Propositional Opinion Diffusion

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[Joint work with Emiliano Lorini and Laurent Perrussel]

Have you ever had an opinion?

What should
we do with Putin?

Have you ever had an opinion?

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Relax Angela!
Let's take a selfie first

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What should
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Relax Angela!
Let's take a selfie first

Have you ever had an opinion?

Remember you are
a Nobel laureate
for peace

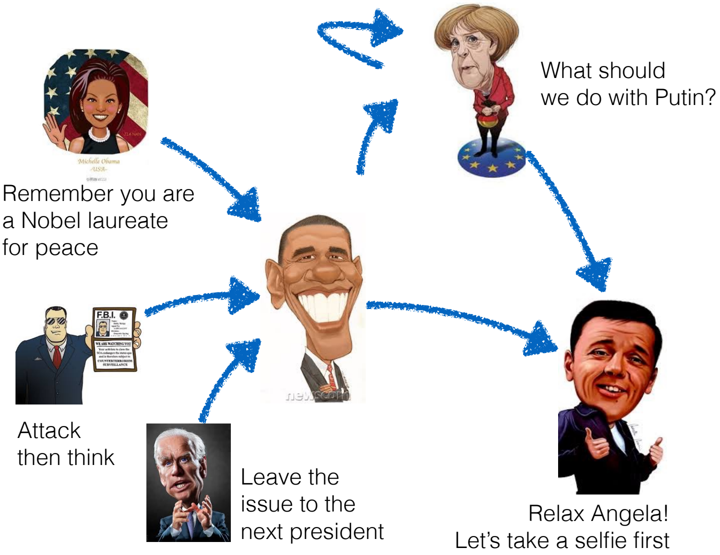
What should
we do with Putin?

Attack
then think

Leave the
issue to the
next president

Relax Angela!
Let's take a selfie first

Have you ever had an opinion?



Opinion diffusion in the literature

Our work is grounded on two research agendas:

1. Opinion diffusion and formation – Social sciences/social network analysis
   - Continuous opinions on a given issue
   - Opinion diffusion as a linear combination of the influencers' opinions
   - Difference: Qualitative study rather than quantitative
   - Difference: We are not obsessed with consensus
2. Formal models of influence – Game theory
   - Extract the influence structure from observed choices
   - Decision restricted to a single binary issue
   - Difference: multi-issue decisions on a given network

De Groot. Reaching a consensus. *Journal of the American Statistical Association*, 1974

Grabisch and Rusinowska. A model of influence in a social network. *Theory and Decision*, 2010.

Opinion diffusion as aggregation

We propose a model of opinion diffusion based on aggregation methods:

- Opinions as 0/1 vectors of binary views
- Iterative revision of individual opinions on a network
- New opinion as the aggregation of neighbours' opinions

This model could be used for:

- Predicting the diffusion of opinions, such as preferences on products or candidates in social networks
- Mechanism design: what is the best network structure to obtain convergence (not consensus!)
- Strategic reasoning in social interaction

Outline

1. A quick summary of related work (done)
2. Basic definitions: opinions, influence network, aggregation
3. Convergence results and algorithms
   - General result
   - Unanimous diffusion
   - Majoritarian diffusion
4. Conclusions and perspectives

Basic definitions I: Opinions

We study the diffusion on individual opinions:

- $\mathcal{I} = \{1, \ldots, m\}$ a finite set of issues
- $\mathcal{N} = \{1, \ldots, n\}$ a finite set of individuals
- An opinion is a yes/no evaluation of the issues $B_i : \mathcal{I} \to \{0, 1\}$

A constraint $\mathrm{IC} \in \mathcal{L}_{PS}$ can be added to model logically related issues.

What is an opinion?

A public expression of an agent's view, like a preference, a judgment, a choice... It is not a belief or an intention, but rather the expression of it.

