

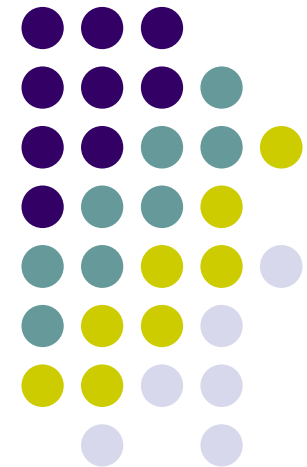
Mark Allen Satterthwaite:

**“Strategy-Proofness and Arrow’s Conditions:
Existence and Correspondence Theorems for
Voting Procedures and Social Welfare Functions”**

(in: *Journal of Economic Theory*, vol. 10, pp. 187-217 (1975))

Stefan Eichinger

June 23, 2009

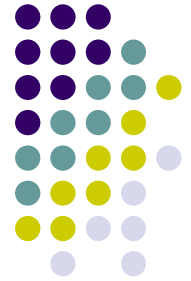


Overview of the presentation



1. Biographical sketch of the author
2. Terminology & key concepts
3. Proof of the existence theorem for strategy-proof strict voting procedures
4. Discussion pointers

Overview of the presentation



1. **Biographical sketch of the author**
2. Terminology & key concepts
3. Proof of the existence theorem for strategy-proof strict voting procedures
4. Discussion pointers

Mark (Allen) Satterthwaite



- Academic career:

- 1973: PhD in Economics from University of Wisconsin, Madison
- since then: faculty at Kellogg School of Management, Northwestern University

- Areas of expertise:

competition in healthcare, healthcare management, strategy, voting systems

- “Strategy-Proofness and Arrow’s Conditions”:

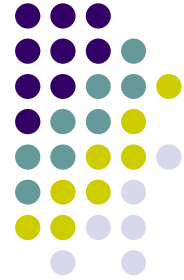
- originated from his PhD thesis: *The Existence of a Strategy Proof Voting Procedure: A Topic in Social Choice Theory* (1973);
 - => **Gibbard-Satterthwaite Theorem** (independently from Allan Gibbard)
- this paper written after reading Gibbard’s proof of the theorem (cf. sec. 4&5)

Overview of the presentation



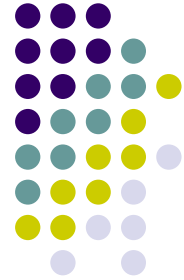
1. Biographical sketch of the author
2. Terminology & key concepts
3. Proof of the existence theorem for strategy-proof strict voting procedures
4. Discussion pointers

The existence theorem for strategy-proof voting procedures



- Terminology & concepts:
 - a **committee** is a set I_n of $n \geq 1$ individuals;
 - an **alternative set** S_m is a set of $m \geq 3$ elements/alternatives;
 - for each individual $i \in I_n$, there is a weak ordering R_i (i.e., reflexive, transitive, complete) on S_m called a **preference ordering**;
 - if $x, y \in S_m, i \in I_n$, then $x R_i y$ and $x \succ_i y$ mean “individual i prefers x over y ” and “individual i prefers x over y or is indifferent between x and y ” respectively;
 - π_m ... the set of all possible preference orderings (with respect to S_m);
 - π_m^n ... the n -ary Cartesian product of π_m ;
 - for each individual $i \in I_n$, there is a weak ordering B_i on S_m (i.e., $B_i \in \pi_m$) called a **ballot**;
 - $B = (B_1, \dots, B_n)$... the **ballot set** composed of ballots B_1, \dots, B_n ;
 - a **voting procedure** for n individuals and m alternatives is a function $v^{nm}: \pi_m^n \rightarrow T_p \subseteq S_m$ (for $1 \leq p \leq m$) [intuitively: v^{nm} selects for each ballot set B the elected alternative $x \in S_m$];
 - $\langle I_n, S_m, v^{nm}, T_p \rangle$... the **committee's structure**.

Terminology & concepts continued ...



- Definition of strategy-proofness:

An individual $i \in I_n$ can **manipulate** a voting procedure v^{nm} at ballot set $B = (B_1, \dots, B_n)$ iff there is a ballot B_i' such that $v^{nm}(B_1, \dots, B_i', \dots, B_n) \mathbf{B}_i v^{nm}(B_1, \dots, B_i, \dots, B_n)$.

