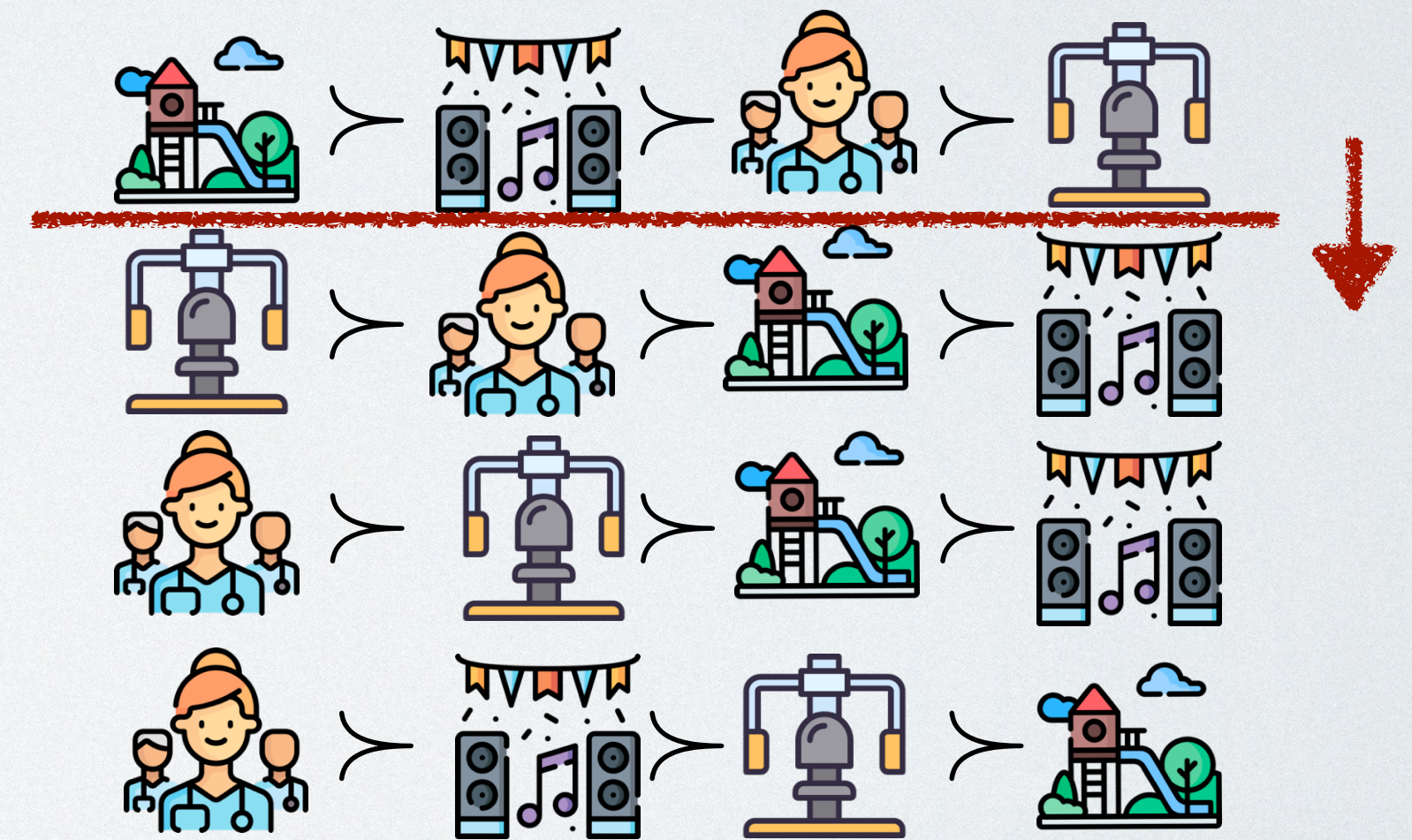
But note that on small number of issues the measures are correlated

