Time and the continuum

Michiel van Lambalgen Riccardo Pinosio

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- focus on notion of dimensionless point/instant

Naive view of temporal continuum

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- what more could one wish for?

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- "In any case, it seems to me that the alternative continuum-discontinuum is a genuine alternative; i.e. there is no compromise here. In [a discontinuum] theory there cannot be space and time, only numbers[...]. It will be especially difficult to elicit something like a spatio-temporal quasi-order from such a schema. I can not picture to myself how the axiomatic framework of such a physics could look[...]. But I hold it as altogether possible that developments will lead there[...]"

Time in philosophy

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> *There is some sense – easier to feel than to state – in which time is an unimportant and superficial characteristic of reality. Past and future must be acknowledged to be as real as the present, and a certain emancipation from the slavery of time is essential to philosophic thought. (Bertrand Russell)*

Russell considers the flow of time to be unreal. Sometimes time itself is considered to be unreal, because contradictory

Aristotle on skepticism w.r.t. time

Next for discussion after the subjects mentioned is Time. The best plan will be to begin by working out the difficulties connected with it, making use of the current arguments. First, does it belong to the class of things that exist or to that of things that do not exist? Then secondly, what is its nature?

To start, then: the following considerations would make one suspect that it either does not exist at all or barely, and in an obscure way. One part of it has been and is not, while the other is going to be and is not yet. Yet time-both infinite time and any time you like to take-is made up of these. One would naturally suppose that what is made up of things which do not exist could have no share in reality. Further, if a divisible thing is to exist, it is necessary that, when it exists, all or some of its parts must exist. But of time some parts have been, while others have to be, and no part of it is though it is divisible. For what is 'now' is not a part: a part is a measure of the whole, which must be made up of parts. Time, on the other hand, is not held to be made up of 'nows'.

Saint Agustine

If any fraction of time be conceived that cannot now be divided even into the most minute momentary point, this alone is what we may call time present. But this flies so rapidly from future to past that it cannot be extended by any delay. For if it is extended, it is then divided into past and future. But the present has no extension whatever.

Human temporal experience: William James' Principles of Psychology 1890

- we have no sense for empty time: no internal clock which is consciously accessible
- the present is intimately related to consciousness, which is not a discrete 'string of beads' of successive 'nows'
- consciousness is the 'specious present', which is responsible for e.g. judgment of difference of events
- apart from the 'specious present', there is no time *intuition*, only symbolization

The ‘specious present’

[T]he practically cognized present [i.e. the specious present] is no knife-edge, but a saddle-back, with a certain breadth of its own on which we sit perched, and from which we look in two directions of time. The unit of composition of our perception of time is a *duration*, with a bow and a stern, as it were – a rearward- and a forward-looking end. It is only as parts of this *duration-block* that the relation of *succession* of one end to the other is perceived. We do not first feel one end and then feel the other after it, and from the perception of the succession infer an interval of time between, but we seem to feel the interval of time as a whole, with its two ends embedded in it. The experience is from the outset a synthetic datum, not a simple one; and to sensible perception its elements are inseparable, although attention looking back may easily decompose the experience, and distinguish its beginning from its end. (William James, Principles of Psychology, p. 574)

The specious present: neurobiology

- ‘slow’ processing cycle: $3s$ (Pöppel)
- example: Necker cube
- example CUBACUBACUBACUBA ...
- within each window of $3s$, percepts are bound together (in working memory, by neural synchrony?)
- after $3s$ the brain asks: ‘what’s new?’
- some percepts are then transferred to long term memory; no discontinuity
- duration estimates for durations less than $3s$ are much more accurate than for those greater than $3s$

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- How to model this mathematically?

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- motions in this sense need not give rise to point mappings

Whitrow's version of Zeno's 'Achilles' paradox

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$$t = \frac{2v_0}{g}(1 + e + e^2 + e^3 + \dots) = \frac{2v_0}{g}(\frac{1}{1-e}).$$

E.g. if $e = \frac{3}{4}$, and $v_0 = \frac{1}{2}g$, then $t = 4s$: but if time is infinitely divisible, there will be infinitely many instantaneous bounces, which are real events!

Changing topology and connectivity of the continuum

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- $\mathbb{R}$ is not ultraconnected!

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- since W is ultraconnected, itis pseudocompact: every continuous $g : W \longrightarrow \mathbb{R}$ is bounded

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- the role of w is to ensure that e is open, not closed, and hence extended

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- the inclusion relation $\subseteq$ on the *Pasts* induces a linear order on the triples (*Past*, *Present*, *Future*) – this is our temporal continuum
- it is impossible to remove a point from this continuum, since nothing would remain

Kant

Refl **4425** Spatium est quantum, sed non compositum. For space does not arise through the positing of its parts, but the parts are only possible through space; likewise with time. The parts may well be considered abstrahendo a caeteris, but cannot be conceived removendo caetera; they can therefore be distinguished, but not separated, and the divisio non est realis, sed logica. Since the divisibility of matter seems to come down to the space that it occupies, and it is as divisible as this space, the question arises whether the divisibility of matter is not as merely logical as that of space

More Kant

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The three modi of time are *persistence*, *succession* and *simultaneity* [...] Only through that which persists does existence in different parts of the temporal series acquire a magnitude, which one calls duration. For in mere sequence alone existence is always disappearing and beginning, and never has the least magnitude. Without that which persists there is therefore no temporal relation. (A177/B219)

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