NATURAL LOGIC IN NATURAL LANGUAGE

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Abstract Which ideology is taught when you 'translate' from plain language into logical formulas? We discuss the possibilities for a 'natural logic' of reasoning within, or closer to, natural language, and relate this research program to the history of logic, linguistics, computer science, and cognitive science.

1 The momentous shift from classical to modern logic

De Morgan's historical example: *"All horses are animals.* So*, all horse tails are animal tails."* Supposed to show the inadequacy of traditional logic: binary relations irreducible/essential. First-order: $\forall x\,(Hx \rightarrow Ax) \Rightarrow \forall x\,((Tx\ \&\ \exists y\,(\mathbf{H}y\ \&\ Rxy)) \rightarrow (Tx\ \&\ \exists y\,(\mathbf{A}y\ \&\ Rxy)))$.

2 Logical deep form versus linguistic surface form

Major rupture in the history of logic? Frege, Russell, Carnap (Geach's 'Kingdom of Darkness'): natural vs. formal language. *Misleading Form Thesis*: natural language syntax is misleading. *Montague's Thesis*: no significant difference in principle between natural & formal languages. (Still, methodology: Formal systems thinking vs. lightweight study of reasoning phenomena.) In reality, a very long history of logical versus linguistic form (see history chapter in GAMUT).

3 Some modern basics of monotonicity inference

The given reasoning pattern is predicate replacement in a simple natural language sentence.

Semantic *monotonicity*: if $\boldsymbol{M}, s \models \varphi(P)$, $P^{\boldsymbol{M}} \subseteq Q^{\boldsymbol{M}}$, then $\boldsymbol{M}, s \models \varphi(Q)$.

Syntactic *positive occurrence*: even number of negations, or BNF: *atoms* | & | $\vee$ | $\forall$ | $\exists$.

Lyndon's Theorem: A first-order formula $\varphi(P)$ is semantically monotone in P iff $\varphi(P)$ is equivalent to a formula whose only occurrences of P are positive.

Direction from right to left also holds in higher-order logics – and natural language.

4 Distribution reasoning in traditional logic

Van Eijck, Sanchez, Hodges: "Dictum de Omni (↑) et Nullo (↓)". Traditional logic accounts for relational reasoning. Difficulty: systematic account complex expressions/quantifier iteration. Modern traditionalists/revivalists of classical logic: Sommers, Syllogistic alternatives to FOL.

5 Monotonicity inferences in natural language

Ladusaw around 1980: negative polarity items triggered by 'downward contexts'.

Generalized Quantifier Theory: $Q\,AB$. Upward, but also downward entailment:

$Q\,AB, \quad A' \subseteq A \quad / \; Q\,A'B$

Various inference types: e.g., the quantifier expression "All" is $\downarrow MON \uparrow$.

Some further basic properties of determiner expressions in natural language:

Conservativity $Q\,AB$ iff $Q\,A(B\cap A)$

Variety if $A\neq\emptyset$, then $Q\,AB$ for some B, and $\neg Q\,AC$ for some C.

Theorem The quantifiers "All", "Some", "No", "Not All" in the Square of Opposition are the only ones satisfying Conservativity, Double Monotonicity, and Variety.

Survey of possible patterns: see Peters & Westerståhl's "Quantifiers" book (2006).

Other inferential properties of specific determiner expressions:

e.g., *Symmetry*: $Q\,AB$ iff $Q\,BA$.

6 Other cultural traditions in logic

China, 5th century B.C. Monotonicity in Moist logic:

"A white horse is a horse. To ride a white horse is to ride a horse."

"Your sister is a woman. To love your sister is not to love a woman."

(See HOLIC webpage for the Handbook of the History of Logic in China.)

Islamic tradition: W. Hodges' recent work on the extended syllogistic reasoning developed by Ibn Sina/Avicenna (three-quantifier iterations, monotonicity, other rules).

(Comment: Hodges' analysis uses insights from formal languages in logic: system close to the 'Guarded Fragment': a decidable fragment of FOL found by formal means.)

7 The 'natural logic' program in the 1980s

Original sources: J. van Benthem, 1986, *Essays in Logical Semantics*, Reidel, Dordrecht, 1987, 'Meaning: Interpretation and Inference', *Synthese* 73:3, 451-470.

