Homework #1

Deadline: Wednesday, 11 February 2015, 11:00

Question 1 (10 marks)

All of the cake-cutting procedures we have discussed in class can be described more systematically in terms of asking the agents to respond to a series of *marking queries* ("given value α and point x, please indicate point y such that the slice $[x, y]$ has value α to you!") and *evaluation queries* ("given points x and y, please report value α such that the slice $[x, y]$ has value α to you!").

Describe a procedure for dividing a cake between four agents that guarantees that each agent obtains at least 1/6 of the cake according to her own valuation and that uses at most three marking queries. (For comparison, recall that both the *last-diminisher* and the *divide-and-conquer* procedures offer better fairness guarantees, namely at least 1/4 to each agent, but require up to four marking queries, one per agent, already in the first round.)

(Adapted from J. Robertson and W. Webb, *Cake-Cutting Algorithms*, A.K. Peters, 1998.)

Question 2 (10 marks)

In voting theory, a *profile* is a vector of strict linear orders on the set of alternatives, one for each voter. Given such a profile, we can compute the corresponding *weighted majority graph* (WMG): a directed graph, with the vertices being the alternatives, for which—for any two alternatives x and y—the edge from x to y is labelled with the margin of victory of x over y:

$$m(x, y) \quad = \quad (\#\text{voters ranking } x \text{ above } y) - (\#\text{voters ranking } y \text{ above } x)$$

Thus, $m(x, y)$ might be negative, namely when a strict majority of voters prefer y over x. Clearly, the WMG contains *less information* than the the original profile: we cannot unambiguously compute the profile from the WMG. Still, for some voting rules, having the WMG is enough to compute the election winner(s), while for others it is not.

(a) Prove that the *Borda* winners can always be computed from the WMG by giving a definition of the Borda rule in terms of the WMG.

(b) Prove that the *plurality* winners cannot always be computed from the WMG.

(Please turn over)

Question 3 (10 marks)

This question is about *uniform quota rules* in judgment aggregation and relates to the basic impossibility theorem due to List and Pettit.

- Consider the uniform quota rule F_λ with quota $\lambda := \frac{2}{3}n$. Which of the following properties are satisfied by F_λ: anonymity, neutrality, independence, completeness, complement-freeness? Justify your answers, writing one sentence per property.

- Consider the agenda $\Phi = \{p, \neg p, q, \neg q, r, \neg r, p \vee q \vee r, \neg(p \vee q \vee r)\}$. Characterise the class of all uniform quota rules (in terms of their quota λ) that are guaranteed to return a consistent judgment set for any admissible (i.e., complete and consistent) profile over Φ. Briefly justify your answer.