Question 1 (10 marks)
Recall the *Numbers Game* we had played in class:

> Every player submits a (rational) number between 0 and 100. We then compute the average (arithmetic mean) of all the numbers submitted and multiply that number with $\frac{2}{3}$. Whoever got closest to this latter number wins the game.

To be clear, both 0 and 100 are legal actions in this game (as are all of the infinitely many numbers in between). In case of a tie for getting the number closest to two thirds of the mean, the prize is shared amongst all winners (so it is better to be the sole winner than to tie with another winner). Suppose the number of players is large (say, $n > 100$).

Note that, while in class we had defined games with *finite* sets of actions only, the definition of what constitutes a pure Nash equilibrium naturally extends to the infinite setting considered here. Answer the following questions about the existence of pure Nash equilibria and provide a short justification in each case:

(a) What are the pure Nash equilibria of this game (if any)?
(b) What changes (if anything) if the players must choose integers?
(c) What changes (if anything) if they must choose integers and we use $\frac{9}{10}$ instead of $\frac{2}{3}$?

Question 2 (10 marks)
Compute all (mixed and pure) Nash equilibria for each of the following normal-form games:

(a) \[
\begin{array}{ccc}
T & L & R \\
1 & 4 & 3 \\
2 & 5 & 0 \\
\end{array}
\]
(b) \[
\begin{array}{ccc}
T & L & R \\
1 & 4 & 5 \\
2 & 5 & 1 \\
\end{array}
\]
(c) \[
\begin{array}{ccc}
T & L & R \\
1 & 2 & 1 \\
2 & 5 & 1 \\
\end{array}
\]

Show your working in sufficient detail so as to be able to demonstrate that the equilibria you found indeed are all (and the only) Nash equilibria.
Question 3 (10 marks)

Recall that in a normal-form game \( \langle N, A, u \rangle \) the set of action profiles \( A \) is assumed to be finite. Of course, we could easily drop this assumption of finiteness. The purpose of this question is to explore the impact of this assumption on the existence of Nash equilibria.

(a) Show that the specific proof of Nash’s Theorem on the existence of Nash equilibria we saw in class breaks down when we drop the finiteness assumption. Do this by pinpointing precisely where in the proof we made use of this assumption. A couple of clear sentences in plain English are all that is required.

(b) Now show that Nash’s Theorem itself ceases to hold when we drop the finiteness assumption (so there cannot be an alternative proof that somehow works for infinite games). Do this by presenting a normal-form game with infinitely many action profiles that does not have any (pure or mixed) Nash equilibria. Briefly explain why this is so.