MOST DIVIDED POPULATION

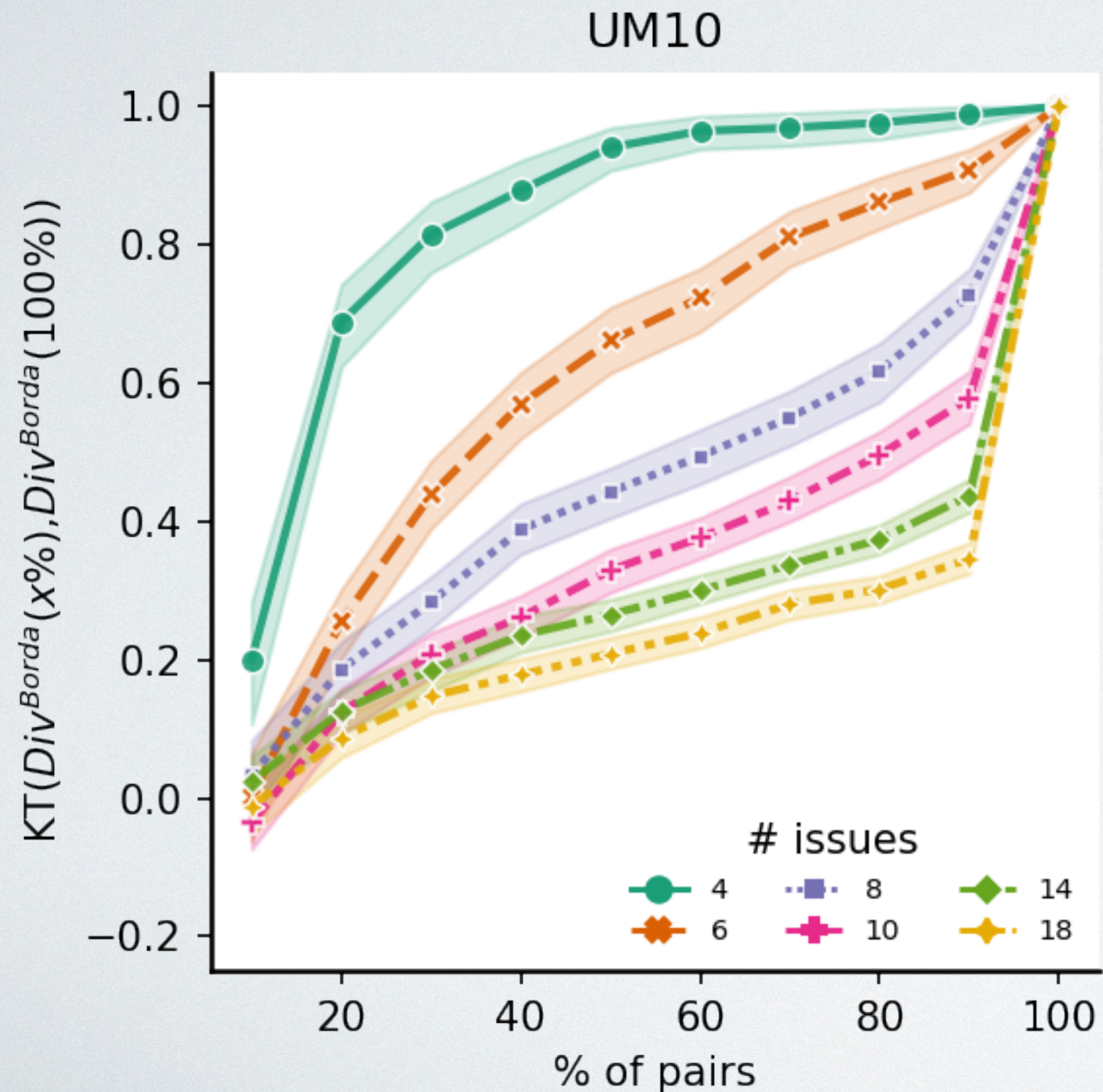Given issue find the subpopulation S that is most divided on

Simple **polynomial** algorithm for Borda score:

- order agents on decreasing ranking of
- any S that maximises divisiveness will be a split of the re-ordered profile
- "moving window" to find the maximal split

Does not seem trivial for the Copeland score

ROBUSTNESS/DISRUPTION

We generated 100 profiles of 100 linear orders using **IC, UM10, UM50**. We deleted X% of pairwise comparisons and computed the ranking of divisiveness

Divisiveness is **not robust (=easy to disrupt)**: deleting between 10/20% of pairs is sufficient to drop KT correlation below 0.5

Curve inversion between less and more than 7 issues

CONTROL BY ADDING USERS (BOTS)

