Mark Allen Satterthwaite:

"Strategy-Proofness and Arrow's Conditions: Existence and Correspondence Theorems for Voting Procedures and Social Welfare Functions"

(in: *Journal of Economic Theory*, vol. 10, pp. 187-217 (1975))

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Overview of the presentation

1. Biographical sketch of the author
2. Terminology & key concepts
3. Proof of the existence theorem for strategy-proof strict voting procedures
4. Discussion pointers

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Mark (Allen) Satterthwaite

- Academic career:

 - 1973: PhD in Economics from University of Wisconsin, Madison
 - since then: faculty at Kellog School of Management, Northwestern University

- Areas of expertise:

competition in healthcare, healthcare management, strategy, voting systems

- “Strategy-Proofness and Arrow’s Conditions”:

- originated from his PhD thesis: *The Existence of a Strategy Proof Voting Procedure: A Topic in Social Choice Theory* (1973);
 => **Gibbard-Sattherthwaite Theorem** (independently from Allan Gibbard)
- this paper written after reading Gibbard’s proof of the theorem (cf. sec. 4&5)

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The existence theorem for strategy-proof voting procedures

- Terminology & concepts:

- a committee is a set I_n of $n \geq 1$ individuals;
- an alternative set S_m is a set of $m \geq 3$ elements/alternatives;
- for each individual $i \in I_n$, there is a weak ordering R_i (i.e., reflexive, transitive, complete) on S_m called a preference ordering;
- if $x, y \in S_m$, $i \in I_n$, then $x \boldsymbol{R}_i y$ and $x R_i y$ mean "individual i prefers x over y" and "individual i prefers x over y or is indifferent between x and y" respectively;
- π_m … the set of all possible preference orderings (with respect to S_m);
- $\pi_m{}^n$ … the n-ary Cartesian product of π_m;
- for each individual $i \in I_n$, there is a weak ordering B_i on S_m (i.e., $B_i \in \pi_m$) called a ballot;
- $B = (B_1, \ldots, B_n)$ … the ballot set composed of ballots $B_1, \ldots, B_n$;
- a voting procedure for n individuals and m alternatives is a function v^{nm}: $\pi_m{}^n \rightarrow T_p \subseteq S_m$ (for $1 \leq p \leq m$) [intuitively: v^{nm} selects for each ballot set B the elected alternative $x \in S_m$];
- $\langle I_n, S_m, v^{nm}, T_p \rangle$ … the committee's structure.

Terminology & concepts continued …

- Definition of strategy-proofness:

  An individual $i \in I_n$ can manipulate a voting procedure v^{nm} at ballot set $B = (B_1, \ldots, B_n)$ iff there is a ballot B_i' such that $v^{nm}(B_1, \ldots, B_i', \ldots, B_n)\ \boldsymbol{B_i}\ v^{nm}(B_1, \ldots, B_i, \ldots, B_n)$.

  A voting procedure v^{nm} is manipulable at ballot set $B = (B_1, \ldots, B_n)$ if there is an individual $i \in I_n$ that can manipulate v^{nm} at B.

  A voting procedure v^{nm} is strategy-proof iff there is no ballot set B at which it is manipulable.

Example: $B_i = R_i$ … sincere strategy vs $B_i' \neq R_i$ … sophisticated strategy

- Restriction D:

  Consider a committee structure $\langle I_n, S_m, v^{nm}, T_p \rangle$. If this structure is subject to Restriction D then only preference sets $R = (R_1, \ldots, R_n) \in \rho_m^{\ n}$ and ballot sets $B = (B_1, \ldots, B_n) \in \rho_m^{\ n}$ are admissible. [Here $\rho_m^{\ n}$ is the n-ary Cartesian product of ρ_m, which is the set of all possible **strong** preference orderings.]

- A committee subject to Restriction D is called a strict committee. The corresponding voting procedure is called a strict voting procedure.

Terminology & concepts continued …

- Three useful functions:

Informally, a choice function Ψ_W is a function (defined for any $W \subset S_m$) which selects for each ballot B_i those alternatives from W ranked highest in the ordering B_i.
Informally, a reduction function θ_W is a function (defined for any $W \subset S_m$) which outputs for each weak ordering $C_i \in \pi_m$ a weak ordering $D_i \in \pi_m$ that is identical with C_i after removing from it any alternative not in W.
Informally, a dictator function $f_T^{\,i}$ is a function which, for any ballot set B, selects from T_p that alternative which individual i has ranked highest on ballot B_i.

