Automated Reasoning

What the dictionaries say:

- **reasoning:** the process by which one judgement is deduced from another or others which are given (Oxford English Dictionary)
- **reasoning:** the drawing of inferences or conclusions through the use of *reason*
  reason: the power of comprehending, inferring, or thinking, esp. in orderly rational ways (cf. *intelligence*) (Merriam-Webster)

The scientific discipline of *Automated Reasoning* is concerned with the study of *reasoning processes* as *computational processes*.

Satisfiability Checking

An article on computational complexity in the *New York Times* from 13 July 1999 starts like this:

> *"Anyone trying to cast a play or plan a social event has come face-to-face with what scientists call a satisfiability problem. Suppose that a theatrical director feels obligated to cast either his ingénue, Actress Alvarez, or his nephew, Actor Cohen, in a production. But Miss Alvarez won't be in a play with Cohen (her former lover), and she demands that the cast include her new flame, Actor Davenport. The producer, with her own favors to repay, insists that Actor Branislavsky have a part. But Branislavsky won't be in any play with Miss Alvarez or Davenport. [...]"*

Is there a possible casting (and if there is, who will play)?

Mathematical Theorem Proving

Four colours are enough to colour any map in such a way that adjacent regions have different colours.

This has first been conjectured by Francis Guthrie, a student of Augustus DeMorgan, in 1852.

The *Four Colour Conjecture* could not be proved for over a century and became one of the most famous open problems in mathematics.

The first correct proof was given in 1976 by Appel and Haken and heavily relied on a computer program. Verifying the details of the proof famously required 1200 hours of computing time.

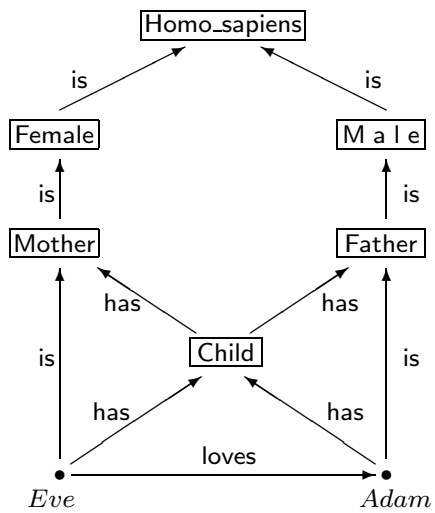
Reasoning with Temporal Constraints

From the operating instructions for a big scary machine:

- The red button has to be pressed ***before*** phase 4711
  — *or it's all going to blow up.*
- The green button has to be pressed ***during*** phase 4711
  — *or it's all going to blow up.*
- The red button has to be pressed ***after*** phase 0815
  — *or it's all going to blow up.*
- Phase 0815 has to ***overlap*** with phase 4711
  — *or it's all going to blow up.*

What if it get's more complicated? Can we use a computer to reason about this kind of information?

Knowledge Representation



We would like to be able to ask a computer questions regarding this knowledge.

Problems: What is the *precise* meaning of this "semantic network", e.g. what is the difference between *Adam* and Male or what is the difference between a loves- and an is-arrow?

Figure taken from: D. Gabbay *et al.*, *Many-Dimensional Modal Logics*, 2002. Forthcoming.

The Big Insight

Before we can use a computer to reason about a problem domain we need to ***formalise*** the problem to be able to represent it adequately within the computer.

In many cases, ***logic*** provides a good representation formalism. It is therefore useful to study logical reasoning mechanisms *in general*, that is, detached from specific applications.

Classical first-order logic is the most widely used logic (but there are many others!).

- FOL has the advantage of being very *general* and *expressive*.
- FOL has the disadvantage of being very *general* and *expressive*.

Arnold Schwarzenegger in *Twins*, to one of the bad guys:

"You have no respect for logic;
I have no respect for people
who have no respect for logic."

[Arnold proceeds to beat up the bad guy.]

Quote found on a slide of a talk by Ph. Kolaitis on *Teaching Logic*.

Early History of Automated Reasoning

1955	Semantic Tableaux (Beth, Hintikka)
1957	Logic Theorist (Newell, Shaw, Simon)
1965	Resolution (Robinson)
1968	Modern Tableaux (Smullyan)
∼1972	Prolog (Kowalski, Colmerauer)

Some Current Directions of Research

- Refinements and optimisations of existing deduction calculi
- Implementation techniques
- Interactive theorem proving
- Deduction in logics other than FOL ("non-classical logics")

Disclaimer: Please note that this and the previous slide are mostly about *Automated Deduction*. While this is probably the most important subarea of *Automated Reasoning*, the latter also includes areas such as *Model Checking*, *Constraint Satisfaction*, and *Term Rewriting*.

Applications

- Hardware verification
- Software verification
- Knowledge representation
- Logic programming (Prolog)
- Deductive databases
- Mathematical theorem proving
- AI Planning
- Natural language processing
- ...