Develop system of inference directly on natural language surface form, see how it cuts the cake differently from the syntax of first-order logic. Where full power of *FOL* really needed?

8 Monotonicity calculus and compositional structure

How to implement the above ideas on a significant functioning part of natural language?

Challenge: iterated quantifiers: $Q_1A\ R\ Q_2B$. Monotonicity (+ Conservativity) w.r.t. both Q_1, Q_2.

Scholastics: new inference patterns for combinations. Dummett: Frege solved it all by single quantifiers + recursion! However, Keenan and others: non-Fregean combinations in *NL*.

Even for simplest case, need theory of *syntactic construction*. For instance,

Categorial Grammar & *Monotonicity Calculus*. Compute positive/negative occurrences words/ subexpressions in tandem with syntactic analysis of an expression. Rules come in two kinds:

Specific information about lexical items: e.g. "All" has type $(e^-, (e^+, t))$.

General effects of composition: function head: positive, lambda body: positive.

Marking along unbroken strings of marked items: + + = – – = +   + – = – + = –

Illustration: compute all markings in the (syntactic parse) of the sentence

"No mortal man can slay every dragon."

Theoretical engine: Lambek Calculus (J. van Benthem, 1991, *Language in Action*, Elsevier, A'dam). Generalize to Boolean type theory, where positive occurrence still implies upward monotonicity ('soundness'). 'Completeness': Is there a Lyndon Theorem? Yes for the Lambek Calculus – but still unknown for type theory in general. Recent work by Icard and Moss.

9 Discussion: inferential and expressive aspects

Syntax is still needed, not pure linguistic surface form (as in 'shallow inference'), and why.

System unthinkable without hybrid of ideas from natural language and modern logic.

Systems border-lines like first-order/higher-order are irrelevant for this view of inference!

Inference takes *free ride on syntax*: low complexity. How far does this phenomenon go?

10 Follow-up: extending the reach of natural logic today

Further ubiquitous features in natural language that allow for fast inferential access?
E.g., Conservativity as domain or *role restriction*, with principles such as

$Q_1A\ R\ Q_2B$ iff $Q_1A\ R\cap(AxB)\ Q_2B$.

Algebraic principles, such as the earlier Symmetry ('conversion'), and so on.

Architecture of simple inferential subsystems (also, *time*): how do they cooperate?

Anaphora: "Everyone with a child owns a garden. Every owner of a garden waters it. *So:?*"

Inference from ambiguous expressions: how much do we get without full construction?

Other forms of inference: natural logic for *default implications* ("birds fly", etc.), instead of/ in cooperation with the monotonic inclusions that we have used as inference triggers?

11 Computational perspectives on natural language

Back to the Scholastics. Logics for iterated quantifier languages (also higher-order "Most"), without full recursion in the language. Completeness and complexity: Ian Pratt, Larry Moss.

Analysis of complexity: is monotonicity inference really easier, decidable than logics like FOL? Apply modern programming techniques (van Eijck et al., CWI Amsterdam).

Natural logic rediscovered around 2010 in work on natural language processing, information retrieval via word search + fast inferencing, and related tasks. Stanford Natural Logic Group.

Excursion 1: (Perhaps somewhat perverse:) It is *undecidable* whether a predicate occurs in a semantically monotone way in a given natural language sentence.

Excursion 2: Monotonicity much used in *fixed-point languages*. Recursive definition $P\boldsymbol{x} \leftrightarrow \varphi(P)(\boldsymbol{x})$ does not make sense in general – but it does when $\varphi(P)$ is semantically monotone.

12 Language use, natural logic, and intelligent agency

Agents have limited powers and resources. Shift emphasis away from simple fragments of a complex language to *simple agents* using only parts of a complex language successfully.

Agents act on soft information and *beliefs* that need revision as new evidence comes in. Inference forms a tandem with *acts of correction*: is there also a natural logic of correction?

Natural logic of agency itself: is there a surface logic of reasoning about agency?

Hybrids: the opposition natural and formal language seems less relevant to human agency than the fact that we constantly create mixtures of both (mathematics, specialized jargon).

13 Cognitive eality of natural logic?

Inference modules in the brain? Current neuroscience experiments on natural logic: various subsystems of human reasoning might have different locations: language, planning, etc.

14 References

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