Homework #6

Deadline: Monday, 9 December 2013, 11:00

Question 1 (10 marks)

Which of the following statements is true? Give either a proof (in the affirmative case) or a counterexample (otherwise).

(a) Any agreement with maximal utilitarian social welfare is Pareto efficient.

(b) No agreement can maximise both utilitarian and egalitarian social welfare.

(c) Any agreement that is optimal with respect to the leximin ordering is both Pareto efficient and maximises egalitarian social welfare.

(d) If preferences are dichotomous (meaning: $u_i(A) = 0$ or $u_i(A) = 1$ for any agent i and any agreement A), then the utilitarian SWO and the leximin ordering coincide.

(e) The egalitarian SWO respects the Pigou-Dalton transfer principle, and it is the only k-rank dictator SWO to do so.

Question 2 (10 marks)

Suppose there are n agents located anywhere on the interval $[0, 1]$. We have to decide where to build an amusement park A, also anywhere on the same interval. The *dis*utility of an agent is its distance to A.

(a) What is the solution selected by the egalitarian CUF?

(b) What is the solution selected by the elitist (n-rank dictator) CUF?

(c) For arbitrary $k \leq n$, give a general algorithm to compute a solution that is optimal with respect to the k-rank dictator CUF. What is the complexity of your algorithm?

Question 3 (10 marks)

Recall the framework for representing utility functions over subsets of PS by means of weighted propositional formulas. Let $n = |PS|$. A *complete cube* is a conjunction of literals of length n that includes exactly one of p and $\neg p$ for every $p \in PS$. Establish the relative succinctness of $\mathcal{L}(pcubes, \mathbb{R})$, the language of positive cubes, and $\mathcal{L}(ccubes, \mathbb{R})$, the language of complete cubes.