Measuring Diversity of Preferences

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joint work with Ulle Endriss

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Introduction

- Real world vs. synthetic preference profiles

- Diverse vs. consensus preferences
  - less diverse: better behavior?
    - fewer paradoxes
    - easier to reach an agreement
    - less disappointment
**Example**

Which one is more diverse?

<table>
<thead>
<tr>
<th>1: $a \succ b \succ c$</th>
<th>2: $a \succ b \succ c$</th>
<th>3: $a \succ b \succ c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: $a \succ c \succ b$</td>
<td>2: $b \succ a \succ c$</td>
<td>3: $c \succ a \succ b$</td>
</tr>
<tr>
<td>1: $b \succ a \succ c$</td>
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</tr>
<tr>
<td>1: $c \succ b \succ a$</td>
<td>2: $b \succ a \succ c$</td>
<td>2: $a \succ c \succ b$</td>
</tr>
</tbody>
</table>
### Example

Which one is more diverse?

<table>
<thead>
<tr>
<th></th>
<th>1: ( \text{a} \succ \text{b} \succ \text{c} )</th>
<th>2: ( \text{b} \succ \text{a} \succ \text{c} )</th>
<th>3: ( \text{c} \succ \text{a} \succ \text{b} )</th>
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<tbody>
<tr>
<td></td>
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<tr>
<td>3</td>
<td>2</td>
<td>6</td>
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<td>2</td>
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</tr>
</tbody>
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---
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Which one is more diverse?

<table>
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<tr>
<th>1 : a ≻ b ≻ c</th>
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</tr>
</thead>
<tbody>
<tr>
<td>2 : b ≻ c ≻ a</td>
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<td>2 : b ≻ a ≻ c</td>
</tr>
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</table>

<table>
<thead>
<tr>
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<th>2</th>
<th>6</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>6</td>
<td>6</td>
<td>5</td>
</tr>
</tbody>
</table>
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<tr>
<td></td>
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</tr>
<tr>
<td></td>
<td>2 : c ≻ a ≻ b</td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
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<td>3</td>
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<tr>
<td></td>
<td>6</td>
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<td>5</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>9</td>
<td>15</td>
<td>12</td>
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</table>
### Example

Which one is more diverse?

<table>
<thead>
<tr>
<th></th>
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<th>2: $a \succ b \succ c$</th>
<th>2: $b \succ a \succ c$</th>
<th>2: $a \succ c \succ b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2: a \succ b \succ c$</td>
<td>3</td>
<td>6</td>
<td>3</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>$2: b \succ c \succ a$</td>
<td>6</td>
<td>6</td>
<td>5</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>$2: c \succ a \succ b$</td>
<td>12</td>
<td>9</td>
<td>15</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>2</td>
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<td>3</td>
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</tr>
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<tr>
<td></td>
<td>$3$</td>
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<td>$6$</td>
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</tr>
<tr>
<td></td>
<td>$2$</td>
<td>$3$</td>
<td>$3$</td>
<td>$2$</td>
</tr>
<tr>
<td>$4(2 + 2 + 2) = 24$</td>
<td>$9 \times 3 = 27$</td>
<td>$\frac{6}{2}(1 + 1 + 2 + 2 + 3) = 27$</td>
<td>$4(1 + 1 + 2) = 16$</td>
<td></td>
</tr>
</tbody>
</table>
Outline

1. Introduction
   - Diversity

2. Measuring Preference Diversity
   - Notation
   - Preference Diversity Orderings and Indices
   - Specific preference diversity indices

