Homework #1

Deadline: Tuesday, 8 April 2025, 13:00

Exercise 1 (10 points)

Recall the *Numbers Game* we played in class:

> *Every player submits a (rational) number between 0 and 100. We then compute the average (arithmetic mean) of all the numbers submitted and multiply that number with $\frac{2}{3}$. Whoever got closest to this latter number wins the game.*

To be clear, both 0 and 100 are legal actions in this game (as are all of the infinitely many numbers in between). In case of a tie for getting the number closest to two thirds of the mean, the prize is shared amongst all winners (so it is better to be the sole winner than to tie with another winner). Suppose the number of players is large (say, $n > 100$).

Note that, while in class we had defined games with *finite* sets of actions only, the definition of what constitutes a pure Nash equilibrium naturally extends to the infinite setting considered here. Answer the following questions about the existence of pure Nash equilibria and provide a short justification in each case:

(a) What are the pure Nash equilibria of this game (if any)?

(b) What changes (if anything) if the players must choose integers?

(c) What changes (if anything) if they must choose integers and we use $\frac{9}{10}$ instead of $\frac{2}{3}$?

Exercise 2 (10 points)

Compute all (mixed and pure) Nash equilibria for both of the following normal-form games:

(a)

	L	R
T	3, 5	2, 7
B	1, 6	8, 4

(b)

	L	R
T	2, 7	2, 5
B	2, 1	2, 7

Show your working in sufficient detail so as to be able to demonstrate that the equilibria you found indeed are all (and the only) Nash equilibria.

Exercise 3 (10 points)

We saw in class that every normal-form game $\langle N, \boldsymbol{A}, \boldsymbol{u} \rangle$ has a (possibly mixed) Nash equilibrium. Recall that for such a normal-form game both the set of agents N and the set of actions A_i available to each agent $i \in N$ are assumed to be finite. Of course, even if we drop this assumption of finiteness, the notion of Nash equilibrium is still well-defined. The purpose of this exercise is to explore the impact of this assumption on the existence of Nash equilibria. Specifically, we are going to explore what happens if we keep assuming that N is finite but allow for the possibility that $A_1 \cup \cdots \cup A_n$ might be infinite (which is a concise way of saying that one or more of the individual action sets might be infinite).

(a) Show that Nash's Theorem ceases to hold when we allow for the possibility that the universe of actions $A_1 \cup \cdots \cup A_n$ might be infinite. Do this by presenting a normal-form game with an infinite universe of actions that does not have any (pure or mixed) Nash equilibria. Briefly explain why this is so.

(b) If indeed Nash's Theorem ceases to hold when $A_1 \cup \cdots \cup A_n$ might be infinite, then of course it must be the case that the proof we discussed in class breaks down somewhere. But it is not immediately obvious what the problem is, not least because we have discussed Brouwer's Theorem only informally. Identify a step in the proof that breaks down under our relaxed assumption and explain your findings. Please note that this is an open-ended question that is deliberately vague. We expect to get more than a one-sentence explanation from you, but also not more than half a page of text.