We discuss, then we decide: Reliability based preference change

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which restaurant to go ?

Alan

Barbara

Chiara

who is the better candidate ?

Eco

Stat

Stat

CS

what this talk is not about !

- dialogues, speech acts, argumentation
- aggregation of preferences
- interplay of knowledge, beliefs and preferences

what is this talk about ?

- modeling discussions in an implicit way
  - public announcement of preference orderings
  - changing of preferences based on some intuitive policies
  - effect of reliability of agents
- decision making
  - attaining unanimity
  - attaining stability

disclaimers !

- ordering assumptions
- preference vs. reliability
- more questions than answers

the semantic model

Let $\mathtt{A}$ be a set of agents.

*A **preference & reliability (PR) frame** is a tuple* $F = \langle W, \{\leq_i, \preccurlyeq_i\}_{i \in \mathtt{A}} \rangle$ *where*

- $W \neq \emptyset$ *is a set of* ***possible worlds***,
- $\leq_i \subseteq (W \times W)$, *a total preorder, is agent* i*'s* ***preference relation*** *over worlds,*
- $\preccurlyeq_i \subseteq (\mathtt{A} \times \mathtt{A})$, *a total order, is agent* i*'s* ***reliability relation*** *over agents.*

$w \leq_i u$: ***" for agent*** i***, world*** w ***is at least as preferable as world*** u***"***

$j \preccurlyeq_i j'$ ***" for agent*** i***, agent*** j' ***is at least as reliable than agent*** j***"***

$\{a, b, c\}$

$\{❶, ❷, ❸, ❹\}$

$\leq_a: \{❶\} \to \{❸\} \to \{❷\} \to \{❹\}$ $\quad \preccurlyeq_a: a \to b \to c$

$\leq_b: \{❶\} \to \{❹, ❷\} \to \{❸\}$ $\quad \preccurlyeq_b: b \to c \to a$

$\leq_c: \{❶\} \to \{❷\} \to \{❸, ❹\}$ $\quad \preccurlyeq_c: a \to b \to c$

more on preference and reliability

Given a **PR** frame $F = \langle W, \{\leq_i, \preccurlyeq_i\}_{i \in \mathtt{A}}\rangle$, define

- i's 'strictly less preferable' relation:

$$w <_i u \quad \textbf{iff}_{def} \quad w \leq_i u \text{ and } u \not\leq_i w$$

- i's 'equally preferable' relation:

$$w \simeq_i u \quad \textbf{iff}_{def} \quad w \leq_i u \text{ and } u \leq_i w$$

- i's most preferred worlds in a set $U \subseteq W$:

$$\text{Max}_i(U) := \{v \in U \mid u \leq_i v \text{ for every } u \in U\}$$

- i's most reliable agent:

$$\text{mr}(i) = j \quad \textbf{iff}_{def} \quad j' \preccurlyeq_i j \text{ for every } j' \in \mathtt{A}$$

possible notions of upgrade

1. **Drastic upgrade**:

   $w \leq'_i u$ iff$_{def}$ $w \leq_{\mathrm{mr}(i)} u$

2. **Radical upgrade**:

   $w \leq'_i u$ iff$_{def}$ $(w <_{\mathrm{mr}(i)} u)$ or $(w \simeq_{\mathrm{mr}(i)} u$ and $w \leq_i u)$

3. **Conservative upgrade**

   $w \leq'_i u$ iff$_{def}$ $(\{w,u\} \cap \mathrm{Max}_{mr(i)}(W) = \{w,u\}$ and $w \leq_i u)$ or $(\{w,u\} \cap \mathrm{Max}_{mr(i)}(W) = \{u\})$
   or $(\{w,u\} \cap \mathrm{Max}_{mr(i)}(W) = \varnothing$ and $w \leq_i u)$

4. **Tie-breaker upgrade**:

   $w \leq'_i u$ iff$_{def}$ $(w <_i u)$ or $(w \simeq_i u$ and $w \leq_{\mathrm{mr}(i)} u)$

general lexicographic upgrade

- *A **lexicographic list** $\mathcal{R}$ over W is a finite non-empty list whose elements are indexes of preference orderings over W ($\mathcal{R}[1]$ has the highest priority).*

