Homework #3

Deadline: Tuesday, 17 November 2026, 10:00

Exercise 1 (10 points)

Recall that a preference R is called *single-peaked with respect to axis* $\triangleright$ in case R ranks alternative x above alternative y whenever, on that axis, x is located somewhere between y and the alternative ranked at the top by R. A profile $\boldsymbol{R}$ then is called *single-peaked with respect to axis* $\triangleright$ in case every preference in $\boldsymbol{R}$ is. Finally, a profile $\boldsymbol{R}$ is called *single-peaked* (for *some* axis) if there exists an axis $\triangleright$ such that $\boldsymbol{R}$ is single-peaked with respect to $\triangleright$.

Single-peakedness is an attractive property, because it allows us to circumvent some of the core problems in social choice. The purpose of this exercise is to form some intuitions about how prevalent a property we should expect this to be. Answer the following questions:

(a) Fix an arbitrary axis $\triangleright$ on a set A of m alternatives. How many different preferences are there that are single-peaked with respect to $\triangleright$?

(b) Show that for sufficiently large numbers n of voters and m of alternatives, the proportion of profiles that are single-peaked (with respect to *some* axis) drops below 1%.

Exercise 2 (10 points)

Suppose we elicit pairwise comparisons of alternatives from individual voters one by one, so at any stage in the elicitation process we have one partial order on the set of alternatives for each voter. Provide a polynomial algorithm to check whether a given alternative is the necessary plurality winner for a given partial profile of this kind.