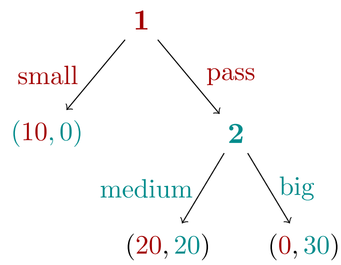
Homework #4

Deadline: Tuesday, 6 May 2025, 13:00

Submit your solutions for (up to) three of the following four exercises. If you solve all four, we will consult a random number generator to decide which three to look at and grade.

Exercise 1 (10 points)

Consider the following game. There are three kinds of prizes: *small* (worth 10 utility points), *medium* (worth 20 points), and *big* (worth 30 points). Player 1 moves first and can either claim a small prize or pass. In the former case, the game ends; in the latter case, it is Player 2's turn and she can either award a medium prize to both players or claim a big prize for herself. Thus, the game has the following extensive form:



Answer the following questions:

(a) Formally define the extensive form of this game.
(b) Compute the set of all (pure) subgame-perfect equilibria of this game.
(c) Represent the same game as a normal-form game.
(d) Compute the set of all (mixed and pure) Nash equilibria of this game.

Exercise 2 (10 points)

In class, we formally defined how to translate any given extensive game (of perfect information) into a normal-form game and we also noted that the exact same method can be used to translate imperfect-information games into the normal form. We furthermore hinted at a method for translating any given normal-form game into an extensive game of imperfect information. Provide a clear description of the latter method for the case of arbitrary normal-form games with two players (and any finite number of actions per player).

Using the methods of translation mentioned, one can find an extensive game G such that, if we first translate G into a normal-form game G' and if we then translate G' into an extensive-form game G'', then G'' need not be the same game as G. Find an example to show that this is indeed so. Then explain in what sense G and G'' are nevertheless equivalent.

Exercise 3 (10 points)

Recall the *Choose Game* played by Jerry and George in this clip from Seinfeld:

`https://bit.ly/seinfeld-the-choose-game`

In the clip, they play 'first to win three'. To keep it simple, let's focus on 'best of three' instead, i.e., let's suppose they play *up to* three rounds of the basic game but stop as soon as one of the players has won two rounds.

Sketch how you would model the full (multi-round) game as an imperfect-information game. You may draw the tree by hand (take a photo and include it in your document), as long as it is easily readable. It is also fine to not provide the full tree, as long as it is clear what the rest of the tree is supposed to look like. Report the following statistics for your game:

- the number $|Z|$ of outcome nodes
- the number $|H_1|$ of choice nodes for the first player, as well as the number $|H_1/{\sim_1}|$ of equivalence classes of choice nodes distinguishable by that player
- the number $|H_2|$ of choice nodes for the second player, as well as the number $|H_2/{\sim_2}|$ of equivalence classes of choice nodes distinguishable by that player

Next, briefly explain what the pure strategies of the two players are (there is no need to provide a complete listing). How many such strategies are there for each of the two players?

Finally, sketch the normal-form game corresponding to our imperfect-information game. When represented as a two-dimensional matrix game, how many rows and how many columns does this matrix have? How can you determine the numbers in the matrix cells?

Hint: Keep in mind that in, say, the second round a player might (but does not have to) condition her choice of action based on what happened during the first round.

Exercise 4 (10 points)

Conduct and report on an experimental study to find out how well people can play Simplified Poker, as defined in the lecture, and how well they fare against the equilibrium strategies we computed. This exercise leaves you a lot of freedom regarding the experiment you want to run and the questions you want to try and answer.

We will grade this exercise as follows: 10 marks for an excellent study that provides some original insight. 8 marks for a very good study that meets all the basic requirements one could reasonably ask for. 6 points for a fair attempt. No points for anything else.