- *Given $\mathcal{R}$, define $\leq_{\mathcal{R}} \subseteq (W \times W)$ as*

$$w \leq_{\mathcal{R}} u \quad \textit{iff}_{def} \quad \underbrace{\left(w \leq_{\mathcal{R}[|\mathcal{R}|]} u \wedge \bigwedge_{k=1}^{|\mathcal{R}|-1} w \simeq_{\mathcal{R}[k]} u\right)}_{1} \vee \underbrace{\bigvee_{k=1}^{|\mathcal{R}|-1} \left(w <_{\mathcal{R}[k]} u \wedge \bigwedge_{\ell=1}^{k-1} w \simeq_{\mathcal{R}[\ell]} u\right)}_{2}$$

key facts

- the **general lexicographic** upgrade **generalizes** the **drastic**, **radical** and **tie breaker** upgrades

- the **general lexicographic** upgrade **preserves** reflexivity, transitivity, antisymmetry, totality and ‘disconnectedness’

- the **conservative** upgrade **is not** an instance of **general lexicographic** upgrade

which restaurant to go ?

Alan

Barbara

Chiara

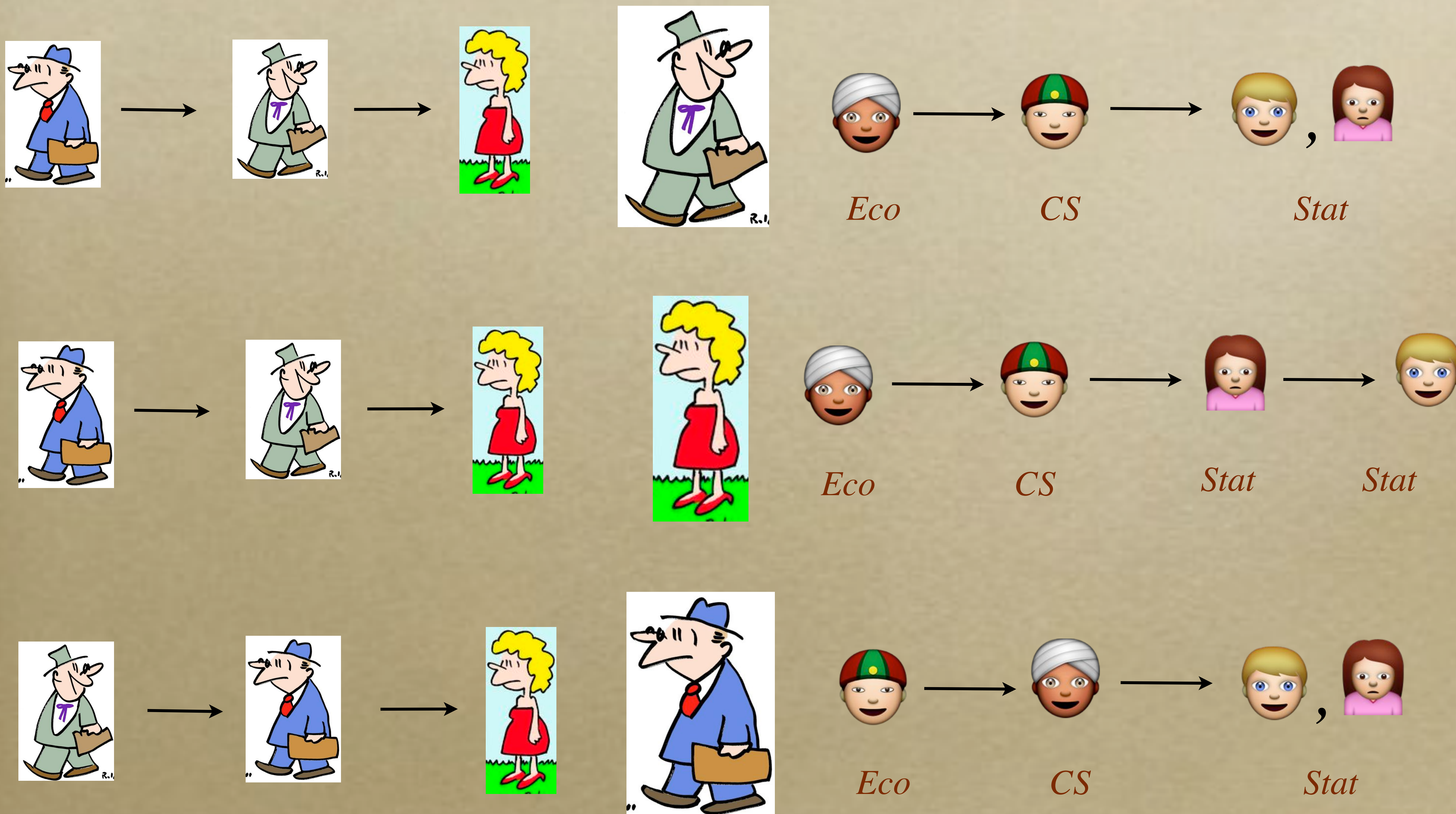
upgrading preferences

Alan

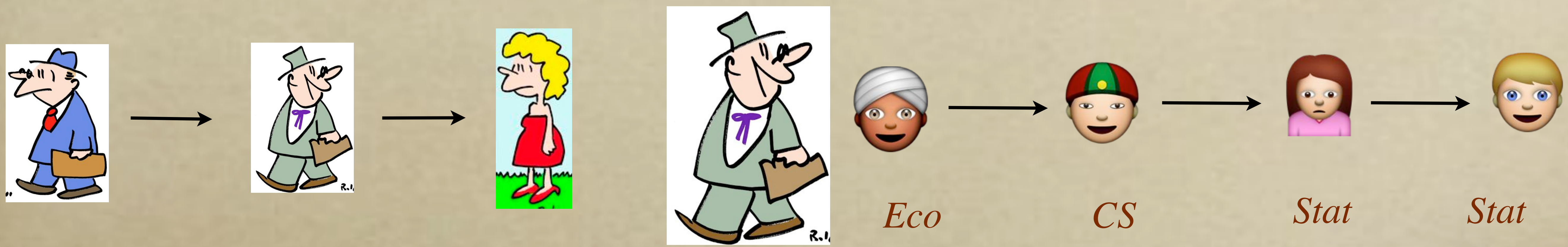
Barbara

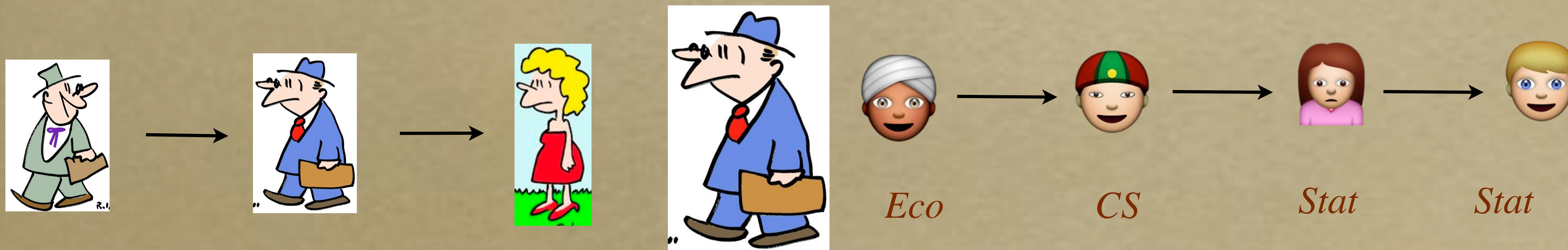
Chiara

who is the better candidate ?

