Homework #3

Deadline: Monday, 18 November 2013, 11:00

Question 1 (10 marks)

An important line of research in social choice theory is aimed at understanding the *frequency* with which certain undesirable situations, e.g., Condorcet cycles or opportunities for strategic manipulation, occur. While classical paradoxes and impossibility theorems show that these situations can never be ruled out entirely, it is conceivable that they might be very infrequent, in which case the situation would not actually be as bleak as the classical results suggest. To measure frequency we have to make assumptions regarding the likelihood of certain profiles of preferences to occur. The standard approach is to assume that every logically possible profile is equally likely to occur. This is known as the *impartial culture* (IC) assumption. Under the closely related *impartial anonymous culture* (IAC) assumption, each *anonymous* profile is taken to be equally likely to occur. For example, if there are two alternatives and two voters, then under the IC assumption each of the four possible profiles $(x \succ y, x \succ y)$, $(x \succ y, y \succ x)$, $(y \succ x, x \succ y)$ and $(y \succ x, y \succ x)$ has the same probability of $\frac{1}{4}$ to occur. Under the IAC assumption, on the other hand, we do not distinguish $(x \succ y, y \succ x)$ and $(y \succ x, x \succ y)$, and thus each of $(x \succ y, x \succ y)$, $(x \succ y, y \succ x)$ and $(y \succ x, y \succ x)$ has the same probability of $\frac{1}{3}$ to occur.

We can use assumptions such as the IC or the IAC assumption to generate a large number of profiles. For a given voting rule, we can then check for each voter whether she would have an incentive to manipulate, if we assume that her true preferences are as indicated by the profile and all other voters' ballots are as indicated by the profile. This approach allows us to compare the *degree of manipulability* of different voting rules. (Although much more difficult, in principle it is also possible to derive these degrees of manipulability using analytical methods, rather than to make use of simulations.) While interesting, this kind of approach has been criticised for being based on arguably unrealistic assumptions: the distribution of preferences in a real electorate will have little in common with either the IC or the IAC assumption.

The purpose of this question is to explore the frequency-based approach further:

(a) Find out about one further approach to generating test data for election simulations proposed in the literature. Explain the approach and briefly discuss its advantages and disadvantages. Write at most one page of text.

(b) Suggest a new approach to automatically generating profiles. Argue why (and under what circumstances) your approach will produce realistic or otherwise interesting data. Also discuss the limitations of your approach. Write at most one page of text.

(Please turn over)

Question 2 (10 marks)

The purpose of this exercise is to explore the boundaries of some of the impossibility theorems we have discussed.

(a) Does the Muller-Satterthwaite Theorem continue to hold when we replace strong monotonicity by weak monotonicity?

(b) Does the Gibbard-Satterthwaite Theorem continue to hold when we drop the condition of surjectivity?

(c) Does the Duggan-Schwartz Theorem continue to hold when we replace the condition of immunity against manipulation by both optimistic and pessimistic voters by immunity against manipulation by optimistic voters only?

(d) Does the Duggan-Schwartz Theorem continue to hold when we replace the condition of immunity against manipulation by both optimistic and pessimistic voters by immunity against manipulation by pessimistic voters only?

(e) Let us call a voter *cautious* if she prefers a set of alternatives A to another set B only if she ranks her least preferred alternative in A above her most preferred alternative in B. That is, such a voter would only consider manipulating if the worst way of tie breaking would yield a better result for her than the best way of tie braking when she votes truthfully. Does the Duggan-Schwartz Theorem continue to hold when we replace the condition of immunity against manipulation by both optimistic and pessimistic voters by immunity against manipulation by cautious voters?

Justify your answers. In case you show that a given theorem ceases to hold under changed conditions by proving a specific voting rule meets all the requirements stated, also indicate why that same voting rule does not constitute a counterexample to the original theorem.