Homework #6

Deadline: Monday, 15 October 2012, 13:00

Question 1 (10 marks)

Which of the following statements is true? Give either a proof (in the affirmative case) or a counterexample (otherwise).

(a) Any agreement with maximal utilitarian social welfare is Pareto efficient.

(b) No agreement can maximise both utilitarian and egalitarian social welfare.

(c) Any agreement that is optimal with respect to the leximin ordering is both Pareto efficient and maximises egalitarian social welfare.

(d) If preferences are dichotomous (meaning: $u_i(A) = 0$ or $u_i(A) = 1$ for any agent i and any agreement A), then the utilitarian SWO and the leximin ordering coincide.

(e) The egalitarian SWO respects the Pigou-Dalton transfer principle, and it is the only k-rank dictator SWO to do so.

Question 2 (10 marks)

Suppose there are n agents located anywhere on the interval $[0, 1]$. We have to decide where to build an amusement park A, also anywhere on the same interval. The *dis*utility of an agent is its distance to A.

(a) What is the solution selected by the egalitarian CUF?

(b) What is the solution selected by the elitist (n-rank dictator) CUF?

(c) For arbitrary $k \leq n$, give a general algorithm to compute a solution that is optimal with respect to the k-rank dictator CUF. What is the complexity of your algorithm?

Question 3 (10 marks)

What is the computational complexity of (the decision variant of) the problem of finding an allocation of indivisible goods to agents that maximises elitist social welfare?

(a) First state your answer (and your proof) with respect to the explicit form of representing utility functions (where the size of the representation of a function is proportional to the number of bundles to which it assigns a non-zero value).

(b) Then repeat the same exercise, this time assuming that utility functions are expressed using the language of weighted goals (without restrictions to the types of formulas used).

Note that both of these languages can express valuation functions that need not be monotonic (that is, simply giving all the items to one agent will usually not yield an allocation with maximal elitist social welfare).