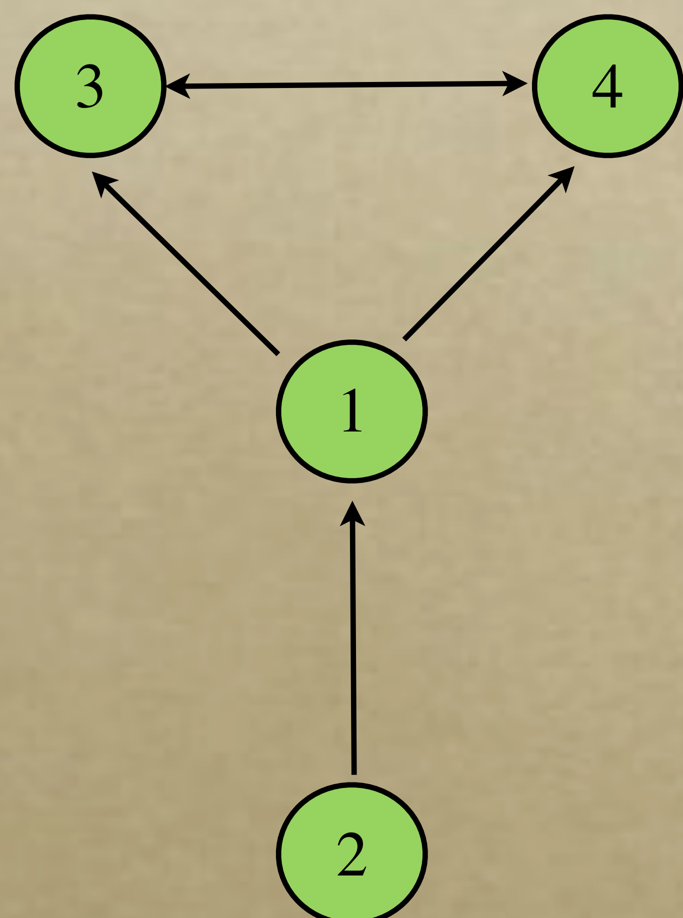


upgrading preferences

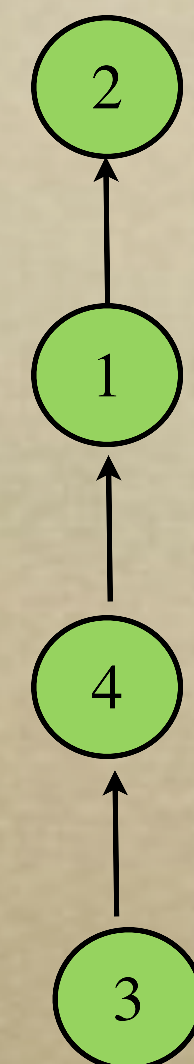




conservative upgrade

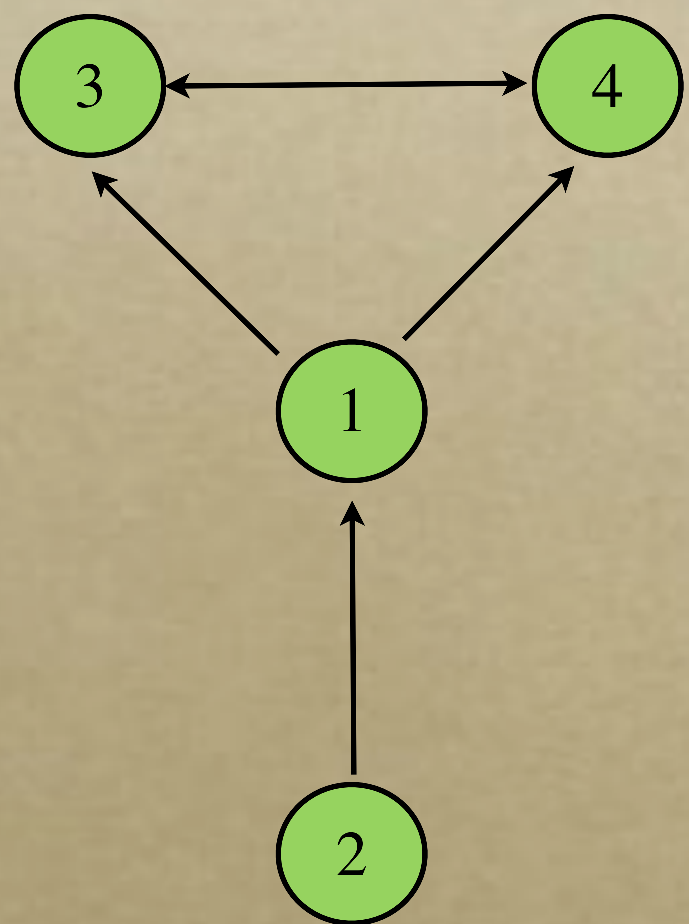


$b : b \longrightarrow a$



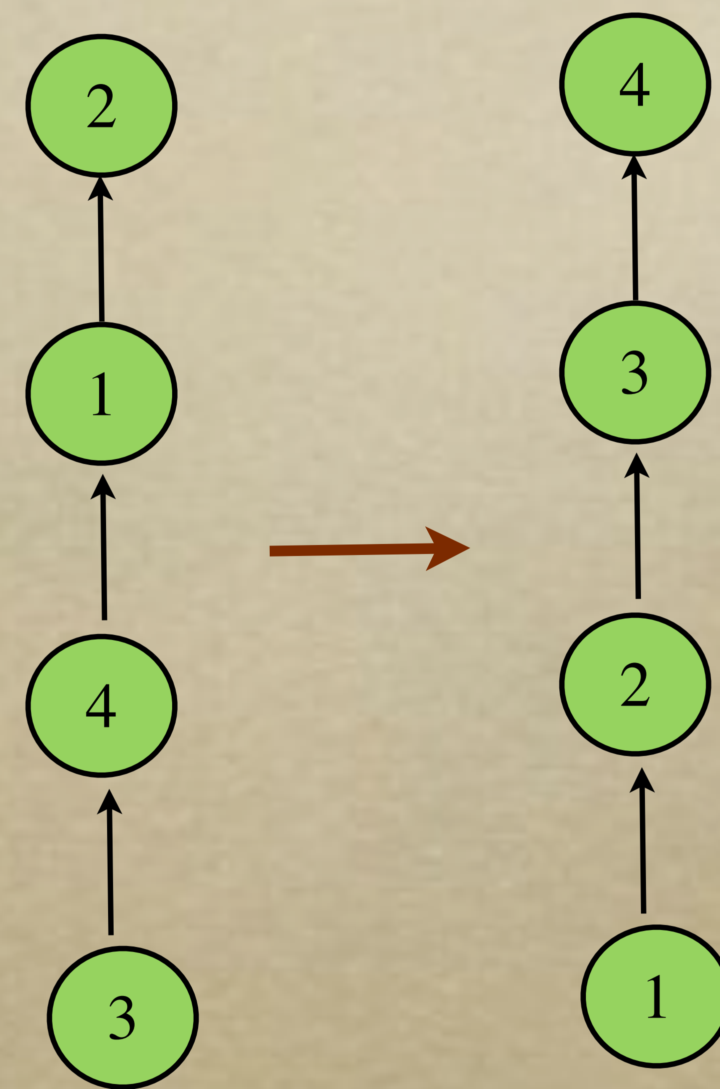
a *b*

conservative upgrade



a

$b : b \longrightarrow a$



b b

general layered upgrade

- *A* ***layered list*** $\mathcal{S}$ *over* W *is a finite (possibly empty) list of pairwise disjoint subsets of* W *together with the index of a preference ordering over* W *(*$\mathcal{S}[1]$ *has the highest priority).*

- *Given* $\mathcal{S}$*, define* $\leq_{\mathcal{S}} \subseteq (W \times W)$ *as*

$$w \leq_{\mathcal{S}} u \quad \textit{iff}_{def} \quad \underbrace{\left(w \leq_{\mathcal{S}_{\mathrm{Def}}} u \wedge \left(\{w,u\} \cap \bigcup_{k=1}^{|\mathcal{S}|} \mathcal{S}[k] = \varnothing \vee \bigvee_{k=1}^{|\mathcal{S}|} \{w,u\} \subseteq \mathcal{S}[k]\right)\right)}_{1}$$
$$\vee \quad \underbrace{\bigvee_{k=1}^{|\mathcal{S}|} \left(u \in \mathcal{S}[k] \wedge w \notin \bigcup_{\ell=1}^{k} \mathcal{S}[\ell]\right)}_{2}$$

key facts

- the **general layered** upgrade **generalizes** the **conservative** upgrade mentioned earlier
