Homework #4

Deadline: Monday, 20 May 2019, 18:00

Question 1 (10 marks)

Consider the following two definitions for the unanimity of a judgment aggregation rule F:

- *Propositionwise unanimity:* F is unanimous if, for every formula $\varphi \in \Phi$ and every profile $\boldsymbol{J} \in \mathcal{J}(\Phi)^n$, it is the case that $\varphi \in J_i$ for all agents $i \in N$ implies $\varphi \in F(\boldsymbol{J})$. That is, if every agent accepts φ, then so should the aggregation rule.
- *Simple unanimity:* F is unanimous if, for every judgment set $J \in \mathcal{J}(\Phi)$, it is the case that $F(J, \ldots, J) = J$. That is, if all agents report exactly the same judgment set, then that same set should also be the output of the aggregation rule.

Show that these two definitions indeed define different concepts. Then show that, in the presence of independence and complement-freeness of outcomes, they coincide.

Question 2 (10 marks)

Recall that the impossibility direction of the agenda characterisation theorem due to Nehring and Puppe proved in class establishes that all neutral, independent, and monotonic judgment aggregation rules that guarantee consistent and complete outcomes (for a sufficiently rich agenda Φ) must be dictatorships. We might consider weakening the requirement of returning judgment sets that are complete and only ask for outcomes that are deductively closed: judgment set $J \subseteq \Phi$ is *deductively closed* (with respect to Φ) if $J \models \varphi$ implies $\varphi \in J$ for every proposition $\varphi \in \Phi$. The purpose of this exercise is to show that this relaxation of our requirements does not significantly improve the situation. Prove the following theorem:

> *Every propositionwise-unanimous, neutral, independent, and monotonic aggregation rule F that guarantees consistent and deductively closed outcomes for an agenda Φ violating the median property must be an oligarchy.*

Here an *oligarchy* is an aggregation rule F for which there exists a coalition $C^\star \subseteq N$ such that $F(\boldsymbol{J}) = \{\varphi \in \Phi \mid C^\star \subseteq N_\varphi^{\boldsymbol{J}}\}$ for every profile $\boldsymbol{J} \in \mathcal{J}(\Phi)^n$. Thus, a proposition gets accepted by the rule if and only if all oligarchs accept it. Observe that dictatorships and the unanimity rule are examples for such oligarchies. The *unanimity axiom* (defined in the previous exercise) is included in our list of assumptions to rule out trivial counterexamples such as the aggregation rule that always returns the empty judgment set.

Hints: Recall that every neutral and independent aggregation rule (for a nontrivial agenda) can be described in terms of a set $\mathcal{W}$ of winning coalitions. Start by establishing some of the structural properties of $\mathcal{W}$, given the assumptions made for our theorem. They will be similar but not identical to the structural properties discussed in class. Then think about what you can say about the intersection of all winning coalitions in $\mathcal{W}$.

Question 3 (10 marks)

Suppose we can divide the agenda into a set of premises and a set of conclusions, each of which is closed under complementation. Recall that the premise-based rule is resolute, i.e., there will always be a single outcome. Analyse the computational complexity of the problem of determining whether a given formula φ in the agenda will belong to this single outcome for a given profile $\boldsymbol{J}$. Do so initially under the assumption that every propositional variable occurring anywhere within one of the formulas in the agenda is itself one of the premises. Then repeat your analysis without making this assumption.