Borda, 8 alternatives, IC
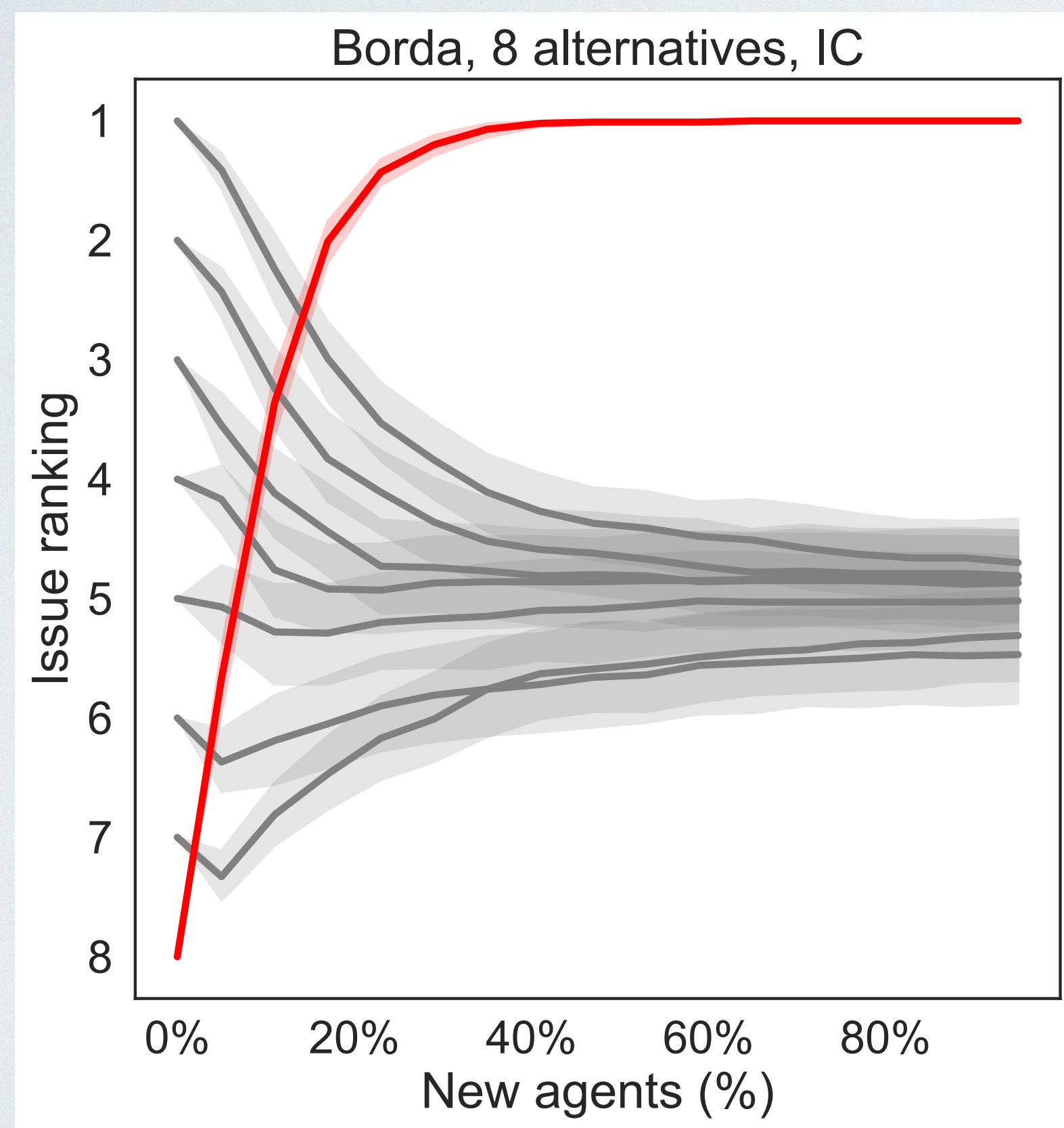
To make issue b the most divisive we tested a simple algorithm INJECT-s: that adds fake rankings:

- Compute the ranking <s given by score s
- add one user with ranking <s moving a to top
- add one user with ranking <s moving a to bottom
- repeat until success

We can prove that INJECT-s terminates in poly time

ALGORITHM:
MANIPULATE RANKING USING BOTS

8 issues, Borda

Issue ranking vs New agents (%)