A voting procedure v^{nm} is **manipulable** at ballot set $B = (B_1, \dots, B_n)$ if there is an individual $i \in I_n$ that can manipulate v^{nm} at B .

A voting procedure v^{nm} is **strategy-proof** iff there is no ballot set B at which it is manipulable.

Example: $B_i = R_i \dots$ **sincere strategy** vs $B_i' \neq R_i \dots$ **sophisticated strategy**

- Restriction D:

Consider a committee structure $\langle I_n, S_m, v^{nm}, T_p \rangle$. If this structure is **subject to Restriction D** then only preference sets $R = (R_1, \dots, R_n) \in \rho_m^n$ and ballot sets $B = (B_1, \dots, B_n) \in \rho_m^n$ are admissible. [Here ρ_m^n is the n-ary Cartesian product of ρ_m , which is the set of all possible **strong** preference orderings.]

- A committee subject to Restriction D is called a **strict committee**. The corresponding voting procedure is called a **strict voting procedure**.

Terminology & concepts continued ...



- Three useful functions:

Informally, a **choice function** Ψ_W is a function (defined for any $W \subset S_m$) which selects for each ballot B_i those alternatives from W ranked highest in the ordering B_i .

Informally, a **reduction function** θ_W is a function (defined for any $W \subset S_m$) which outputs for each weak ordering $C_i \in \pi_m$ a weak ordering $D_i \in \pi_m$ that is identical with C_i after removing from it any alternative not in W .

Informally, a **dictator function** f_T^i is a function which, for any ballot set B , selects from T_p that alternative which individual i has ranked highest on ballot B_i .

- Definition of dictatorship:

A voting procedure is **dictatorial** iff there is an individual $i \in I_n$ such that $v^{nm}(B) = f_T^i(B)$ for any $B \in \pi_m^n$.

Two variants of dictatorship: $T_p = S_m$ (**fully dictatorial** v.p.), $T_p \subset S_m$ (**partially dictatorial** v.p.).

Overview of the presentation



1. Biographical sketch of the author
2. Terminology & key concepts
3. Proof of the existence theorem for strategy-proof strict voting procedures
4. Discussion pointers



The existence theorem

- Theorem 1(Gibbard-Satterthwaite):

Consider a strict committee structure $\langle I_n, S_m, v^{nm}, T_p \rangle$ where $n \geq 1$ and $m \geq p \geq 3$. The voting procedure v^{nm} is strategy-proof iff it is dictatorial.

Proof (outline): (\Leftarrow): immediate.

(\Rightarrow): using Lemmas 5&6.

- Lemma 5:

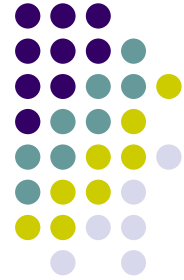
Consider a strict committee structure $\langle I_n, S_m, v^{nm}, T_p \rangle$ where $n \geq 1$, $m \geq 3$ and $p \geq 1$. If v^{nm} is strategy-proof, then it is either fully dictatorial or strongly alternative.

Proof (outline): 3 parts: - base step ($n = 1$, $m = 3$)

- induction on n [only a sketch]

- induction on m [not in the paper]

\Rightarrow We will start with the base case (Lemmas 1&2) and then do induction on n (Lemmas 3&4).



The base step ($n = 1, m = 3$):

- Definition of weak and strong alternative-exclusion:

A voting procedure v^{nm} is **weak alternative-excluding** iff $T_p \subset S_m$.

Given a strict committee structure $\langle I_n, S_m, v^{nm}, T = T_p \rangle$, the strict voting procedure v^{nm} **satisfies Condition U** iff, for any $B = (B_1, \dots, B_n) \in \rho_m^n$ such that $\Psi_T(B_1) = \Psi_T(B_2) = \dots = \Psi_T(B_n)$, then $v^{nm}(B) = \Psi_T(B_1)$. [= a Pareto optimality condition]

A voting procedure v^{nm} is **strong alternative-excluding** iff it is weak-alternative excluding and satisfies Condition U.