- Definition of dictatorship:

  A voting procedure is dictatorial iff there is an individual $i \in I_n$ such that $v^{nm}(B) = f_T^{\,i}(B)$ for any $B \in \pi_m^{\,n}$.

Two variants of dictatorship: $T_p = S_m$ (fully dictatorial v.p.), $T_p \subset S_m$ (partially dictatorial v.p.).

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The existence theorem

- Theorem 1(Gibbard-Satterthwaite):

  Consider a strict committee structure $\langle I_n, S_m, v^{nm}, T_{\mathrm{p}}\rangle$ where $n \geq 1$ and $m \geq p \geq 3$. The voting procedure v^{nm} is strategy-proof iff it is dictatorial.

  Proof (outline): (<=): immediate.

  (=>): using Lemmas 5&6.

- Lemma 5:

  Consider a strict committee structure $\langle I_n, S_m, v^{nm}, T_{\mathrm{p}}\rangle$ where $n \geq 1$, $m \geq 3$ and $p \geq 1$. If v^{nm} is strategy-proof, then it is either fully dictatorial or strongly alternative.

  Proof (outline): 3 parts: - base step ($n = 1$, $m = 3$)

  - induction on n [only a sketch]

  - induction on m [not in the paper]

=> We will start with the base case (Lemmas 1&2) and then do induction on n (Lemmas 3&4).

The base step ($n = 1$, $m = 3$):

- <u>Definition of weak and strong alternative-exclusion:</u>

  A voting procedure v^{nm} is weak alternative-excluding iff $T_p \subset S_m$.

  Given a strict committee structure $\langle I_n, S_m, v^{nm}, T = T_\mathrm{p}\rangle$, the strict voting procedure v^{nm} satisfies Condition U iff, for any $B = (B_1, \ldots, B_\mathrm{n}) \in \rho_m{}^n$ such that $\Psi_T(B_1) = \Psi_T(B_2) = \ldots = \Psi_T(B_n)$, then $v^{nm}(B) = \Psi_T(B_1)$. [= a Pareto optimality condition]

  A voting procedure v^{nm} is strong alternative-excluding iff it is weak-alternative excluding and satisfies Condition U.

- <u>Lemma 1:</u>

  Consider a strict committee structure $\langle I_n, S_m, v^{nm}, T = T_\mathrm{p}\rangle$ where $n \geq 1$, $m \geq 3$ and $p \geq 1$. If v^{nm} is strategy-proof, then it satisfies Condition U.

  <u>Proof:</u>

  Suppose that v^{nm} is strategy-proof and does not satisfy Condition U. Then there is an $x \in T_\mathrm{p}$ and $C \in \rho_m{}^n$ such that $\Psi_T(\mathrm{C}_1) = \Psi_T(\mathrm{C}_2) = \ldots = \Psi_T(\mathrm{C}_n)$, but $v^{nm}(\mathrm{C}) \neq \Psi_T(\mathrm{C}_1)$.

Lemma 1:
Consider a strict committee structure $\langle I_n, S_m, v^{nm}, T = T_p \rangle$ where $n \geq 1$, $m \geq 3$ and $p \geq 1$. If v^{nm} is strategy-proof, then it satisfies Condition U.

We have: $\Psi_T(C_1) = \Psi_T(C_2) = \ldots = \Psi_T(C_n)$, but $v^{nm}(C) \neq \Psi_T(C_1)$.

=> since $\Psi_T(C_1) \in T_p$, there is a $D \in \rho_m{}^n$ such that $v^{nm}(D) = \Psi_T(C_1)$.

=> Consider the following sequence $S(C, D)$:

$$v^{nm}(C_1, C_2, \ldots, C_n) \neq \Psi_T(C_1).$$
$$v^{nm}(D_1, C_2, \ldots, C_n)$$
$$\ldots$$
$$v^{nm}(D_1, \ldots, D_{i-1}, C_i, C_{i+1} \ldots, C_n)$$
$$v^{nm}(D_1, \ldots, D_{i-1}, D_i, C_{i+1} \ldots, C_n) \ldots$$
$$v^{nm}(D_1, \ldots, D_{n-1}, C_n)$$
$$v^{nm}(D_1, \ldots, D_{n-1}, D_n) = \Psi_T(C_1).$$

=> At some point in $S(C, D)$, the outcome must change from $\neg\Psi_T(C_1)$ to $\Psi_T(C_1)$. Let's say this happens at individual $i \in I_n$:

$$v^{nm}(D_1, \ldots, D_{i-1}, C_i, C_{i+1} \ldots, C_n) = x \neq \Psi_T(C_1).$$
$$v^{nm}(D_1, \ldots, D_{i-1}, D_i, C_{i+1} \ldots, C_n) = \Psi_T(C_1).$$

Lemma 1:
Consider a strict committee structure $\langle I_n, S_m, v^{nm}, T = T_p\rangle$ where $n \geq 1$, $m \geq 3$ and $p \geq 1$. If v^{nm} is strategy-proof, then it satisfies Condition U.