Methods
— IC
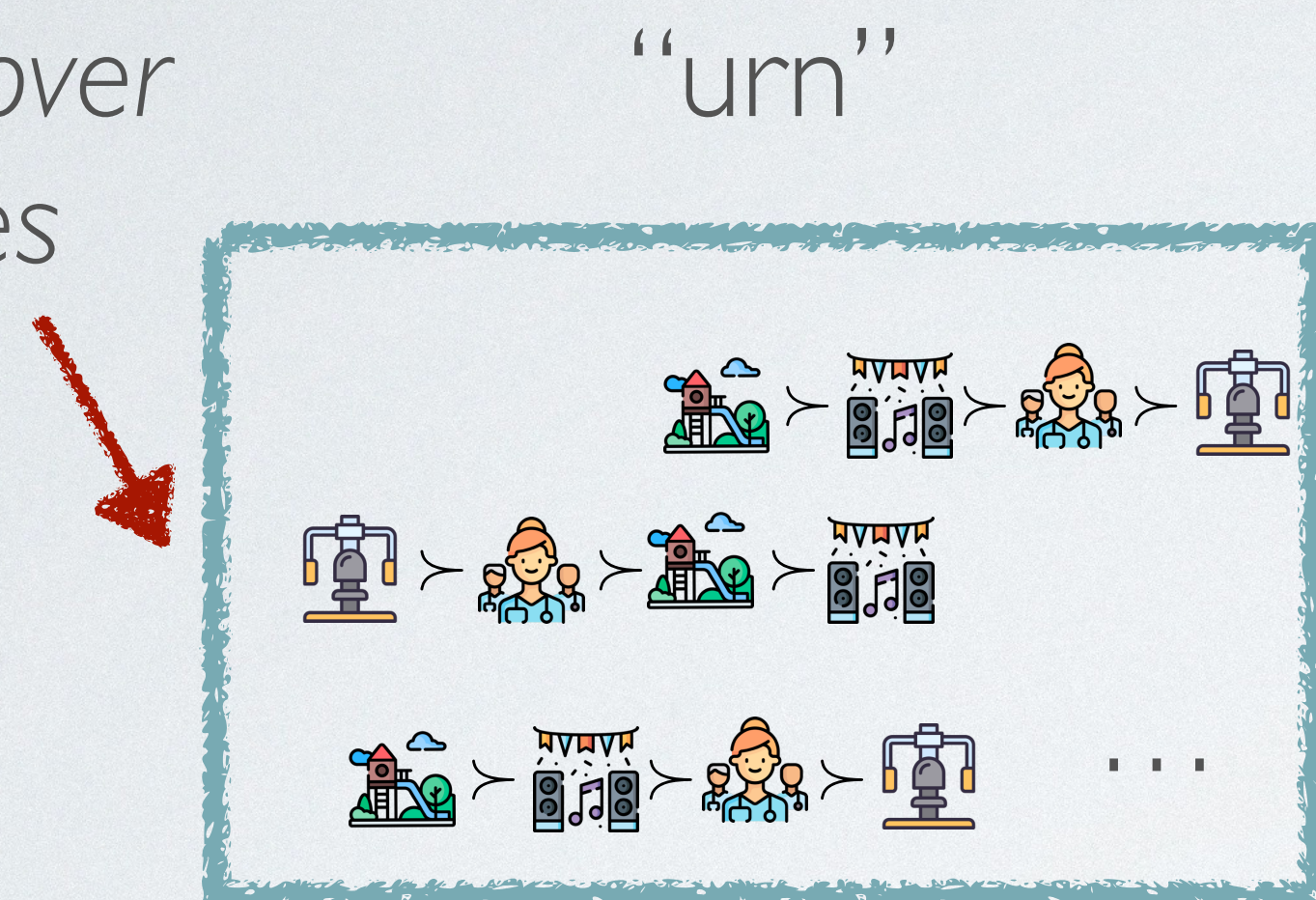-- UM50
