Truth Tables

Negation ('not')

A	$\neg A$
$\top$	$\bot$
$\bot$	$\top$

Conjunction ('and')

A	B	$A \wedge B$
$\top$	$\top$	$\top$
$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\bot$
$\bot$	$\bot$	$\bot$

We write $\top$ for *true* and $\bot$ for *false*.

Truth Tables (continued)

Disjunction ('or')

A	B	$A \vee B$
$\top$	$\top$	$\top$
$\top$	$\bot$	$\top$
$\bot$	$\top$	$\top$
$\bot$	$\bot$	$\bot$

Implication ('if ... then')

A	B	$A \Rightarrow B$
$\top$	$\top$	$\top$
$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\top$
$\bot$	$\bot$	$\top$

Example

	P	Q	R	$\neg P$	$(\neg P \Rightarrow Q)$	$(\neg P \Rightarrow Q) \vee R$
1	$\top$	$\top$	$\top$	$\bot$	$\top$	$\top$
2	$\top$	$\top$	$\bot$	$\bot$	$\top$	$\top$
3	$\top$	$\bot$	$\top$	$\bot$	$\top$	$\top$
4	$\top$	$\bot$	$\bot$	$\bot$	$\top$	$\top$
5	$\bot$	$\top$	$\top$	$\top$	$\top$	$\top$
6	$\bot$	$\top$	$\bot$	$\top$	$\top$	$\top$
7	$\bot$	$\bot$	$\top$	$\top$	$\bot$	$\top$
8	$\bot$	$\bot$	$\bot$	$\top$	$\bot$	$\bot$

Exercise

Show that the following two formulas are logically equivalent:

- $A \wedge B$
- $\neg(\neg A \vee \neg B)$

Definitions

Tautology. A formula for which all rows in the truth table are $\top$ is called a *tautology.* That is, a tautology always holds, whatever the truth values of the propositional variables.

Contradiction. A formula for which all rows evaluate to $\bot$ is called a *contradiction* (or an *inconsistent* formula).

Consistency. A formula for which *at least one* row evaluates to $\top$ is called a *consistent* formula.

Precedence Hierarchy of Connectives

What's the truth value of the following formula if the values of A, B, C are all $\bot$?

$$A \Rightarrow B \wedge C$$

Precedence hierarchy. $\neg$ binds stronger than $\wedge$ binds stronger than $\vee$ binds stronger than $\Rightarrow$. Hence,

$$A \Rightarrow (B \wedge C) \quad = \quad A \Rightarrow B \wedge C \quad \neq \quad (A \Rightarrow B) \wedge C$$