- the **general layered** upgrade **preserves** reflexivity, transitivity, antisymmetry, totality and (under an extra condition) ‘disconnectedness’
- under totality, any ordering generated by a **general lexicographic** upgrade can be generated by a **general layered** upgrade, but in general this is not the case.

general lexicolayered upgrade

- *A **lexicolayered list** $\mathcal{RS}$ over W is a finite non-empty list whose elements are layered lists over W ($\mathcal{RS}[1]$ has the highest priority).*
- *Given $\mathcal{RS}$, define $\leq_{\mathcal{RS}} \subseteq (W \times W)$ as*

$$w \leq_{\mathcal{RS}} u \quad \textit{iff}_{def} \quad \underbrace{\left(w \leq_{\mathcal{RS}[\,|\mathcal{RS}|\,]} u \wedge \bigwedge_{k=1}^{|\mathcal{RS}|-1} w \simeq_{\mathcal{RS}[k]} u\right)}_{1} \vee \underbrace{\bigvee_{k=1}^{|\mathcal{RS}|-1} \left(w <_{\mathcal{RS}[k]} u \wedge \bigwedge_{\ell=1}^{k-1} w \simeq_{\mathcal{RS}[\ell]} u\right)}_{2}$$

key facts

- the **general lexicolayered** upgrade **generalizes** both **general lexicographic** upgrade and **general layered** upgrade
- the **general lexicolayered** upgrade **preserves** reflexivity, transitivity, antisymmetry, totality and (under an extra condition) ‘disconnectedness’

from frames to models

Let $\mathbf{P}$ be a set of atomic propositions.

