Computational Social Choice 2020

Ulle Endriss
Institute for Logic, Language and Computation
University of Amsterdam

[http://www.illc.uva.nl/~ulle/teaching/comsoc/2020/]

Plan for Today

It is not always in the best interest of voters to truthfully reveal their preferences when voting. This is called *strategic manipulation*.

We are going to see two theorems that show that this can't be avoided:

- *Gibbard-Satterthwaite Theorem* (1973/1975)
- *Duggan-Schwartz Theorem* (2000)

The latter generalises the former by considering irresolute voting rules, where voters have to strategise w.r.t. *sets* of winners.

We then are going to (briefly) review three approaches for addressing the challenges raised by strategic manipulation:

- *Domain restrictions:* excluding problematic profiles
- *Computational barriers:* making manipulation intractable
- *Informational barriers:* hiding information from manipulators

Example

Recall that under the *plurality rule* (used in most political elections) the candidate ranked first most often wins the election.

Assume the preferences of the people in, say, Florida are as follows:

49%:	Bush $\succ$ Gore $\succ$ Nader
20%:	Gore $\succ$ Nader $\succ$ Bush
20%:	Gore $\succ$ Bush $\succ$ Nader
11%:	Nader $\succ$ Gore $\succ$ Bush

So even if nobody is cheating, Bush will win this election.

> *It would have been in the interest of the Nader supporters to manipulate, i.e., to misrepresent their preferences.*

Is there a better voting rule that avoids this problem?

Truthfulness, Manipulation, Strategyproofness

For now, we only deal with *resolute* voting rules $F : \mathcal{L}(A)^n \to A$.

Unlike for all earlier results discussed, we now have to distinguish:

- the *ballot* a voter reports
- her actual *preference* order

Both are elements of $\mathcal{L}(A)$. If they coincide, then the voter is *truthful*.

F is *strategyproof* (or *immune to manipulation*) if for no voter $i \in N$ there exist a profile $\boldsymbol{R}$ (including i's *truthful preference* R_i) and an *untruthful ballot* R'_i for i such that R_i ranks $F(R'_i, \boldsymbol{R}_{-i})$ above $F(\boldsymbol{R})$.

Thus: Nobody has an incentive to misrepresent their preferences.

Notation: $(R'_i, \boldsymbol{R}_{-i})$ is the profile obtained by replacing R_i in $\boldsymbol{R}$ by R'_i.

Importance of Strategyproofness

Why do we want voting rules to be strategyproof?

- "Thou shalt not bear false witness against thy neighbour."
- Voters should not have to waste resources pondering over what other voters will do and trying to figure out how best to respond.
- If everyone strategises (and makes mistakes when guessing how others will vote), then the final ballot profile will be very far from the electorate's true preferences and thus the election winner may not be representative of their wishes at all.

The Gibbard-Satterthwaite Theorem

Recall: A resolute SCF F is *surjective* if for every alternative $x \in A$ there exists a profile $\boldsymbol{R}$ such that $F(\boldsymbol{R}) = x$.

Gibbard (1973) and Satterthwaite (1975) independently proved:

Theorem 1 (Gibbard-Satterthwaite) *Any resolute SCF for $\geqslant 3$ alternatives that is surjective and strategyproof is a dictatorship.*

Remarks:

- a *surprising* result + not applicable in case of *two* alternatives
- The opposite direction is clear: *dictatorial* $\Rightarrow$ *strategyproof*
- *Random* rules don't count (but might be "strategyproof").

A. Gibbard. Manipulation of Voting Schemes: A General Result. *Econometrica*, 1973.

M.A. Satterthwaite. Strategy-proofness and Arrow's Conditions. *Journal of Economic Theory*, 1975.

Proof

We shall prove the Gibbard-Satterthwaite Theorem to be a corollary of the Muller-Satterthwaite Theorem (even if, historically, G-S came first).

Recall the *Muller-Satterthwaite Theorem:*

- Any *resolute* SCF for $\geqslant 3$ alternatives that is *surjective* and *strongly monotonic* must be a *dictatorship*.

