Homework #3

Deadline: Wednesday, 23 April 2025, 18:00

Submit your solutions for (up to) two of the following three exercises. If you solve all three, we will consult a random number generator to decide which two to look at and grade.

Exercise 1 (10 points)

Recall that a *constant-sum game* is a two-player normal-form game $\langle N, \boldsymbol{A}, \boldsymbol{u}\rangle$ for which there exists a constant $c \in \mathbb{R}$ such that $u_i(\boldsymbol{a}) + u_{-i}(\boldsymbol{a}) = c$ for all action profiles $\boldsymbol{a} \in \boldsymbol{A}$. Here, for either player $i \in N = \{1, 2\}$, we use $-i$ to refer to her opponent. In case $c = 0$, the game in question is a *zero-sum game*. For a given constant-sum game $\langle N, \boldsymbol{A}, \boldsymbol{u}\rangle$ with constant c, we can define a 'corresponding' zero-sum game $\langle N, \boldsymbol{A}, \boldsymbol{u'}\rangle$, i.e., a game with the same players and action profiles, for which $u'_i(\boldsymbol{a}) = u_i(\boldsymbol{a}) - \frac{c}{2}$ for all $i \in N$ and all $\boldsymbol{a} \in \boldsymbol{A}$. Intuitively, we would expect these two games to be 'similar' in many respects.

(a) The purpose of this first part of the exercise is to make the above intuition concrete as far as Nash equilibria are concerned. Show that the set of mixed Nash equilibria of a given constant-sum game does not change when we turn it into a zero-sum game by applying the transformation defined above.

(b) Give an example for why the strategies adopted by the players might change when we transform a constant-sum game as described above. This could be absolutely anything. You do not need to restrict yourself to concepts we have defined in class.

Exercise 2 (10 points)

We have seen that for zero-sum games fictitious play always converges: after a certain number of iterations, the profile of empirical mixed strategies will be (very close to) a Nash equilibrium. But we have said nothing about the *rate of convergence*. How long will this actually take? Write a program to simulate fictitious play for arbitrary 2×2 zero-sum games (and more general classes of games if you like). Then use your program to produce a (short) report that offers some insight into the question of the rate of convergence.

This exercise leaves you a lot of freedom for how to approach the problem and what questions to provide answers to exactly. Here are some of the issues you will have to engage with: How do you generate games to test your program on? How do you measure the 'distance' between two consecutive profiles of empirical mixed strategies, so as to be able to assert convergence? How do you best visualise the data you generate?

Exercise 3 (10 points)

Consider the following normal-form game, where α might be either 0 or 2:

	L	R
T	$5, 5$	$\alpha, 10$
B	$10, \alpha$	$1, 1$

Suppose that, *a priori*, these two scenarios ($\alpha = 0$ and $\alpha = 2$) are equally likely to occur. But by the time the two players actually have to choose which actions to play, the row player *knows* the value of α, while the column player does not. Define this game formally as a Bayesian game and compute all of its pure Bayes-Nash equilibria.