Tutorial #7

Exercise 1

Consider the following hedonic game with three players:

$$\begin{array}{ll} \underline{\text{Player 1:}} & \{1,3\} \succ_1 \{1,2,3\} \succ_1 \{1\} \succ_1 \{1,2\} \\ \underline{\text{Player 2:}} & \{1,2,3\} \succ_2 \{1,2\} \succ_2 \{2,3\} \succ_2 \{2\} \\ \underline{\text{Player 3:}} & \{1,3\} \succ_3 \{3\} \succ_3 \{1,2,3\} \succ_3 \{2,3\} \end{array}$$

For each of the following notions of stability, either find a coalition structure that satisfies it or explain why no such coalition structure exists:

(a) Nash stability
(b) Individual stability
(c) Contractual stability

Exercise 2

Show that every hedonic game has a coalition structure that is contractually stable.

Hint: There is a very simple proof that starts out from the same basic idea as the (more involved) proof we saw for the fact that every hedonic game with a symmetric profile of additively separable preferences has a coalition structure that is Nash stable.

Exercise 3

Ali, Amy, and Ann are looking for jobs. Bea, Ben, and Bob own small companies with one job opening each. Everyone's preferences are as follows:

$$\begin{array}{llll} \underline{\text{Ali:}} & \text{Bea} \succ \text{Ben} \succ \text{Bob} & \underline{\text{Bea:}} & \text{Amy} \succ \text{Ann} \succ \text{Ali} \\ \underline{\text{Amy:}} & \text{Ben} \succ \text{Bea} \succ \text{Bob} & \underline{\text{Ben:}} & \text{Ali} \succ \text{Ann} \succ \text{Amy} \\ \underline{\text{Ann:}} & \text{Bea} \succ \text{Bob} \succ \text{Ben} & \underline{\text{Bob:}} & \text{Ann} \succ \text{Ali} \succ \text{Amy} \end{array}$$

Use the deferred-acceptance algorithm to determine a stable matching for this scenario.