We shall prove a lemma showing that strategyproofness implies strong monotonicity (and we'll be done). ✓ (Details are in my review paper.)

For other short proofs of G-S, see Barberà (1983) and Benoît (2000).

S. Barberà. Strategy-Proofness and Pivotal Voters: A Direct Proof the Gibbard-Satterthwaite Theorem. *International Economic Review*, 1983.

J.-P. Benoît. The Gibbard-Satterthwaite Theorem: A Simple Proof. *Economic Letters*, 2000.

U. Endriss. Logic and Social Choice Theory. In A. Gupta and J. van Benthem (eds.), *Logic and Philosophy Today*, College Publications, 2011.

Strategyproofness implies Strong Monotonicity

Lemma 2 *Any resolute SCF that is strategyproof (SP) must also be strongly monotonic (SM).*

- SP: no incentive to vote untruthfully
- SM: $F(\boldsymbol{R}) = x \;\Rightarrow\; F(\boldsymbol{R'}) = x$ if $N^{\boldsymbol{R}}_{x \succ y} \subseteq N^{\boldsymbol{R'}}_{x \succ y}$ for all y

Proof: We'll prove the contrapositive. So assume F is *not* SM.
So there exist $x, x' \in A$ with $x \neq x'$ and profiles $\boldsymbol{R}, \boldsymbol{R'}$ such that:

- $N^{\boldsymbol{R}}_{x \succ y} \subseteq N^{\boldsymbol{R'}}_{x \succ y}$ for all alternatives y, including x' $(\star)$
- $F(\boldsymbol{R}) = x$ and $F(\boldsymbol{R'}) = x'$

Moving from $\boldsymbol{R}$ to $\boldsymbol{R'}$, there must be a *first* voter affecting the winner.
So w.l.o.g., assume $\boldsymbol{R}$ and $\boldsymbol{R'}$ differ only w.r.t. voter i. Two cases:

- $i \in N^{\boldsymbol{R'}}_{x \succ x'}$: if i's true preferences are as in $\boldsymbol{R'}$, she can benefit from voting instead as in $\boldsymbol{R}$ $\Rightarrow$ ↯ [SP]
- $i \notin N^{\boldsymbol{R'}}_{x \succ x'} \Rightarrow^{(\star)} i \notin N^{\boldsymbol{R}}_{x \succ x'} \Rightarrow i \in N^{\boldsymbol{R}}_{x' \succ x}$: if i's true preferences are as in $\boldsymbol{R}$, she can benefit from voting as in $\boldsymbol{R'}$ $\Rightarrow$ ↯ [SP]

The Bigger Picture

We have by now seen three impossibility theorems for *resolute* SCF's, all of which apply in case there are at least *three alternatives:*

Gibbard-Satterthwaite Theorem
[surjective + strategyproof $\Rightarrow$ dictatorial]

$\Uparrow$

Muller-Satterthwaite Theorem
[surjective + strongly monotonic $\Rightarrow$ dictatorial]

$\Uparrow$

Arrow's Theorem
[Paretian + independent $\Rightarrow$ dictatorial]

We proved Arrow's Theorem by analysing when a coalition can force a pairwise ranking. The other two results followed by comparing axioms.

Shortcomings of Resolute Voting Rules

The Gibbard-Satterthwaite Theorem only applies to *resolute* rules.
But the restriction to resolute rules is problematic:

- No "natural" voting rule is resolute (w/o tie-breaking rule).
- We can get very basic impossibilities for resolute rules:

  We've seen already that *no resolute* voting rule for *two voters* and *two alternatives* can be both *anonymous* and *neutral*.

We therefore should really be analysing *irresolute* voting rules ...

Manipulability w.r.t. Psychological Assumptions

To analyse manipulability when we might get a set of winners, we need to make assumptions on how voters rank *sets of alternatives*, e.g.:

- A voter is an *optimist* if she prefers X over Y whenever she prefers her favourite $x \in X$ over her favourite $y \in Y$.
- A voter is a *pessimist* if she prefers X over Y whenever she prefers her least preferred $x \in X$ over her least preferred $y \in Y$.