A **PR** *model is a tuple* $M = \langle W, \{\leq_i, \preccurlyeq_i\}_{i \in \mathsf{A}}, V\rangle$ *where*

- $\langle W, \{\leq_i, \preccurlyeq_i\}_{i \in \mathsf{A}}\rangle$ *is a* **PR** *frame,*
- $V : \mathbf{P} \to \wp(W)$ *is an* ***atomic valuation****.*

The pair (M, w) with $w \in W$ is a **PR** *state*.

the static language

Formulas ($\varphi, \psi, \ldots$) *and relational expressions* ($\pi, \sigma, \ldots$) *in* $\mathcal{L}$ *are given, respectively, by*

$$\varphi, \psi ::= p \mid j \sqsubseteq_i j' \mid \neg\varphi \mid \varphi \vee \psi \mid \langle\pi\rangle\, \varphi$$
$$\pi, \sigma ::= 1 \mid \leq_i \mid \geq_i \mid ?(\varphi, \psi) \mid -\pi \mid \pi \cup \sigma \mid \pi \cap \sigma$$

where $p \in \mathtt{P}$ *and* $i, j, j' \in \mathtt{A}$.

Define

- the constants $\top, \bot$ and the connectives $\wedge, \rightarrow, \leftrightarrow$ as usual.
- for every π, the modal operator $[\pi]$ as usual:

$$[\pi]\,\varphi := \neg\langle\pi\rangle\,\neg\varphi$$

- for every π, the modal operator $\boxed{\pi}$ (the'window' operator) as:

$$\boxed{\pi}\,\varphi := [-\pi]\,\neg\varphi$$

the semantic interpretation

Let (M, w) *be a* **PR** *state with* $M = \langle W, \{\leq_i, \preccurlyeq_i\}_{i \in \mathtt{A}}, V \rangle$. *Define, simultaneously for every* φ *and every* π, *the* ***satisfaction relation*** $\Vdash \subseteq$ (*'states'* $\times$ *'formulas'*) *and the* ***relation*** $R_\pi \subseteq (W \times W)$ *as*

$$
\begin{array}{lll}
(M, w) \Vdash p & \text{iff} & w \in V(p) \\
(M, w) \Vdash j \sqsubseteq_i j' & \text{iff} & j \preccurlyeq_i j' \\
(M, w) \Vdash \neg \varphi & \text{iff} & (M, w) \nVdash \varphi \\
(M, w) \Vdash \varphi \vee \psi & \text{iff} & (M, w) \Vdash \varphi \text{ or } (M, w) \Vdash \psi \\
(M, w) \Vdash \langle \pi \rangle \varphi & \text{iff} & \text{there is } u \in W \text{ such that } R_\pi wu \text{ and } (M, u) \Vdash \varphi
\end{array}
$$

and

$$
\begin{array}{rcl}
R_1 & := & W \times W \\
R_{\leq_i} & := & \leq_i \\
R_{\geq_i} & := & \{(u, w) \mid w \leq_i u\} \\
R_{?(\varphi, \psi)} & := & \{(w, u) \mid (M, w) \Vdash \varphi \text{ and } (M, u) \Vdash \psi\}
\end{array}
\qquad
\begin{array}{rcl}
R_{-\pi} & := & (W \times W) \setminus R_\pi \\
R_{\pi \cup \sigma} & := & R_\pi \cup R_\sigma \\
R_{\pi \cap \sigma} & := & R_\pi \cap R_\sigma
\end{array}
$$

observe how ...

$(M, w) \Vdash \langle 1 \rangle\, \varphi$	iff	there is $u \in W$ such that $(M, u) \Vdash \varphi$
$(M, w) \Vdash [\pi]\, \varphi$	iff	for every $u \in W$, $R_\pi wu$ implies $(M, u) \Vdash \varphi$
$(M, w) \Vdash \boxed{\pi}\, \varphi$	iff	for every $u \in W$, $(M, u) \Vdash \varphi$ implies $R_\pi wu$

the dynamic language

Language $\mathcal{L}_{\{\mathrm{fx},\mathrm{fy},\mathrm{fxy}\}}$ *extends* $\mathcal{L}$ *with modalities* $\langle \mathrm{fx}^i_{\mathcal{R}} \rangle$, $\langle \mathrm{fy}^i_{\mathcal{S}} \rangle$ *and* $\langle \mathrm{fxy}^i_{\mathcal{RS}} \rangle$ *for every lexicographic list* $\mathcal{R}$*, layered list* $\mathcal{S}$*, lexicolayered list* $\mathcal{RS}$ *and every agent* $i \in \mathtt{A}$*. Given a* **PR** *state* (M, w)*,*

