Homework #1

Deadline: Wednesday, 12 April 2023, 10:00

Exercise 1 (10 points)

Recall the *Numbers Game* we played in class:

> *Every player submits a (rational) number between 0 and 100. We then compute the average (arithmetic mean) of all the numbers submitted and multiply that number with $\frac{2}{3}$. Whoever got closest to this latter number wins the game.*

To be clear, both 0 and 100 are legal actions in this game (as are all of the infinitely many numbers in between). In case of a tie for getting the number closest to two thirds of the mean, the prize is shared amongst all winners (so it is better to be the sole winner than to tie with another winner). Suppose the number of players is large (say, $n > 100$).

Note that, while in class we had defined games with *finite* sets of actions only, the definition of what constitutes a pure Nash equilibrium naturally extends to the infinite setting considered here. Answer the following questions about the existence of pure Nash equilibria and provide a short justification in each case:

(a) What are the pure Nash equilibria of this game (if any)?

(b) What changes (if anything) if the players must choose integers?

(c) What changes (if anything) if they must choose integers and we use $\frac{9}{10}$ instead of $\frac{2}{3}$?

Exercise 2 (10 points)

Compute all (mixed and pure) Nash equilibria for both of the following normal-form games:

(a)

	L	R
T	5, 9	6, 3
B	2, 2	9, 5

(b)

	L	R
T	4, 7	6, 3
B	6, 3	1, 3

Show your working in sufficient detail so as to be able to demonstrate that the equilibria you found indeed are all (and the only) Nash equilibria.

Exercise 3 (10 points)

We saw in class that every normal-form game $\langle N, \boldsymbol{A}, \boldsymbol{u} \rangle$ has a (possibly mixed) Nash equilibrium. Recall that for such a normal-form game both the set of agents N and the set of actions A_i available to each agent $i \in N$ are assumed to be finite. Of course, even if we drop this assumption of finiteness, the notion of Nash equilibrium is still well-defined. The purpose of this exercise is to explore the impact of this assumption on the existence of Nash equilibria. Specifically, we are going to explore what happens if we keep assuming that N is finite but allow for the possibility that $A_1 \cup \cdots \cup A_n$ might be infinite (which is a concise way of saying that one or more of the individual action sets might be infinite).

Does every normal form game $\langle N, \boldsymbol{A}, \boldsymbol{u} \rangle$ have a Nash equilibrium, even when $A_1 \cup \cdots \cup A_n$ might be infinite? Either prove that this is the case or provide a counterexample.