Now we can speak about manipulability by certain types of voters:

- F is called *immune to manipulation by optimistic voters* if no optimistic voter can ever benefit from voting untruthfully.
- F is called *immune to manipulation by pessimistic voters* if no pessimistic voter can ever benefit from voting untruthfully.

Axiom: Nonimposition

Let F be an *irresolute* voting rule/SCF $F : \mathcal{L}(A)^n \to 2^A \setminus \{\emptyset\}$.

- F is *nonimposed* if for every alternative x there exists a profile $\boldsymbol{R}$ under which x is the unique winner: $F(\boldsymbol{R}) = \{x\}$.

For comparison, *surjectivity* means that for every element in the co-domain of F there is an input producing that element. Thus:

$$\text{resolute} \Rightarrow (\text{nonimposed} = \text{surjective})$$

Dictatorships for Irresolute Rules

Let F be an *irresolute* voting rule/SCF $F : \mathcal{L}(A)^n \to 2^A \setminus \{\emptyset\}$.

There are two natural notions of dictatorship for such rules:

- Voter $i \in N$ is called a (strong) *dictator* if $F(\boldsymbol{R}) = \{\text{top}(R_i)\}$ for every profile $\boldsymbol{R} \in \mathcal{L}(A)^n$.
- Voter $i \in N$ is called a *weak dictator* if $\text{top}(R_i) \in F(\boldsymbol{R})$ for every profile $\boldsymbol{R} \in \mathcal{L}(A)^n$. (Such a voter is also called a *nominator*.)

F is called *weakly dictatorial* if it has a weak dictator. Otherwise F is called *strongly nondictatorial*.

The Duggan-Schwartz Theorem

There are several extensions of the Gibbard-Satterthwaite Theorem for irresolute voting rules. The Duggan-Schwartz Theorem is usually regarded as the most important of these results.

Our statement of the theorem follows Taylor (2002):

Theorem 3 (Duggan and Schwartz, 2000) *Any voting rule for $\geqslant 3$ alternatives that is nonimposed and immune to manipulation by both optimistic and pessimistic voters is weakly dictatorial.*

Proof: Omitted.

Note that the Gibbard-Satterthwaite Theorem is a direct corollary.

J. Duggan and T. Schwartz. Strategic Manipulation w/o Resoluteness or Shared Beliefs: Gibbard-Satterthwaite Generalized. *Social Choice and Welfare*, 2000.

A.D. Taylor. The Manipulability of Voting Systems. *The American Mathematical Monthly*, 2002.

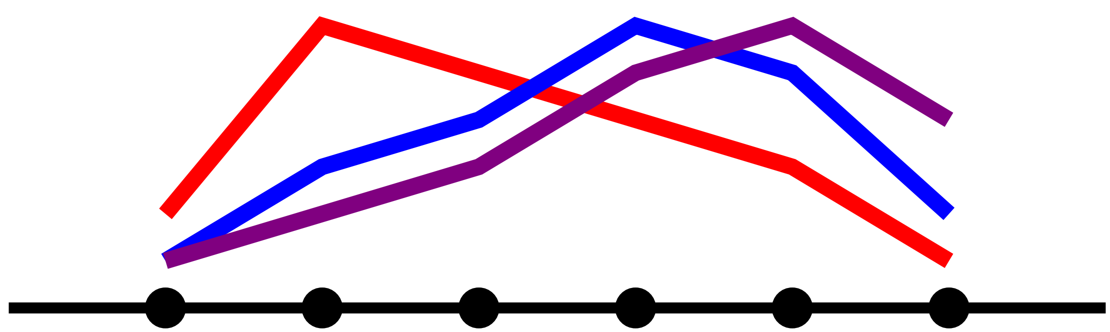
Barriers to Strategic Manipulation

Is that the end of it? No! Next we are going to briefly review three kinds of barriers against strategic manipulation . . .