$$
\begin{array}{lcl}
(M,w) \Vdash \langle \mathrm{fx}^i_{\mathcal{R}} \rangle \varphi & \textit{iff} & \left(fx^i_{\mathcal{R}}(M), w\right) \Vdash \varphi \\
(M,w) \Vdash \langle \mathrm{fy}^i_{\mathcal{S}} \rangle \varphi & \textit{iff} & \left(fy^i_{\mathcal{S}}(M), w\right) \Vdash \varphi \\
(M,w) \Vdash \langle \mathrm{fxy}^i_{\mathcal{RS}} \rangle \varphi & \textit{iff} & \left(fxy^i_{\mathcal{RS}}(M), w\right) \Vdash \varphi
\end{array}
$$

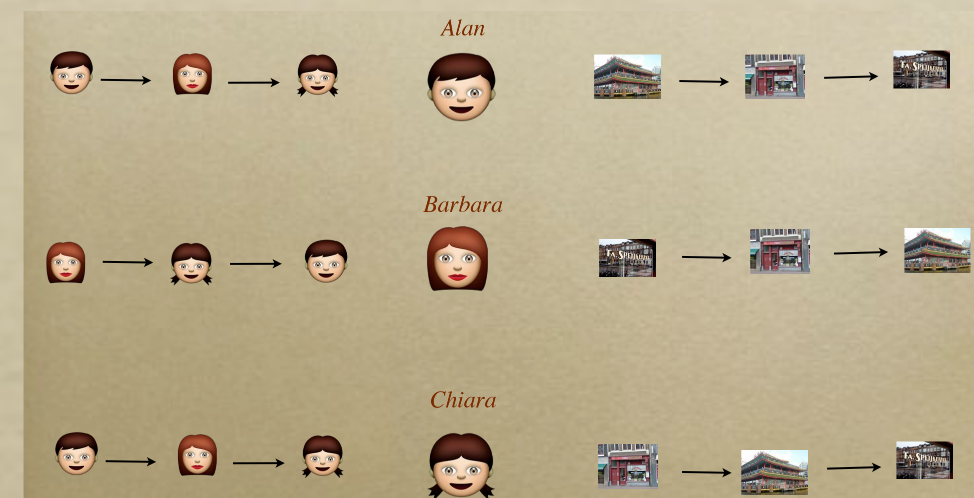
where

- *the* **PR** *model* $fx^i_{\mathcal{R}}(M)$ *is exactly as* M *except in* $\leq_i$*, which is now given by* $\leq_{\mathcal{R}}$*,*
- *the* **PR** *model* $fy^i_{\mathcal{S}}(M)$ *is exactly as* M *except in* $\leq_i$*, which is now given by* $\leq_{\mathcal{S}}$*.*
- *the* **PR** *model* $fxy^i_{\mathcal{RS}}(M)$ *is exactly as* M *except in* $\leq_i$*, which is now given by* $\leq_{\mathcal{RS}}$*.*

expressing the restaurant situation

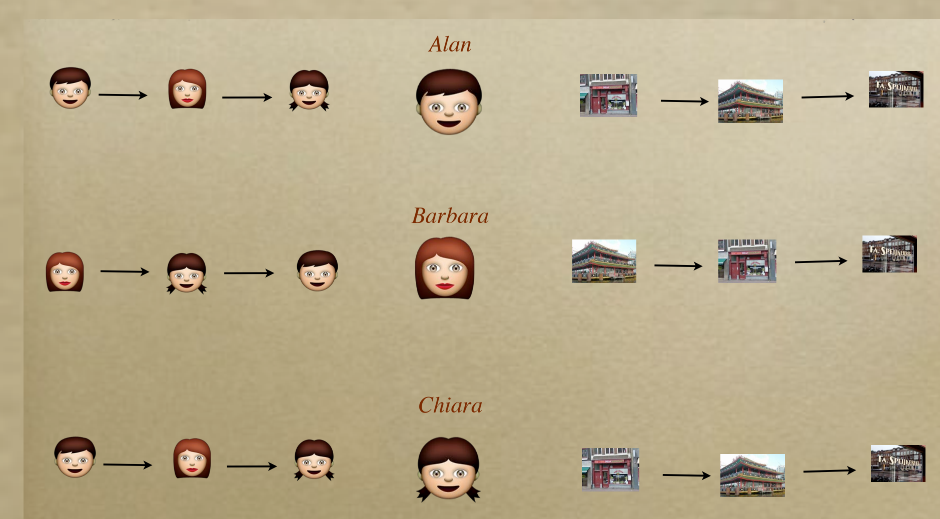
$\mathbf{M} \models \langle \leq_{\mathsf{Barbara}} \rangle$ 