- Lemma 1:

Consider a strict committee structure $\langle I_n, S_m, v^{nm}, T = T_p \rangle$ where $n \geq 1, m \geq 3$ and $p \geq 1$. If v^{nm} is strategy-proof, then it satisfies Condition U.

Proof:

Suppose that v^{nm} is strategy-proof and does not satisfy Condition U. Then there is an $x \in T_p$ and $C \in \rho_m^n$ such that $\Psi_T(C_1) = \Psi_T(C_2) = \dots = \Psi_T(C_n)$, but $v^{nm}(C) \neq \Psi_T(C_1)$.



Lemma 1:

Consider a strict committee structure $\langle I_n, S_m, v^{nm}, T = T_p \rangle$ where $n \geq 1$, $m \geq 3$ and $p \geq 1$. If v^{nm} is strategy-proof, then it satisfies Condition U.

We have: $\Psi_T(C_1) = \Psi_T(C_2) = \dots = \Psi_T(C_n)$, but $v^{nm}(C) \neq \Psi_T(C_1)$.

\Rightarrow since $\Psi_T(C_1) \in T_p$, there is a $D \in \rho_m^n$ such that $v^{nm}(D) = \Psi_T(C_1)$.

\Rightarrow Consider the following sequence $S(C, D)$:

$$\begin{aligned}
 &v^{nm}(C_1, C_2, \dots, C_n) \neq \Psi_T(C_1). \\
 &v^{nm}(D_1, C_2, \dots, C_n) \\
 &\quad \dots \\
 &v^{nm}(D_1, \dots, D_{i-1}, C_i, C_{i+1}, \dots, C_n) \\
 &v^{nm}(D_1, \dots, D_{i-1}, D_i, C_{i+1}, \dots, C_n) \dots \\
 &v^{nm}(D_1, \dots, D_{n-1}, C_n) \\
 &v^{nm}(D_1, \dots, D_{n-1}, D_n) = \Psi_T(C_1).
 \end{aligned}$$

\Rightarrow At some point in $S(C, D)$, the outcome must change from $\neg\Psi_T(C_1)$ to $\Psi_T(C_1)$. Let's say this happens at individual $i \in I_n$:

$$\begin{aligned}
 &v^{nm}(D_1, \dots, D_{i-1}, C_i, C_{i+1}, \dots, C_n) = x \neq \Psi_T(C_1). \\
 &v^{nm}(D_1, \dots, D_{i-1}, D_i, C_{i+1}, \dots, C_n) = \Psi_T(C_1).
 \end{aligned}$$



Lemma 1:

Consider a strict committee structure $\langle I_n, S_m, v^{nm}, T = T_p \rangle$ where $n \geq 1$, $m \geq 3$ and $p \geq 1$. If v^{nm} is strategy-proof, then it satisfies Condition U.

We have: $v^{nm}(D_1, \dots, D_{i-1}, C_i, C_{i+1}, \dots, C_n) = x \neq \Psi_T(C_1)$.
 $v^{nm}(D_1, \dots, D_{i-1}, D_i, C_{i+1}, \dots, C_n) = \Psi_T(C_1)$.

\Rightarrow Then the following scenario is possible: $C_i = R_i$ (i.e., C_i represents i 's sincere strategy and D_i represents i 's sophisticated strategy).

\Rightarrow Thus, v^{nm} is manipulable and consequently is not strategy-proof. Contradiction. So, v^{nm} satisfies Condition U.



Using Lemma 1 we obtain ...

- Lemma 2:

Consider a strict committee structure $\langle I_l, S_3, v^{l,3}, T = T_p \rangle$ where $1 \leq p \leq 3$. If $v^{l,3}$ is strategy-proof, then it is either fully dictatorial or strong alternative-excluding.

Proof: By contradiction & case distinctions. Lemma 1 covers the case where $v^{l,3}$ does not satisfy Condition U.