Example

Take preference "I like Nikon more than Canon, Sony is the worst":

- *Issues:* p_{nc} *for "I prefer Nikon to Canon",* p_{ns} *and* p_{cs} *accordingly*
- *Integrity constraint:* $p_{nc} \wedge p_{cs} \rightarrow p_{ns}$*, repeated for all pairs of alternatives*
- *My opinion is* $B = (1, 1, 1)$

Basic definitions II: Influence network and aggregation procedures

Individuals are related to each other:

- An influence network $E \subseteq \mathcal{N} \times \mathcal{N}$
- If $(i, j) \in E$ then agent i influences agent j
- The network is directed!

Opinion diffusion as aggregation

The opinion of an agent i is the aggregated opinion of its influencers $Inf(i) = \{j \in N \mid (j, i) \in E\}$.

Several aggregation procedures for binary issues exist:

- The *majority rule*: accept issue i if a (strict) majority accept it
- The *unanimity rule*: take opinion B if all influencers have opinion B
- *Distance-based rules* (future work)

Basic definitions III: The iterative process

Let us sum up all the ingredients:

- F_i is an aggregation procedure
- $B_i^t \in \{0,1\}^{\mathcal{I}}$ is the opinion of agent i at time t
- $\boldsymbol{B}^t = (B_1^t, \dots, B_n^t)$ is the profile of individual opinions at time t

Propositional opinion diffusion (POD)

Consider the following iterative process:

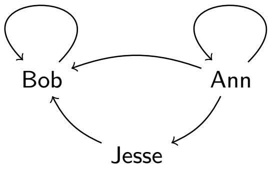
$$B_i^t = \begin{cases} B_i^{t-1} & \textit{if } Inf(i) = \emptyset \\ F_i(\boldsymbol{B}^{t-1}_{Inf(i)}) & \textit{otherwise} \end{cases}$$

Where $\boldsymbol{B}^{t-1}_{Inf(i)}$ is $\boldsymbol{B}^{t-1}$ restricted to the set $Inf(i)$ of influencers of agent i.

If all individuals use the same aggregator we call the process uniform-POD.

A classical example revisited

An influence network between Ann, Bob and Jesse:



The three agents need to decide whether to approve the building of a swimming pool (first issue) and a tennis court (second issue) in the residence where they live. Here are their initial opinions and their evolution following POD using the majority rule:

Initial opinions	Profile $\boldsymbol{B}^1$	Profile $\boldsymbol{B}^2$
$B_A^0 = (0,1)$	$B_A^1 = (0,1)$	$B_A^2 = (0,1)$
$B_B^0 = (0,0)$	$B_B^1 = (0,0)$	$B_B^2 = (0,1)$
$B_J^0 = (1,0)$	$B_J^1 = (0,1)$	$B_J^2 = (0,1)$

Properties of aggregators

Not all aggregation procedures make sense!

We do not consider negative influence (doing the opposite of some influencers):

Ballot-Monotonicity: for all profiles $\boldsymbol{B} = (B_1, \ldots, B_n)$, if $F(\boldsymbol{B}) = B^*$ then for any $1 \leq i \leq n$ we have that $F(\boldsymbol{B}_{-i}, B^*) = B^*$.

And black sheep change their mind:

Unanimity: for all profiles $\boldsymbol{B} = (B_1, \ldots, B_n)$, if $B_i = B$ for all $1 \leq i \leq n$ then $F(\boldsymbol{B}) = B$.

The majority rule, the unanimity rule, (some) distance-based procedures all satisfy ballot-monotonicity and unanimity.

What are we looking for?
Convergence, not consensus

We look for properties of the network structure (=classes of graphs) that guarantee the convergence of POD on any vector of initial opinions:

Definition

Given a class of graphs $\mathcal{E} \subseteq 2^{E^2}$, we say that POD converges on $\mathcal{E}$ if for all graphs $E \in \mathcal{E}$ and for all profiles of initial opinions $\boldsymbol{B}^0 \in X^{\mathcal{N}}$ there is a convergence time $\bar{t} \in \mathbb{N}$ such that $\boldsymbol{B}^t = \boldsymbol{B}^{\bar{t}}$ for all $t \geq \bar{t}$.

A very simple case: Uniform POD on complete graphs

What happens if everybody is influenced by everybody (including themselves) and the aggregation rule for uniform-POD is unanimous?

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Theorem (simple)

If F is unanimous, then uniform-POD converges on the class of complete graphs in two steps.