$\mathbf{M}$



$\mathbf{M} \models \langle \mathrm{fx}_R \rangle \langle \leq_{\mathsf{Barbara}} \rangle$ 

$\mathbf{M}'$



unanimity and stability

Let $\mathbf{F} = \langle W, \{\leq_i, \preccurlyeq_i\}_{i \in \mathtt{A}} \rangle$ *be a* **PR** *frame and* $\mathbf{B} = \{a_1, \ldots, a_m\} \subseteq \mathtt{A}$ *a set of agents.*

- *There is* ***unanimity*** *among agents in* **B** *at* **F** *when*

$$\leq_{a_1} = \cdots = \leq_{a_m}$$

- *There is* ***stability*** *among agents in* **B** *at* **F** ***under a given preference upgrade policy f*** *when*

$$F_\gamma|_B = F_{\gamma+1}|_B \quad \text{for every } \gamma \geq 1$$

with $\mathbf{F_1} := \mathbf{F}$ *and* $\mathbf{F_{\gamma+1}} := f(\mathbf{F_{\gamma+1}})$.

simple general results

- under **general layered** upgrade, **unanimity** does **not** imply **stability**
- under **general lexicographic** upgrade, **unanimity implies stability**

the drastic upgrade case

Drastic upgrade: $w \leq'_i u$ **iff**$_{def}$ $w \leq_{\mathrm{mr}(i)} u$.

Let $F = \langle W, \{\leq_i, \preccurlyeq_i\}_{i \in \mathtt{A}} \rangle$ *be a* **PR** *frame; let* i *be an agent. An* i ***reliability stream from*** F *is a function* $\alpha_i : \mathbb{N} \to \mathtt{A}$ *given by*

$$\alpha_i[0] := i$$
$$\alpha_i[\ell + 1] := \mathrm{mr}(\alpha_i[\ell]) \quad \text{for every } \ell \geq 0$$

Let $F = \langle W, \{\leq_i, \preccurlyeq_i\}_{i \in \mathtt{A}} \rangle$ *be a* **PR** *frame where the* $\leq_i$ *are all different. The iterative application of drastic upgrade over the agents' individual preference starting from* F *reaches unanimity (and hence stability) if and only if*

$$\text{there is } \ell \in \mathbb{N} \text{ such that } \alpha_{a_1}[\ell] = \cdots = \alpha_{a_n}[\ell]$$

with α_a *agent* a*'s reliability stream from* F.

the lexicographic upgrade case

Lexicographic upgrade: if agent i's reliability ordering is given by $a_1 \preceq_i \cdots \preceq_i a_n$, then

$$w \leq'_i u \quad \textbf{iff}_{def} \quad (w <_{a_n} u) \text{ or } (w \simeq_{a_n} u \text{ and } w <_{a_{n-1}} u) \text{ or } \cdots$$
$$\text{or } (w \simeq_{a_n} u \text{ and } \cdots \text{ and } w \simeq_{a_2} u \text{ and } w \leq_{a_1} u)$$

Let $F = \langle W, \{\leq_i, \preccurlyeq_i\}_{i \in A} \rangle$ be a **PR** *frame; let $F' = \langle W, \{\leq'_i, \preccurlyeq_i\}_{i \in A} \rangle$ be the result of lexicographic upgrades at F. If $u \simeq'_j v$ for some agent $j \in A$, then such 'tie' will not be broken by further applications of such upgrade.*

After applying the lexicographic upgrade once, further applications behave exactly as the drastic upgrade.

which restaurant to go : original situation

Alan

Barbara

Chiara

which restaurant to go : upgrading once

Alan

Barbara

Chiara

which restaurant to go : upgrading twice

Alan

Barbara

Chiara

conclusion

- preference and reliability models
- preference upgrades based on reliability
- logical language to express these notions
- unanimity and stability

future work

- characterizing unanimity and stability
- weakening the relational properties
- reliability dynamics
- knowledge - belief - manipulation
- combining deliberative and aggregative perspectives

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What can we logicians offer ?