We have: $v^{nm}(D_1, \ldots, D_{i-1}, C_i, C_{i+1} \ldots, C_n) = x \neq \Psi_T(C_1)$.
$v^{nm}(D_1, \ldots, D_{i-1}, D_i, C_{i+1} \ldots, C_n) = \Psi_T(C_1)$.

=> Then the following scenario is possible: $C_i = R_i$ (i.e., C_i represents i's sincere strategy and D_i represents i's sophisticated strategy).

=> Thus, v^{nm} is manipulable and consequently is not strategy-proof. Contradiction. So, v^{nm} satisfies Condition U.

■

Using Lemma 1 we obtain …

- Lemma 2:
Consider a strict committee structure $\langle I_1, S_3, v^{1,3}, T = T_p\rangle$ where $1 \leq p \leq 3$. If $v^{1,3}$ is strategy-proof, then it is either fully dictatorial or strong alternative-excluding.

Proof: By contradiction & case distinctions. Lemma 1 covers the case where $v^{1,3}$ does not satisfy Condition U.

■

The inductive step on n:

As a preliminary step, observe that any $v^{n,3}$ can be represented as an n-dimensional table. For example, for $n = 2$ we could have Table I:

TABLE I

$v^{2,3}(B_1, B_2)$

		B_1					
		$(x\ y\ z)$	$(x\ z\ y)$	$(y\ x\ z)$	$(y\ z\ x)$	$(z\ x\ y)$	$(z\ y\ x)$
	$(x\ y\ z)$	x	x	y	y	y	y
	$(x\ z\ y)$	x	x	y	y	y	y
B_2	$(y\ x\ z)$	y	y	x	x	x	x
	$(y\ z\ x)$	y	z	x	x	x	x
	$(z\ x\ y)$	y	y	x	x	x	x
	$(z\ y\ x)$	y	y	x	x	x	x

The inductive step continued …

Alternatively, the same information can be represented as shown in Table II:

TABLE II

$v^{2,3}(B_1, B_2)$

$$
v^{2,3}(B_1, B_2) = \begin{cases} v_1^{1,3}(B_1) \text{ if } B_2 = (x\ y\ z) \\ v_2^{1,3}(B_1) \text{ if } B_2 = (x\ z\ y) \\ v_3^{1,3}(B_1) \text{ if } B_2 = (y\ x\ z) \\ v_4^{1,3}(B_1) \text{ if } B_2 = (y\ z\ x) \\ v_5^{1,3}(B_1) \text{ if } B_2 = (z\ x\ y) \\ v_6^{1,3}(B_1) \text{ if } B_2 = (z\ y\ x) \end{cases}
$$

Where

		$v_1^{1,3}$	$v_2^{1,3}$	$v_3^{1,3}$	$v_4^{1,3}$	$v_5^{1,3}$	$v_6^{1,3}$
	$(x\ y\ z)$	x	x	y	y	y	y
	$(x\ z\ y)$	x	x	y	z	y	y
B_1	$(y\ x\ z)$	y	y	x	x	x	x
	$(y\ z\ x)$	y	y	x	x	x	x
	$(z\ x\ y)$	y	y	x	x	x	x
	$(z\ y\ x)$	y	y	x	x	x	x

The inductive step continued …

- Lemma 3:

  Consider a strict committee structure $\langle I_{n+1}, S_3, v^{n+1,3}, T_p \rangle$ where $n \geq 1$ and $1 \leq p \leq 3$. Let $B = (B_1, \ldots, B_n)$. Then $v^{n+1,3}$ can be represented as shown below, where $v_1^{n,3}, \ldots, v_6^{n,3}$ are strict voting procedures for committees with n members.