Proof. For all individuals i the set of $Inf(i) = \mathcal{N}$. Hence at step 1 $B_i = F(B_1, \ldots, B_n)$ for all $i \in \mathcal{N}$ (all individuals have the same opinion). Since the rule is unanimous, from step 2 onwards the result will not change.

General convergence result

A directed-acyclic graph (DAG) with loops is a directed graphs that does not contain cylcles involving more than one node.

Theorem

If F_i satisfies ballot-monotonicity for all $i \in \mathcal{N}$, then POD converges on the class of DAG with loops after at most diam$(E) + 1$ number of steps.

Proof. Start from the sources and propagate opinions.

Observations:

- The proof gives us an algorithm to compute the result at convergence in a number of steps bounded by the diameter of the graph
- The theorem is not easy to generalise: take the example of a circle
- The theorem works for any aggregator F_i, even if they are all different

The unanimous case

Suppose an individual changes her mind only if all her influencers agree on it:

Theorem – unanimous POD

Let E be an influence network without loops such that

- *all cycles contained in E are vertex-disjoint*
- *for each cycle in E, there exists $i \in \mathcal{N}$ belonging to the cycle such that $|Inf(i)| \geqslant 2$, i.e., it has at least one external influencer*

Then U-POD converges on E after at most $|\mathcal{N}|$ steps.

Open question: what is a qualitative version of the small-world assumption?

The majoritarian case

Suppose that an individual changes her mind on a single issue if a majority of her influencers agree on it:

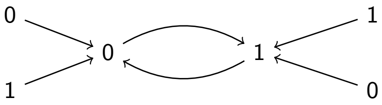
Theorem – majoritarian POD

Let E be an influence network such that

- *all cycles contained in E are vertex-disjoint*
- *if a node i belongs to a cycle, then $|Inf(i)|$ is of even cardinality*

Then maj-POD converges on E after at most $|\mathcal{N}|$ steps.

Not an easy condition to relax:



Static computation of majoritarian POD

We found a closed form to compute the result of majoritarian POD as a "linear combination" of the initial opinion of the sources:

Theorem

Let E be a resolute DAG (=every node has an odd number of influencers) and let $\boldsymbol{B}^*$ be the opinion profile at convergence of maj-POD . Then:

$$B_i^* = maj(\alpha(s_1, i)B_{s_1}^0, \ldots, \alpha(s_m, i)B_{s_m}^0)$$

Where $s_1, \ldots, s_m$ are the sources of E, and $\alpha(s_j, i)$ is the sum over all paths from s_j to i, of the products of the degrees of nodes outside the path (almost).

Two observations:

- A polynomial algorithm for the computation of *maj-POD*
- An interesting measure of influence of a source node

Algorithmic summary

We showed algorithms for the computation of POD at convergence:

Aggregation	Class of graphs	Time bound
Any aggregator	DAG with loops	$\textit{diam}(E) \times \textit{Time}(F)$
Unanimity rule	No loops, disjoint cycles, $\lvert \mathit{Inf}(i) \rvert > 1$ for one node	$O(n^2 m)$
Majority rule	Disjoint cycles $\lvert \mathit{Inf}(i) \rvert$ even on cycles	$O(n^2 m)$
Majority rule	Resolute DAG	$O(k(n+m))$

Where $n = \lvert \mathcal{N} \rvert$ is the number of individuals, $m = \lvert E \rvert$ the number of arcs in the network, and k is the number of sources of E.

Conclusions and perspectives

We proposed a model of opinion diffusion through aggregation:

- Iterative process with discrete time
- Opinion update as the aggregation of the influencers' opinions
- Characterisations of convergence on classes of graphs
- Tractable algorithms for the computation of opinions at convergence

Many avenues for future work:

- Deal with uncertainty: study belief propagation with more realistic models of knowledge/belief
- Study the effect of opinion diffusion on collective decisions: comparison with voter models
- Qualitative social network analysis: what is a qualitative version of the small-world assumption?

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Thank you for your attention!

U. Grandi, E. Lorini, L. Perrussel. Propositional Opinion Diffusion. Proceedings of AAMAS-2015.