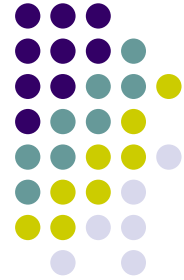
The inductive step on n :

As a preliminary step, observe that any $v^{n,3}$ can be represented as an n -dimensional table. For example, for $n = 2$ we could have Table I:

TABLE I

$v^{2,3}(B_1, B_2)$

		B_1					
		$(x\ y\ z)$	$(x\ z\ y)$	$(y\ x\ z)$	$(y\ z\ x)$	$(z\ x\ y)$	$(z\ y\ x)$
B_2	$(x\ y\ z)$	x	x	y	y	y	y
	$(x\ z\ y)$	x	x	y	y	y	y
	$(y\ x\ z)$	y	y	x	x	x	x
	$(y\ z\ x)$	y	z	x	x	x	x
	$(z\ x\ y)$	y	y	x	x	x	x
	$(z\ y\ x)$	y	y	x	x	x	x



The inductive step continued ...

Alternatively, the same information can be represented as shown in Table II:

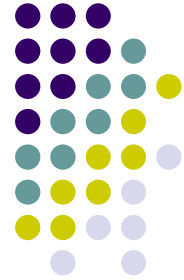
TABLE II

$v^{2,3}(B_1, B_2)$

$$v^{2,3}(B_1, B_2) = \begin{cases} v_1^{1,3}(B_1) & \text{if } B_2 = (x y z) \\ v_2^{1,3}(B_1) & \text{if } B_2 = (x z y) \\ v_3^{1,3}(B_1) & \text{if } B_2 = (y x z) \\ v_4^{1,3}(B_1) & \text{if } B_2 = (y z x) \\ v_5^{1,3}(B_1) & \text{if } B_2 = (z x y) \\ v_6^{1,3}(B_1) & \text{if } B_2 = (z y x) \end{cases}$$

Where

	$v_1^{1,3}$	$v_2^{1,3}$	$v_3^{1,3}$	$v_4^{1,3}$	$v_5^{1,3}$	$v_6^{1,3}$
B_1 (x y z)	x	x	y	y	y	y
(x z y)	x	x	y	z	y	y
(y x z)	y	y	x	x	x	x
(y z x)	y	y	x	x	x	x
(z x y)	y	y	x	x	x	x
(z y x)	y	y	x	x	x	x



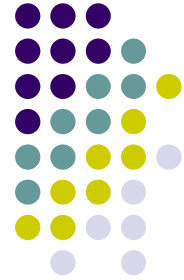
The inductive step continued ...

- Lemma 3:

Consider a strict committee structure $\langle I_{n+1}, S_3, v^{n+1,3}, T_p \rangle$ where $n \geq 1$ and $1 \leq p \leq 3$. Let $B = (B_1, \dots, B_n)$. Then $v^{n+1,3}$ can be represented as shown below, where $v_1^{n,3}, \dots, v_6^{n,3}$ are strict voting procedures for committees with n members.

No ballot set $(B, B_{n+1}) \in \pi_m^{n+1}$ exists at which any individual $i \in I_n$ (individual $n+1$ being excluded) can manipulate $v^{n+1,3}$ iff each of each of $v_1^{n,3}, \dots, v_6^{n,3}$ is strategy-proof.

$$v^{n+1,3}(B, B_{n+1}) = \begin{cases} v_1^{n,3}(B) & \text{if } B_{n+1} = (x y z) \\ v_2^{n,3}(B) & \text{if } B_{n+1} = (x z y) \\ \dots & \\ v_6^{n,3}(B) & \text{if } B_{n+1} = (z y x) \end{cases}$$



To show: no ballot set $(B_n, B_{n+1}) \in \pi_m^{n+1}$ exists at which any individual $i \in I_n$ (individual $n+1$ being excluded) can manipulate $v^{n+1,3}$ iff each of each of $v_1^{n,3}, \dots, v_6^{n,3}$ is strategy-proof.

Proof:

(\Rightarrow) By contradiction.

Suppose $v^{n+1,3}$ is strategy-proof for all individuals $j \in I_n$, but some $v_k^{n,3}$ ($1 \leq k \leq 6$) is not strategy-proof for some individual $i \in I_n$. Suppose that $k = 1$.

\Rightarrow There is some ballot set $B = (B_1, \dots, B_n)$ and ballot B_i' such that $v_1^{n,3}(B_1, \dots, B_i', \dots, B_n) > v_1^{n,3}(B_1, \dots, B_i, \dots, B_n)$.

