Homework #4

Deadline: Wednesday, 4 March 2015, 11:00

Question 1 (10 marks)

Consider the following two definitions for the unanimity of a judgment aggregation rule F:

- *Propositionwise unanimity:* F is unanimous if, for every formula $\varphi \in \Phi$ and every profile $\boldsymbol{J} \in \mathcal{J}(\Phi)^n$, it is the case that $\varphi \in J_i$ for all agents $i \in \mathcal{N}$ implies $\varphi \in F(\boldsymbol{J})$. That is, if every agent accepts φ, then so should the aggregation rule.
- *Simple unanimity:* F is unanimous if, for every judgment set $J \in \mathcal{J}(\Phi)$, it is the case that $F(J, \ldots, J) = J$. That is, if all agents report exactly the same judgment set, then that same set should also be the output of the aggregation rule.

Show that these two definitions indeed define different concepts. Then show that, in the presence of independence and complement-freeness, they coincide.

Question 2 (10 marks)

Prove that, when restricted to agendas including a tautology, every judgment aggregation rule that is neutral, consistent, and complete also satisfies the axiom of simple unanimity.

Question 3 (10 marks)

In binary aggregation, the *Hamming distance* between two ballots $B, B' \in \{0,1\}^m$ is the number of issues on which they differ:

$$H(B, B') \ := \ \sum_{j=1}^{m} |b_j - b'_j|$$

Recall that the *average-voter rule* returns, from amongst the ballots submitted by the agents, those ballots that minimise the sum of the Hamming distances to the other ballots. Define the *plurality-voter rule* as the aggregation rule that returns, from amongst the ballots submitted by the agents, those ballots that have been submitted most often. (Note that both of these rules are irresolute and would have to be combined with a tie-breaking rule, if we always required a unique model as the outcome. This is irrelevant for this question.)

Both of these rules are generalised dictatorships. Clearly, across all generalised dictatorships, the average-voter rule minimises the sum of the Hamming distances between the outcome and the individual ballots (that is exactly how it has been defined!). The purpose of this question is to illustrate how much worse the plurality-voter rule is in this respect. Describe a profile for which the average-voter rule returns a model that is more than 100 times as close to the profile as the model returned by the plurality-voter rule.