  No ballot set $(B, B_{n+1}) \in \pi_m^{n+1}$ exists at which any individual $i \in I_n$ (individual $n+1$ being excluded) can manipulate $v^{n+1,3}$ iff each of each of $v_1^{n,3}, \ldots, v_6^{n,3}$ is strategy-proof.

$$v^{n+1,3}(B, B_{n+1}) = \begin{cases} v_1^{n,3}(B) & \text{if} \quad B_{n+1} = (x\ y\ z) \\ v_2^{n,3}(B) & \text{if} \quad B_{n+1} = (x\ z\ y) \\ \quad \cdots & \\ v_6^{n,3}(B) & \text{if} \quad B_{n+1} = (z\ y\ x) \end{cases}$$

To show: no ballot set $(B_n, B_{n+1}) \in \pi_m^{n+1}$ exists at which any individual $i \in I_n$ (individual $n+1$ being excluded) can manipulate $v^{n+1,3}$ iff each of each of $v_1^{n,3}, \ldots, v_6^{n,3}$ is strategy-proof.

Proof:

(=>) By contradiction.

Suppose $v^{n+1,3}$ is strategy-proof for all individuals $j \in I_n$, but some $v_k^{n,3}$ $(1 \leq k \leq 6)$ is not strategy-proof for some individual $i \in I_n$. Suppose that $k = 1$.

=> There is some ballot set $B = (B_1, \ldots, B_n)$ and ballot B_i' such that $v_1^{n,3}(B_1, \ldots, B_i', \ldots, B_n)$ $\boldsymbol{B_i}$ $v_1^{n,3}(B_1, \ldots, B_i, \ldots, B_n)$.

=> Let individual $n+1$ cast the ballot $B_{n+1}(x, y, z)$.

=> Repeated substitution yields $v^{n+1,3}(B', B_{n+1})$ $\boldsymbol{B_i}$ $v^{n+1,3}(B, B_{n+1})$. Hence, $v^{n+1,3}$ is manipulable
at (B, B_{n+1}). Contradiction. This concludes the necessary part.

(<=) By contradiction.

Suppose all of $v_1^{n,3}, \ldots, v_6^{n,3}$ $(1 \leq k \leq 6)$ are strategy-proof for all individuals $j \in I_n$, but $v^{n+1,3}$ is not strategy-proof for some individual $i \in I_n$ [where $v_1^{n,3}, \ldots, v_6^{n,3}$ are the constituents of $v^{n+1,3}$].

We have: Suppose all of $v_1^{n,3}, \ldots, v_6^{n,3}$ $(1 \leq k \leq 6)$ are strategy-proof for all individuals $j \in I_n$, but $v^{n+1,3}$ is not strategy-proof for some individual $i \in I_n$ [where $v_1^{n,3}$, …, $v_6^{n,3}$ are the constituents of $v^{n+1,3}$].

=> There is some ballot set $(B, B_{n+1}) = (B_1, \ldots, B_i, \ldots, B_n, B_{n+1})$ and ballot B_i' such that $v^{n+1,3}(B', B_{n+1})$ $\boldsymbol{B_i}$ $v^{n+1,3}(B, B_{n+1})$.

=> Let individual $n+1$ cast the ballot $B_{n+1}(x, y, z)$.

=> Repeated substitution yields $v_1^{n,3}(B')$ $\boldsymbol{B_i}$ $v^{n,3}(B)$. Hence, $v_1^{n,3}$ is not strategy-proof. Contradiction. Thus, the sufficiency part is proved too.

■

However, Lemma 3 gave us only a necessary condition for constructing a strategy-proof voting procedure $v^{n+1,3}$. Under certain circumstances, individual $n+1$ can manipulate the voting procedure.

=> Lemma 4 establishes a necessary condition for $v^{n+1,3}$ to be strategy-proof.

The inductive step continued …

- Lemma 4:
  Consider a strict committee $\langle I_{n+1}, S_3, v^{n+1,3}, T_{\mathrm{p}}\rangle$ where $n \geq 1$ and $1 \leq p \leq 3$. If every strategy-proof strict voting procedure $v^{n,3}$ is either fully dictatorial or strong alternative-excluding, then a necessary condition for $v^{n+1,3}$ to be strategy-proof is that it be either fully dictatorial or strong alternative-excluding.

Proof: Let us first fix some terminology:

$V^{\mathrm{n+1}}$ … the set of all strict voting procedures $v^{n+1,3}$;

$H^{n+1} \subset V^{\mathrm{n+1}}$ … the set of all strict voting procedures $v^{n+1,3}$ that are fully dictatorial or strong alternative-excluding;

Define V^{n} and H^{n} accordingly for the case of n individuals.