3. Axiomatic Analysis
   - Axioms
   - Results

4. Experimental Analysis
   - Diversity distribution across cultures
   - Impact on social choice-theoretic effects

5. Conclusion
Basic Definitions

**Individuals** \( N = \{1, 2, \ldots, n\} \), finite set of \( n \) individuals (voters)

**Alternatives** \( X = \{x_1, \ldots, x_m\} \), finite set of \( m \) alternatives (candidates)

**Preferences** Members of \( L(X) \) (the set of strict linear orders over \( X \))

**Profile** \( R = (R_1, \ldots, R_n) \in L(X)^n \), vector of preference orders

**Example**

For \( X = \{a, b, c\} \) and 5 voters, a possible profile is:

\[
R = (abc, abc, acb, cab, cba)
\]
Preference Diversity Orderings and Indices

PDO & PDI

**Definition (Preference diversity index)**

A *preference diversity index* (PDI) is a function \( \Delta : \mathcal{L}(\mathcal{X})^n \rightarrow \mathbb{R}^+ \cup \{0\} \), mapping profiles to the nonnegative reals, that respects \( \Delta(R, \ldots, R) = 0 \) for any \( R \in \mathcal{L}(\mathcal{X}) \).

A PDI \( \Delta \) is *normalised* if it maps any given profile to the interval \([0, 1]\), and the maximum of 1 is reached for at least one profile, i.e., \( \max \{ \Delta(R) \mid R \in \mathcal{L}(\mathcal{X})^n \} = 1 \).

**Definition (Preference diversity order)**

A *preference diversity order* (PDO) is a weak order \( \succeq \) declared on the space of preference profiles \( \mathcal{L}(\mathcal{X})^n \) that respects \( R \succeq (R, \ldots, R) \) for all \( R \in \mathcal{L}(\mathcal{X})^n \) and all \( R \in \mathcal{L}(\mathcal{X}) \).
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Specific preference diversity indices

### Definition (support-based PDI)

\[ \Delta^\ell_{\supp}^k (R) : \text{number of ordered } k\text{-tuples of alternatives occurring in at least one individual preference in profile } R. \]

\[ \Delta^\ell_{\supp}^m (R) : \text{simple support-based PDI, counts number of different preferences in } R. \]

### Definition (distance-based PDI)

\[ \Delta^{\Phi, \delta}_{\dist} (R) : \text{aggregated (e.g., } \Phi = \Sigma) \text{ distance (} \delta \text{) between all pairs of individual preferences in profile } R. \]

Kendall tau distance:

\[ K(R, R') = \frac{1}{2} \cdot |\{(x, y) \mid xRy \text{ and } yR'x\}| \]

### Definition (compromise-based PDI)

\[ \Delta^{\Phi, F}_{\com} (R) : \text{aggregated (e.g., } \Phi = \Sigma) \text{ Kendall tau distance of individual preferences in } R \text{ to a compromise preference } F(R) \text{ (e.g., } F = \text{Borda rule}). \]

### Example

\[ \Delta^\ell_{\supp}^m (abc, abc, acb, cab, cba) = 4 \]

\[ \Delta^{\Sigma, K}_{\dist} (abc, abc, acb, cab, cba) = 0 + 1 + 2 + 3 + 1 + 2 + 3 + 1 + 2 + 1 = 16 \]

\[ \Delta^{\Sigma, \text{Borda}}_{\com} (abc, abc, acb, cab, cba) = \sum_{r \in R} K(acb, r) = 1 + 1 + 0 + 1 + 2 = 5 \]
Specific preference diversity indices

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## Example

<table>
<thead>
<tr>
<th></th>
<th>2: (abc)</th>
<th>3: (abc)</th>
<th>1: (abc)</th>
<th>2: (abc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta^\ell=m) (\supp)</td>
<td>3</td>
<td>2</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>(\Delta^\ell=2) (\supp)</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>(\Delta^{\Sigma,D}) (\dist)</td>
<td>12</td>
<td>9</td>
<td>15</td>
<td>12</td>
</tr>
<tr>
<td>(\Delta^{\Sigma,K}) (\dist)</td>
<td>24</td>
<td>27</td>
<td>27</td>
<td>16</td>
</tr>
<tr>
<td>(\Delta^{\Sigma,S}) (\dist)</td>
<td>24</td>
<td>18</td>
<td>24</td>
<td>16</td>
</tr>
<tr>
<td>(\Delta^{\max,K}) (\dist)</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>
Axioms are used to evaluate/categorize methods.