Domain Restriction: Single-Peaked Preferences

We only discuss the oldest and most famous domain restriction . . .

A profile $\boldsymbol{R} = (R_1, \ldots, R_n)$ is *single-peaked* if we can arrange the alternatives from left to right along some dimension $\gg$ such that R_i ranks x above y whenever x is between y and $\mathrm{top}(R_i)$ according to $\gg$.



Sometimes a natural assumption: traditional political parties, agreeing on a number (e.g., legal drinking age), . . .

Strategyproofness of the Median-Voter Rule

For a given dimension $\gg$, the *median-voter rule* asks each voter for her top alternative and elects the alternative proposed by the voter corresponding to the median w.r.t. $\gg$.

Theorem 4 *If an odd number of voters have preferences that are single-peaked w.r.t. $\gg$, then the median-voter rule is strategyproof.*

Proof: W.l.o.g., our manipulator's top alternative is *to the right* of the median (the winner). If she declares a peak further to the right, nothing will change. If she declares a peak further to the left, either nothing will change, or the new winner will be even worse. ✓

This is closely related to Black's *Median Voter Theorem*, showing that under the same conditions a Condorcet winner exists and is elected.

D. Black. On the Rationale of Group Decision-Making. *The Journal of Political Economy*, 1948.

Computational Barriers to Manipulation

Every voting rule can be manipulated in some profiles. But even when it is *possible* to manipulate, maybe actually doing so is *difficult*?

Tools from *complexity theory* can help make this idea precise.

> *If manipulation is computationally intractable for F, then we might F consider resistant (but not immune) to manipulation.*

Does not work for most rules, but STV manipulation is NP-hard. Some nice results for resistance to group manipulation for other rules.

Discussion: Practical significance of these results is debatable.

J.J. Bartholdi III, C.A. Tovey, and M.A. Trick. The Computational Difficulty of Manipulating an Election. *Social Choice and Welfare*, 1989.

V. Conitzer and T. Walsh. Barriers to Manipulation in Voting. In F. Brandt et al. (eds.), *Handbook of Computational Social Choice*. CUP, 2016.

Informational Barriers to Manipulation

Suppose voter i has only *partial information* about the profile. If π is a function mapping any truthful profile $\boldsymbol{R}$ to the information $\pi(\boldsymbol{R})$ given to i, then i must consider possible any profile in this set:

$$\mathcal{W}_i^{\pi(\boldsymbol{R})} \quad = \quad \{\, \boldsymbol{R}' \in \mathcal{L}(A)^n \mid \pi(\boldsymbol{R}) = \pi(\boldsymbol{R}') \text{ and } R_i = R'_i \,\}$$

<u>Example:</u> π might be an *opinion poll* that returns, say, the winner of the election, or the plurality score of every alternative.

Now i will manipulate using R'_i only if doing so is *strictly better* for her in at least one profile in $\mathcal{W}_i^{\pi(\boldsymbol{R})}$ and *not worse* in any of the others.

Limited positive results to date. For instance, the *antiplurality rule* is strategyproof when voters only have *winner information*.

<u>Remark:</u> Interesting, still very much underexplored research direction.

A. Reijngoud and U. Endriss. Voter Response to Iterated Poll Information. AAMAS-2012.

Summary

We saw that *strategic manipulation* is a major problem in voting:

- *Gibbard-Satterthwaite:* SP + surjectivity $\Rightarrow$ dictatorship
- *Duggan-Schwartz:* dropping resoluteness does not help much

But we also saw that there are approaches for tackling this problem:

- *Domain restrictions*
- *Computational barriers*
- *Informational barriers*

The study of strategic manipulation is very much at the intersection of social choice theory with *game theory* and *mechanism design*.

Other forms of strategic behaviour that may occur in the context of elections include *bribery* and *gerrymandering*.

What next? Moving away from the classical model of voting, we will start looking into new ideas for democratic decision making.