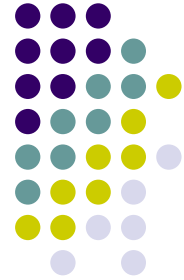
\Rightarrow Let individual $n+1$ cast the ballot $B_{n+1}(x, y, z)$.

\Rightarrow Repeated substitution yields $v^{n+1,3}(B', B_{n+1}) > v^{n+1,3}(B, B_{n+1})$. Hence, $v^{n+1,3}$ is manipulable at (B, B_{n+1}) . Contradiction. This concludes the necessary part.

(\Leftarrow) By contradiction.

Suppose all of $v_1^{n,3}, \dots, v_6^{n,3}$ ($1 \leq k \leq 6$) are strategy-proof for all individuals $j \in I_n$, but $v^{n+1,3}$ is not strategy-proof for some individual $i \in I_n$ [where $v_1^{n,3}, \dots, v_6^{n,3}$ are the constituents of $v^{n+1,3}$].

We have: Suppose all of $v_1^{n,3}, \dots, v_6^{n,3}$ ($1 \leq k \leq 6$) are strategy-proof for all individuals $j \in I_n$, but $v^{n+1,3}$ is not strategy-proof for some individual $i \in I_n$ [where $v_1^{n,3}, \dots, v_6^{n,3}$ are the constituents of $v^{n+1,3}$].



\Rightarrow There is some ballot set $(B, B_{n+1}) = (B_1, \dots, B_i, \dots, B_n, B_{n+1})$ and ballot B_i' such that $v^{n+1,3}(B', B_{n+1}) \mathbf{B}_i v^{n+1,3}(B, B_{n+1})$.

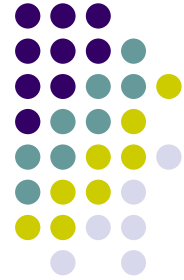
\Rightarrow Let individual $n+1$ cast the ballot $B_{n+1}(x, y, z)$.

\Rightarrow Repeated substitution yields $v_1^{n,3}(B') \mathbf{B}_i v^{n,3}(B)$. Hence, $v_1^{n,3}$ is not strategy-proof.
Contradiction. Thus, the sufficiency part is proved too.



However, Lemma 3 gave us only a necessary condition for constructing a strategy-proof voting procedure $v^{n+1,3}$. Under certain circumstances, individual $n+1$ can manipulate the voting procedure.

\Rightarrow Lemma 4 establishes a necessary condition for $v^{n+1,3}$ to be strategy-proof.



The inductive step continued ...

- Lemma 4:

Consider a strict committee $\langle I_{n+1}, S_3, v^{n+1,3}, T_p \rangle$ where $n \geq 1$ and $1 \leq p \leq 3$. If every strategy-proof strict voting procedure $v^{n,3}$ is either fully dictatorial or strong alternative-excluding, then a necessary condition for $v^{n+1,3}$ to be strategy-proof is that it be either fully dictatorial or strong alternative-excluding.

Proof: Let us first fix some terminology:

V^{n+1} ... the set of all strict voting procedures $v^{n+1,3}$;

$H^{n+1} \subset V^{n+1}$... the set of all strict voting procedures $v^{n+1,3}$ that are fully dictatorial or strong alternative-excluding;

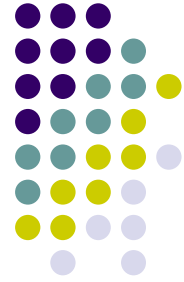
Define V^n and H^n accordingly for the case of n individuals.

$W^{n+1} \subset V^{n+1}$... the set of all strict voting procedures $v^{n+1,3} \in V^{n+1}$ that are constructed from voting procedures $v^{n,3} \in H^n$, that is, all constituents $v^{n,3}$ are in H^n ;

$V^{n+1*} \subset V^{n*}$... the sets of all strategy-proof strict voting procedures contained in V^{n+1} and V^n respectively.

Lemma 4: If every strategy-proof $v^{n,3}$ is either fully dictatorial or strong alternative-excluding, then: if $v^{n+1,3}$ is strategy-proof then it is either fully dictatorial or strong alternative-excluding.