PDO’s are easier to deal with analytically. The results will also apply to PDI’s indirectly.

A PDO $\succeq$ is **anonymous** if, for every permutation $\sigma : \mathcal{N} \to \mathcal{N}$, we have $(R_1, \ldots, R_n) \sim (R_{\sigma(1)}, \ldots, R_{\sigma(n)})$.

A PDO $\succeq$ is **neutral** if, for every permutation $\tau : \mathcal{X} \to \mathcal{X}$, we have $(R_1, \ldots, R_n) \sim (\tau(R_1), \ldots, \tau(R_n))$.

A PDO $\succeq$ is **strongly discernible** if no two profiles are equally diverse, unless due to anonymity and neutrality.

A PDO $\succeq$ is **weakly discernible** if $R$ being unanimous and $R'$ not being unanimous together imply $R' \succ R$.

A PDO $\succeq$ is **support-invariant** if $\text{Supp}(R) = \text{Supp}(R')$ implies $R \sim R'$.

Support-invariance $\implies$ anonymity.

A PDO $\succeq$ is **independent** if it is the case that $R \succeq R'$ if and only if $R \oplus R \succeq R' \oplus R$ for every two profiles $R, R' \in \mathcal{L}(\mathcal{X})^n$ and every preference $R \not\in \text{Supp}(R) \cup \text{Supp}(R')$. 
Theoretical results

Basic axioms are satisfied by most PDO's:

**Fact**

*Every PDO induced by a PDI of the form* $\Delta_{\text{supp}}^k$, $\Delta_{\text{dist}}^\Phi$, $\delta_{\text{com}}$, *or* $\Delta_{\text{com}}^\Phi$, *with* $k \in \{1, \ldots, m\}$, $\Phi \in \{\Sigma, \text{max}\}$, $\delta \in \{K, S, D\}$, *and* $F$ *being an anonymous and neutral social welfare function is anonymous, neutral, and weakly discernible.*

Other axioms lead to impossibilities or narrow characterisations:

**Proposition**

*For* $m > 2$ *and* $n > m!$, *no PDO can be both support-invariant and strongly discernable.*

**Proposition**

*A PDO is support-invariant, independent, and weakly discernible if and only if it is the simple support-based PDO.*
Theoretical results

Basic axioms are satisfied by most PDO’s:

**Fact**

*Every PDO induced by a PDI of the form \( \Delta_{\text{supp}}^k, \Delta_{\text{dist}}^{\Phi, \delta}, \) or \( \Delta_{\text{com}}^{\Phi, F} \) with \( k \in \{1, \ldots, m\}, \Phi \in \{\Sigma, \text{max}\}, \delta \in \{K, S, D\}, \) and \( F \) being an anonymous and neutral social welfare function is anonymous, neutral, and weakly discernible.*

Other axioms lead to impossibilities or narrow characterisations:

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**Proposition**

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### Results

#### Table of Results

<table>
<thead>
<tr>
<th></th>
<th>$\Delta_{\ell=k}^{\text{supp}}$</th>
<th>$\Delta_{\text{dist}}^{\Sigma,\delta}$</th>
<th>$\Delta_{\text{dist}}^{\text{max},\delta}$</th>
<th>$\Delta_{\text{com}}^{\Sigma,F}$</th>
<th>$\Delta_{\text{com}}^{\text{max},F}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Anonymity</strong></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td><strong>Neutrality</strong></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td><strong>Strong discernibility</strong></td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td><strong>Weak discernibility</strong></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td><strong>Support-invariance</strong></td>
<td>✓</td>
<td>X</td>
<td>✓</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td><strong>Nonlocality</strong></td>
<td>$n \leq k!$</td>
<td>✓</td>
<td>X</td>
<td>✓</td>
<td>X</td>
</tr>
<tr>
<td><strong>Independence</strong></td>
<td>$k = m$</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td><strong>Monotonicity</strong></td>
<td>✓</td>
<td>X</td>
<td>✓</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td><strong>Swap-monotonicity</strong></td>
<td>✓</td>
<td>$\delta = K$</td>
<td>$\delta = K$</td>
<td>$\Delta_{\text{com}}^{\Sigma,F} = F$ is Arrovian</td>
<td></td>
</tr>
</tbody>
</table>

