Homework #5

Question 1 (10 marks)

In the context of judgment aggregation, consider these definitions:

- An aggregator F is called *deductively closed* for agenda Φ, if $\varphi \in \Phi$, $\Delta \subseteq F(\boldsymbol{J})$, and $\Delta \models \varphi$ together imply $\varphi \in F(\boldsymbol{J})$. Note that in the presence of consistency, completeness implies deductive closure, but not *vice versa*.

- An aggregator F is called *oligarchic* if there exists a set of judges $O \subseteq \mathcal{N}$ such that an agenda formula φ is accepted if and only if everyone in that set accepts it, i.e., $\varphi \in F(\boldsymbol{J})$ if and only if $\varphi \in F(J_i)$ for all $i \in O$. Dictatorships and the unanimous rule are both (extreme) examples for oligarchic aggregators.

Let Φ be an arbitrary agenda that violates the median property. Sketch a proof showing that any aggregator F that is unanimous, neutral, independent, monotonic as well as consistent and deductively closed for Φ must be oligarchic.

Hint: Compare the claim to the (impossibility direction of the) agenda characterisation result due to Nehring and Puppe proved in class. It is sufficient if you explain how the proof given in class needs to be altered to obtain the desired result.

Question 2 (10 marks)

While strategic manipulation is a central topic in voting theory, it has received relatively little attention in judgment aggregation. The reason might be that manipulation in judgment aggregation is not a straightforward concept to define. In voting theory, an agent is said to be able to manipulate if she can obtain a preferred outcome by misrepresenting her input to the aggregation mechanism. While it is clear how to define misrepresentation also in judgment aggregation, there is no notion of preference to refer to to define an agent's incentives to engage in such an act of misrepresentation.

Suggest a definition of strategic manipulation in judgment aggregation (different from the "main definition" sketched in class) and discuss its advantages and shortcomings. Discuss either a proposal from the literature or your own ideas. Write at most two pages of text.