Homework #4

Deadline: Thursday, 7 May 2020, 18:00

Question 1 (10 marks)

Your main task for this exercise is to design a resolute multiwinner voting rule F with ranked ballots for $n \in \mathbb{N}$ voters and a finite set $A = \{a_1, a_2, \ldots, a_m\}$ of alternatives to elect a committee of size $k = 3$ such that F is strategyproof under the following two assumptions. The first assumption is that the true preferences of the voters regarding individual alternatives are single-peaked with respect to the dimension $a_1 \gg a_2 \gg \cdots \gg a_m$. The second assumption is that any given voter will prefer committee S to committee S' if and only if she prefers her second most preferred member of S to her second most preferred member of S'. Your voting rule should be as 'reasonable' as possible.

Start by writing down formal definitions of single-peakedness and strategyproofness—specifically for the scenarios of interest here (that is, do not simply recall the standard definitions, but adapt them to the setting considered here). Then define your rule, show that it is strategyproof given our assumptions, and argue (briefly) why you consider it a reasonable rule. Finally, (again, briefly) point out one shortcoming of your rule.

Question 2 (10 marks)

A *weak Condorcet winner* is an alternative that wins or draws against any other alternative in pairwise majority contests. Just like a (normal) Condorcet winner, a weak Condorcet winner need not exist. Unlike a Condorcet winner, however, when it does exist, a weak Condorcet winner need not be unique. In the context of voting in combinatorial domains, show that when voters model their preferences using the language of prioritised goals and each voter specifies only a single goal, then there always is a weak Condorcet winner.