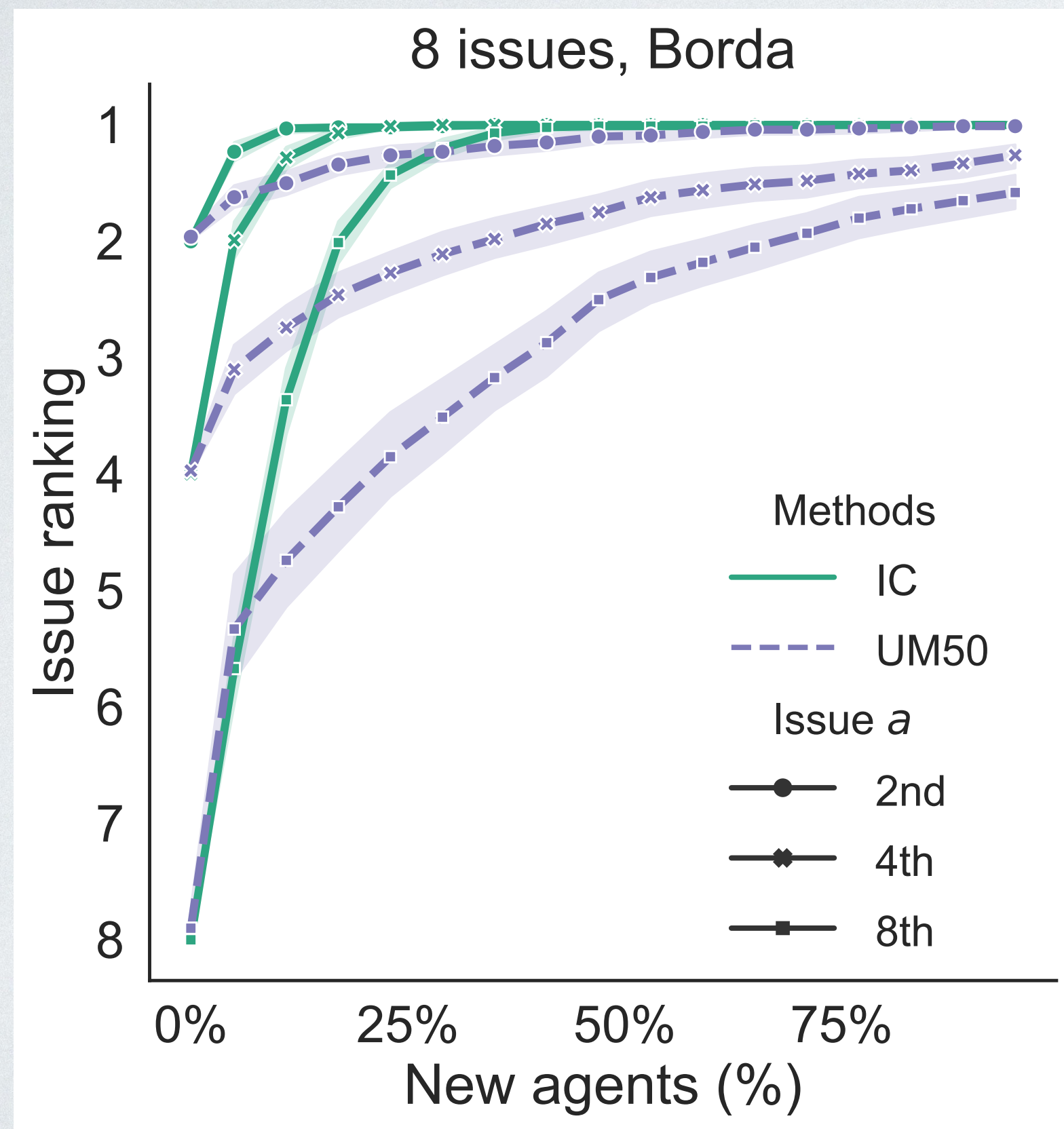
Issue *a*
● 2nd
✕ 4th
■ 8th

