Tutorial #1

Exercise 1

Watch this clip from Seinfeld, in which Jerry and George play the *Choose Game:*

`https://bit.ly/seinfeld-the-choose-game`

They repeatedly play a more basic game, until one player wins the basic game three times. Let us consider that basic game here. Beware that you may need to watch the clip more than once before you are able to infer the rules of the game.

(a) Provide a formal definition of the basic game as a normal-form game $\langle N, \boldsymbol{A}, \boldsymbol{u} \rangle$. That is, make suitable choices for N and $\boldsymbol{A}$, and then specify $\boldsymbol{u}$ in terms of a game matrix.

(b) Compute all Nash equilibria of this game.

You may also want to think about how you can model the 'first to win three' version of the game. We are going to see one way of doing this later on in the course—although, in principle, you already have all the tools you need to model such games now.

Exercise 2

Compute all (mixed and pure) Nash equilibria for each of the following normal-form games:

(a)

	L	R
T	1, 4	2, 3
B	0, 2	3, 5

(b)

	L	R
T	1, 4	2, 5
B	7, 1	2, 2

(c)

	L	R
T	5, 3	2, 8
B	2, 6	2, 2

Exercise 3

To help you understand Nash's Theorem regarding the existence of a Nash equilibrium for every normal-form game, review the following details of the proof presented in class:

(a) On the slide motivating the 'heuristic improvement dynamics' (intended to give an intuitive motivation for the formal construction used in the proof), we have made use of the term $f_i(\boldsymbol{s})(a)$, which might look a little unusual a first (with an opening bracket right after a closing bracket). For each of the subterms of this term (thus for all of i, a, $\boldsymbol{s}$, f_i, $f_i(\boldsymbol{s})$, and $f_i(\boldsymbol{s})(a)$ itself) say (*i*) what it stands for intuitively in the context of the proof and (*ii*) what type it has formally. For example, a stands for any of the actions available to player i and thus is of type A_i.

(b) In the proof of Lemma 2, one step is justified by saying that "[i]f [in a given profile] it is impossible [for a given player] to improve [her payoff] via a pure strategy, then also via a mixed strategy". This might seem counterintuitive at first, given that there are many more mixed strategies our player could try than there are pure strategies at her disposal. Why is this a valid argument nonetheless?

(c) Still in the proof of Lemma 2, there is the claim that we can obtain the equation $s_i(a) = g_i(\boldsymbol{s}, a) \,/\, \sum_{a' \in A_i} g_i(\boldsymbol{s}, a')$ from the definition of f_i. Verify that this is correct.

(d) The last part of the proof is about showing that the conditions of Brouwer's Fixed-Point Theorem are applicable. In this context, on the slides there is the claim that we can think of every strategy s_i as a stochastic vector of length $|A_i|$. If you are not sure what a stochastic vector is, look it up. Then explain how one would construct the corresponding stochastic vector for any given strategy s_i.