H^{n+1} ... fully dictatorial or s.a.e. // W^{n+1} ... constructed // V^{n+1*} ... strategy-proof



Assume that $V^{n*} \subset H^n$.

=> By Lemma 3, $V^{n+1*} \subset W^{n+1}$.

=> Every $v^{n+1,3} \in V^{n+1*}$ can be identified by repeatedly partitioning W^{n+1} and discarding those subsets which are disjoint with V^{n+1*} .

=> The partitioning is done into 7 classes:

$$v^{n,3}(B) = f_T^i(B) \quad \text{where } T = S_3 \text{ and } i \in I_n, \quad (14)$$

$$v^{n,3}(B) = h_K^{n,3}(B) = x, \quad (15)$$

$$v^{n,3}(B) = h_L^{n,3}(B) = y, \quad (16)$$

$$v^{n,3}(B) = h_M^{n,3}(B) = z, \quad (17)$$

$$v^{n,3}(B) = h_N^{n,3}(B), \quad (18)$$

$$v^{n,3}(B) = h_P^{n,3}(B), \quad \text{and} \quad (19)$$

$$v^{n,3}(B) = h_Q^{n,3}(B), \quad (20)$$



Lemma 4: If every strategy-proof $v^{n,3}$ is either fully dictatorial or strong alternative-excluding, then: if $v^{n+1,3}$ is strategy-proof then it is either fully dictatorial or strong alternative-excluding.

H^{n+1} ... fully dictatorial or s.a.e. // W^{n+1} ... constructed // V^{n+1*} ... strategy-proof & contained

=> Thus, we get the following classes at the first level of W^{n+1} :

$$\mathcal{W}_1^{n+1} = \{v^{n+1,3} \mid v^{n+1,3} \in \mathcal{W}^{n+1} \ \& \ v^{n+1,3}[B, (x y z)] = f_T^i(B) \text{ where } T = S_3 \text{ and } i \in I_n\}, \quad (21)$$

$$\mathcal{W}_2^{n+1} = \{v^{n+1,3} \mid v^{n+1,3} \in \mathcal{W}^{n+1} \ \& \ v^{n+1,3}[B, (x y z)] = h_K^{n,3}(B)\}, \quad (22)$$

$$\mathcal{W}_3^{n+1} = \{v^{n+1,3} \mid v^{n+1,3} \in \mathcal{W}^{n+1} \ \& \ v^{n+1,3}[B, (x y z)] = h_L^{n,3}(B)\}, \quad (23)$$

...

$$\mathcal{W}_7^{n+1} = \{v^{n+1,3} \mid v^{n+1,3} \in \mathcal{W}^{n+1} \ \& \ v^{n+1,3}[B, (x y z)] = h_O^{n,3}(B)\}. \quad (24)$$



Lemma 4: If every strategy-proof $v^{n,3}$ is either fully dictatorial or strong alternative-excluding, then: if $v^{n+1,3}$ is strategy-proof then it is either fully dictatorial or strong alternative-excluding.

H^{n+1} ... fully dictatorial or s.a.e. // W^{n+1} ... constructed // V^{n+1*} ... strategy-proof & contained

=> We then have to establish for every such class whether it is disjoint with V^{n+1*} . For example,

$$W^{n+1}_{27} = \{v^{n+1,3} \mid v^{n+1,3} \in W^{n+1}_2 \ \& \ v^{n+1,3} [B, (x, y, z)] = h^{n,3}_Q(B)\}.$$

Let individual $n+1$ have preferences and sincere strategy $R_{n+1} = (x \ z \ y)$ and let the other individuals cast ballots $B_1 = B_2 = \dots = B_n = (z \ y \ x)$.

=> $v^{n+1,3} [B, (x, y, z)] = h^{n,3}_Q(B) = y$. This is the least favourable outcome for $n+1$.

=> By employing the sophisticated strategy $B_{n+1}' = (x \ y \ z)$, we get $v^{n+1,3} [B, (x, y, z)] = h^{n,3}_K(B) = x$.

=> Thus, every $v^{n+1,3} \in W^{n+1}_{27}$ is not strategy-proof. So, W^{n+1}_{27} will be discarded.