$W^{n+1} \subset V^{\mathrm{n+1}}$ … the set of all strict voting procedures $v^{n+1,3} \in V^{\mathrm{n+1}}$ that are constructed from voting procedures $v^{n,3} \in H^{\mathrm{n}}$, that is, all constituents $v^{n,3}$ are in H^{n};

$V^{\mathrm{n+1}*} \subset V^{n*}$ … the sets of all strategy-proof strict voting procedures contained in $V^{\mathrm{n+1}}$ and V^{n} respectively.

Lemma 4: If every strategy-proof $v^{n,3}$ is either fully dictatorial or strong alternative-excluding, then: if $v^{n+1,3}$ is strategy-proof then it is either fully dictatorial or strong alternative-excluding.

H^{n+1}… fully dictatorial or s.a.e. // W^{n+1} … constructed // V^{n+1*} … strategy-proof

Assume that $V^{n*} \subset H^{n}$.

=> By Lemma 3, $V^{n+1*} \subset W^{n+1}$.

=> Every $v^{n+1,3} \in V^{n+1*}$ can be identified by repeatedly partitioning W^{n+1} and discarding those subsets which are disjoint with V^{n+1*}.

=> The partitioning is done into 7 classes:

$$v^{n,3}(B) = f_T^i(B) \quad \text{where} \quad T = S_3 \quad \text{and} \quad i \in I_n, \tag{14}$$

$$v^{n,3}(B) = h_K^{n,3}(B) = x, \tag{15}$$

$$v^{n,3}(B) = h_L^{n,3}(B) = y, \tag{16}$$

$$v^{n,3}(B) = h_M^{n,3}(B) = z, \tag{17}$$

$$v^{n,3}(B) = h_N^{n,3}(B), \tag{18}$$

$$v^{n,3}(B) = h_P^{n,3}(B), \quad \text{and} \tag{19}$$

$$v^{n,3}(B) = h_Q^{n,3}(B), \tag{20}$$

Lemma 4: If every strategy-proof $v^{n,3}$ is either fully dictatorial or strong alternative-excluding, then: if $v^{n+1,3}$ is strategy-proof then it is either fully dictatorial or strong alternative-excluding.

H^{n+1}… fully dictatorial or s.a.e. // W^{n+1} … constructed // V^{n+1*} … strategy-proof & contained

=> Thus, we get the following classes at the first level of W^{n+1}:

$$\mathscr{W}_1^{n+1} = \{v^{n+1,3} \mid v^{n+1,3} \in \mathscr{W}^{n+1} \;\&\; v^{n+1,3}[B, (x\ y\ z)] = f_T^i(B) \text{ where } T = S_3 \text{ and } i \in I_n\}, \tag{21}$$

$$\mathscr{W}_2^{n+1} = \{v^{n+1,3} \mid v^{n+1,3} \in \mathscr{W}^{n+1} \;\&\; v^{n+1,3}[B, (x\ y\ z)] = h_K^{n,3}(B)\}, \tag{22}$$

$$\mathscr{W}_3^{n+1} = \{v^{n+1,3} \mid v^{n+1,3} \in \mathscr{W}^{n+1} \;\&\; v^{n+1,3}[B, (x\ y\ z)] = h_L^{n,3}(B)\}, \tag{23}$$

$$\cdots,$$

$$\mathscr{W}_7^{n+1} = \{v^{n+1,3} \mid v^{n+1,3} \in \mathscr{W}^{n+1} \;\&\; v^{n+1,3}[B, (x\ y\ z)] = h_Q^{n,3}(B)\}. \tag{24}$$

Lemma 4: If every strategy-proof $v^{n,3}$ is either fully dictatorial or strong alternative-excluding, then: if $v^{n+1,3}$ is strategy-proof then it is either fully dictatorial or strong alternative-excluding.

H^{n+1}… fully dictatorial or s.a.e. // W^{n+1} … constructed // V^{n+1*} … strategy-proof & contained

=> We then have two establish for every such class whether it is disjoint with V^{n+1*}. For example,

$$W^{n+1}{}_{27} = \{v^{n+1,3} \mid v^{n+1,3} \in W^{n+1}{}_2 \;\&\; v^{n+1,3}[B, (x, y, z)] = h^{n,3}{}_Q(B)\}.$$

Let individual n+1 have preferences and sincere strategy $R_{n+1} = (x\ z\ y)$ and let the other individuals cast ballots $B_1 = B_2 = \ldots = B_n = (z\ y\ x)$.