s = Borda

We generated 100 profiles of 100 linear orders using IC, UM10, UM50. We considered **three objectives**: make most divisive the issue ranked 2nd, 4th, last

Result depend on size of the population (**adding 25%/35% fake profiles** could be easy to detect over large populations)

More correlated profiles (UM50) are harder to control

GENERATION OF RANKING PROFILES

all m! possible rankings over m issues

"urn"

IC - impartial culture

Draw *n* rankings
uniformly at random
with replacement

UMX - Urn model X% correlation

Draw rankings uniformly at random
replacing m!/9 (for UM10) copies of
the drawn ranking in the urn

Libraries at <u>preflib.org</u>

WHY RANKINGS?

Ordinal information is arguably easier to elicit (e.g. via **pairwise comparisons**)

Easier **user interfaces** = more data, citizen engagement, improved participation

It is also the classical data format of social choice theory (because of assumptions on little interpersonal comparison of utility)

NOT ENOUGH RAISED HANDS?

size O($n \times m$)

Computational problem

INPUT: a profile of rankings, a proposal *b*, a partition S of the users
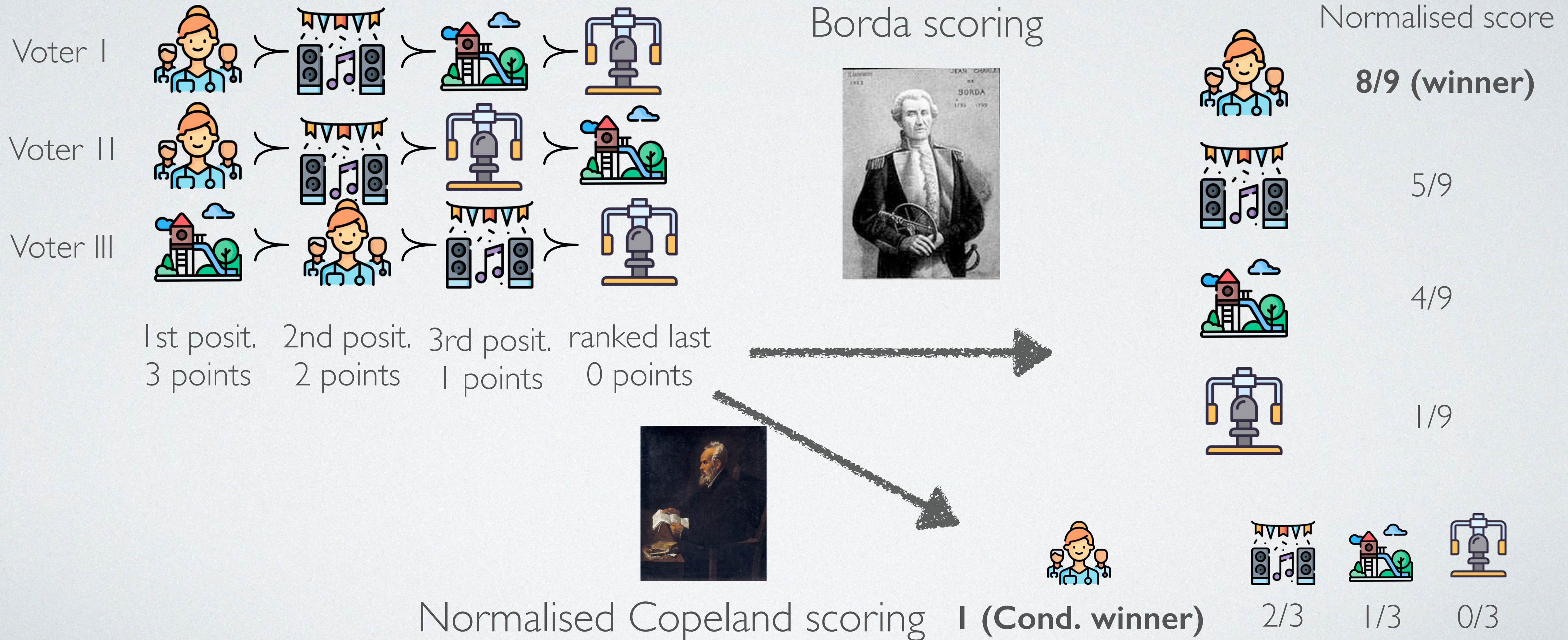OUTPUT: is S the maximally divisive partition?

Polynomial time solvable

There is a polynomial *p(X)* such that the answer to the problem
can be computed in time *p(INPUT SIZE)*

Most divisive population: the algorithm considers n partitions, the
Borda score can be computed in linear time, and we need to
consider m partition of users to compute the average

The brute-force algorithm would consider all possible partitions of n users ($2^{(n-1)}$, not poly)

NOT ENOUGH RAISED HANDS?

Voter I

Voter II

Voter III

1st posit.
3 points

2nd posit.
2 points

3rd posit.
1 points

ranked last
0 points

Borda scoring

Normalised score

8/9 (winner)

5/9

4/9

1/9

Normalised Copeland scoring   **I (Cond. winner)**   2/3   1/3   0/3

WHAT NEXT? INTERDISCIPLINARY APPROACH

Human computer interaction

Learning of preferences

Definition of public opinion

Manipulation and incentives

Field studies in Brazil and France