Lemma 4: If every strategy-proof $v^{n,3}$ is either fully dictatorial or strong alternative-excluding, then: if $v^{n+1,3}$ is strategy-proof then it is either fully dictatorial or strong alternative-excluding.

H^{n+1} ... fully dictatorial or s.a.e. // W^{n+1} ... constructed // V^{n+1*} ... strategy-proof



\Rightarrow Satterthwaite claims that this procedure yields 17 subsets of W_{n+1} that are not disjoint with V^{n+1*} . Furthermore, it can be checked that the elements of these subsets are all either fully dictatorial or strong alternative-excluding.

\Rightarrow Lemma 5:

Consider a strict committee structure $\langle I_n, S_m, v^{nm}, T_p \rangle$ where $n \geq 1$, $m \geq 3$ and $p \geq 1$. If v^{nm} is strategy-proof, then it is either fully dictatorial or strongly alternative-excluding.

\Rightarrow Lemma 6:

Consider a strict committee structure $\langle I_n, S_m, v^{nm}, T = T_p \rangle$ where $n \geq 2$, $m \geq 3$ and $p \geq 1$, and $m \geq p$. If v^{nm} is a strategy-proof and two ballot sets $C, D \in \rho_m^n$ have the property that, for all $i \in I_n$, $\theta_T(C_i) = \theta_T(D_i)$, then $v^{nm}(C) = v^{nm}(D)$.

Theorem 1 (Gibbard-Satterthwaite): Consider a strict committee structure $\langle I_n, S_m, v^{nm}, T_p \rangle$ where $n \geq 1$ and $m \geq p \geq 3$. The voting procedure v^{nm} is strategy-proof iff it is dictatorial.



(\Rightarrow): Suppose v^{nm} is strategy-proof. By Lemma 5, if v^{nm} is strategy-proof, then it is either fully dictatorial or strong alternative-excluding.

To show: If v^{nm} is strategy-proof and strong alternative-excluding then it is partially dictatorial.

\Rightarrow Assume that v^{nm} is strategy-proof, strong alternative-excluding and has range $T = T_p$. Rewrite each ballot $B_i \in \rho_m^n$ as $B_i^* \in \rho_p^n$ (where $B_i^* = \theta_T(B_i)$).

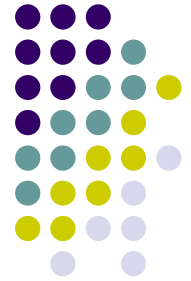
\Rightarrow Consider any distinct $C, D \in \rho_m^n$ such that $[\theta_T(C_1), \dots, \theta_T(C_n)] = [\theta_T(D_1), \dots, \theta_T(D_n)]$. By Lemma 6, $v^{nm}(C) = v^{nm}(D)$.

\Rightarrow There exists a v^{np} such that, for all $B \in \rho_m^n$, $v^{np}[\theta_T(B_1), \dots, \theta_T(B_n)] = v^{nm}(B_1, \dots, B_n)$.

\Rightarrow Since v^{nm} is strategy-proof, so is v^{np} . By Lemma 5, it is either dictatorial or strong alternative-excluding. It cannot be the latter. Thus, v^{np} is dictatorial. From this it follows that v^{nm} is partially dictatorial.



Overview of the presentation



1. Biographical sketch of the author
2. Terminology & key concepts
3. Proof of the existence theorem for strategy-proof strict voting procedures
4. Discussion pointers

Points for discussion



- For the proof of Lemma 4, a great number of partition classes need to be checked. (Satterthwaite refers us to his thesis.) Is there any principled means of eliminating candidate classes?
- In section 3 (pp. 193-4), Satterthwaite briefly considers the case of S_2 and mentions two further strategy-proof voting procedures. Are these all or can there be others?
- In section 4 (pp. 207-8) Satterthwaite expresses the opinion that the correspondence theorem establishes a new conceptual foundation/justification for Arrow's condition: constructing SWFs satisfying rationality, (IIA), (CS), (NNR) is equivalent to constructing a strategy-proof voting procedure. How does this observation relate to arguments against Arrow's condition (e.g. rationality or (IIA))?