- ✓: Satisfied
- X: Not satisfied
- $\Delta_{\ell=k}^{\text{supp}}$: Distance by the support
- $\Delta_{\text{dist}}^{\Sigma,\delta}$: Distance by the distribution
- $\Delta_{\text{dist}}^{\text{max},\delta}$: Maximum distance
- $\Delta_{\text{com}}^{\Sigma,F}$: Distance by the composite function
- $\Delta_{\text{com}}^{\text{max},F}$: Maximum distance by the composite function
Experimental analysis

- Compare diversity of synthetic vs. real preference profiles
  - Impartial Culture assumption (IC): every possible profile is equally likely to occur
  - Course selection dataset (AGH): complete preferences of 153 students over 7 courses

- Relation between diversity and social choice-theoretic properties
  - Condorcet winner/cycle
  - agreement between voting rules
  - voter satisfaction

All profiles are preferences of 50 voters over 5 alternatives.

For each experiment we have drawn 1 million profiles from the relevant distribution.

Note that the number of all possible distinct profiles is: \((5!)^{50} > 10^{100}\)
Diversity distribution across cultures

Preference diversity (x-axis) against frequency (y-axis) in IC and AGH. \([n = 50, m = 5]\)

<table>
<thead>
<tr>
<th>PDI</th>
<th>IC</th>
<th>AGH</th>
<th>PDI</th>
<th>IC</th>
<th>AGH</th>
<th>PDI</th>
<th>IC</th>
<th>AGH</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta \ell = m)</td>
<td>22</td>
<td>13</td>
<td>(\Delta \Sigma, \text{com} )</td>
<td>84</td>
<td>85</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Delta \ell = 2)</td>
<td>1</td>
<td>2</td>
<td>(\Delta \Sigma, \text{dist} )</td>
<td>462</td>
<td>1170</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Delta \ell = 3)</td>
<td>4</td>
<td>12</td>
<td>(\Delta \Sigma, \text{dist} )</td>
<td>660</td>
<td>1561</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Observed number of levels \((n = 50, m = 5)\)
Impact on social choice-theoretic effects

As diversity increases:

- the probability of encountering Condorcet cycles (winners) increases (decreases)
- average degree of agreement decreases
  - degree of agreement: \( \frac{|W_1 \cap W_2|}{|W_1| \times |W_2|} \)
  - plurality rule has much more disagreement with other rules and it becomes worse as diversity increases
- average voter satisfaction decreases
  - voter satisfaction: number of alternatives below the (Borda) winner in the voter's preference
  - normalised to percent: average value is in the range of 50% – 100%
Conclusion

- Preference diversity
  - Concept
  - Formal model
  - Axioms
  - Experiments
    - support our intuition/expectation
Future work

- Other options for measuring diversity
  - other distances and other aggregation operators (e.g., max-of-min)
  - for a given $\ell$, maximum number of preferences with a common subpreference of length $\ell$
  - for a given $k$, maximum length of a common subpreference of any $k$ preferences
  - covering distance of the profile: how close a profile is to covering the full space of possibilities
  - measuring the distance from a single-peaked profile

- Normalization
  - Ratio
  - Percentile
  - Levels

- New axioms
Future work

- Distinguish (real data) profiles
  - Objective
  - Subjective

- Structure of profiles
  - Polarized/Divided
  - Central