=> $v^{n+1,3}[B, (x, y, z)] = h^{n,3}{}_Q(B) = y$. This is the least favourable outcome for $n+1$.

=> By employing the sophisticated strategy $B_{n+1}' = (x\ y\ z)$, we get $v^{n+1,3}[B, (x, y, z)] = h^{n,3}{}_K(B) = x$.

=> Thus, every $v^{n+1,3} \in W^{n+1}{}_{27}$ is not strategy-proof. So, $W^{n+1}{}_{27}$ will be discarded.

Lemma 4: If every strategy-proof $v^{n,3}$ is either fully dictatorial or strong alternative-excluding, then: if $v^{n+1,3}$ is strategy-proof then it is either fully dictatorial or strong alternative-excluding.

H^{n+1}… fully dictatorial or s.a.e. // W^{n+1} … constructed // V^{n+1*} … strategy-proof

⇒ Satterthwaite claims that this procedure yields 17 subsets of W_{n+1} that are not disjoint with V^{n+1*}. Furthermore, it can be checked that the elements of these subsets are all either fully dictatorial or strong alternative-excluding.

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=> Lemma 5:

Consider a strict committee structure $\langle I_n, S_m, v^{nm}, T_p \rangle$ where $n \geq 1$, $m \geq 3$ and $p \geq 1$. If v^{nm} is strategy-proof, then it is either fully dictatorial or strongly alternative-excluding.

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=> Lemma 6:

Consider a strict committee structure $\langle I_n, S_m, v^{nm}, T = T_p \rangle$ where $n \geq 2$, $m \geq 3$ and $p \geq 1$, and $m \geq p$. If v^{nm} is a strategy-proof and two ballot sets $C, D \in \rho_m{}^n$ have the property that, for all $i \in I_n$, $\theta_T(C_i) = \theta_T(D_i)$, then $v^{nm}(C) = v^{nm}(D)$.

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Theorem 1(Gibbard-Satterthwaite): Consider a strict committee structure $\langle I_n, S_m, v^{nm}, T_p\rangle$ where $n \geq 1$ and $m \geq p \geq 3$. The voting procedure v^{nm} is strategy-proof iff it is dictatorial.

(=>): Suppose v^{nm} is strategy-proof. By Lemma 5, if v^{nm} is strategy-proof, then it is either fully dictatorial or strong alternative-excluding.

To show: If v^{nm} is strategy-proof and strong alternative-excluding then it is partially dictatorial.

=> Assume that v^{nm} is strategy-proof, strong alternative-excluding and has range $T = T_p$. Rewrite each ballot $B_i \in \rho_m{}^n$ as $B_i{}^* \in \rho_p{}^n$ (where $B_i{}^* = \theta_T(B_i)$).

=> Consider any distinct $C, D \in \rho_m{}^n$ such that $[\theta_T(C_1), \ldots, \theta_T(C_n)] = [\theta_T(D_1), \ldots, \theta_T(D_n)]$. By Lemma 6, $v^{nm}(C) = v^{nm}(D)$.

=> There exists a v^{np} such that, for all $B \in \rho_m{}^n$, $v^{np}[\theta_T(B_1), \ldots, \theta_T(B_n)] = v^{nm}(B_1, \ldots, B_n)$.

=> Since v^{nm} is strategy-proof, so is v^{np}. By Lemma 5, it is either dictatorial or strong alternative-excluding. It cannot be the latter. Thus, v^{np} is dictatorial. From this it follows that v^{nm} is partially dictatorial.

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Overview of the presentation

1. Biographical sketch of the author

2. Terminology & key concepts

3. Proof of the existence theorem for strategy-proof strict voting procedures

4. Discussion pointers

Points for discussion

- For the proof of Lemma 4, a great number of partition classes need to be checked. (Satterthwaite refers us to his thesis.) Is there any principled means of eliminating candidate classes?

- In section 3 (pp. 193-4), Satterthwaite briefly considers the case of S_2 and mentions two further strategy-proof voting procedures. Are these all or can there be others?

- In section 4 (pp. 207-8) Satterthwaite expresses the opinion that the correspondence theorem establishes a new conceptual foundation/justification for Arrow's condition: constructing SWFs satisfying rationality, (IIA), (CS), (NNR) is equivalent to constructing a strategy-proof voting procedure. How does this observation relate to arguments against Arrow's condition (e.g. rationality or (IIA))?