

Homework #6

Deadline: Tuesday, 20 May 2025, 13:00**Exercise 1** (10 points)

In the context of some of the basic structural properties often used to describe TU games, either prove or disprove each of the following three statements:

- (a) Cohesiveness implies monotonicity.
- (b) Monotonicity implies cohesiveness.
- (c) All simple games are monotonic.

Exercise 2 (10 points)

Consider the following claim regarding properties of TU games:

Every TU game that is balanced must also be cohesive.

This exercise is about providing two different proofs for this claim:

- (a) Explain how the claim follows immediately from results stated formally in class.
- (b) Prove the claim directly from first principles, i.e., without appealing to any of the formal results stated or proved in class.

Exercise 3 (10 points)

Recall that we saw an example showing that the Banzhaf value for TU games does not satisfy the axiom of efficiency: sometimes the payoffs it suggests add up to a sum that exceeds the value generated by the grand coalition. We might attempt to fix this problem by instead using what we shall call the *normalised Banzhaf value*, where we first compute the Banzhaf payoffs and then normalise by multiplying each individual payoff with the ratio between the value of the grand coalition and the sum of all Banzhaf payoffs.

Start by providing a formal definition of this solution concept. Then, for each of the four axioms characterising the Shapley value for TU games, establish whether it is also satisfied by the normalised Banzhaf value.