Homework #2

Deadline: Monday, 17 September 2012, 13:00

Question 1 (10 marks)

This question concerns two alternative definitions of the property of *strong monotonicity* of a *resolute* voting rule $F : \mathcal{L}(\mathcal{X})^{\mathcal{N}} \to \mathcal{X}$. Recall the definition given in class:

(a) F is called strongly monotonic if $x^\star = F(\boldsymbol{R})$ implies $x^\star = F(\boldsymbol{R}')$ for any alternative $x^\star$ and any two profiles $\boldsymbol{R}$ and $\boldsymbol{R}'$ with $N^{\boldsymbol{R}}_{x^\star \succ y} \subseteq N^{\boldsymbol{R}'}_{x^\star \succ y}$ for all alternatives $y \in \mathcal{X} \setminus \{x^\star\}$.

An alternative definition to be found in the literature is the following:

(b) F is called strongly monotonic if $F(\boldsymbol{R}') = F(\boldsymbol{R})$ or $F(\boldsymbol{R}') = x^\star$ for any alternative $x^\star$ and any two profiles $\boldsymbol{R}$ and $\boldsymbol{R}'$ satisfying $N^{\boldsymbol{R}}_{x^\star \succ y} \subseteq N^{\boldsymbol{R}'}_{x^\star \succ y}$ and $N^{\boldsymbol{R}}_{y \succ z} = N^{\boldsymbol{R}'}_{y \succ z}$ for all alternatives $y, z \in \mathcal{X} \setminus \{x^\star\}$.

Explain each definition in plain English and briefly argue why it is a reasonable definition. Then check whether the two definitions are equivalent (proof or counterexample).

Notation: Recall that $N^{\boldsymbol{R}}_{x \succ y}$ is the set of individuals who rank alternative x above alternative y under profile $\boldsymbol{R}$.

Question 2 (10 marks)

In analogy to the definition of Condorcet winners, a *Condorcet loser* is a candidate that would lose against any other candidate in a pairwise contest.

(a) Give an example that shows that the plurality rule *can* elect a Condorcet loser.

(b) Prove that the Borda rule *never* elects a Condorcet loser.

Remark: It is in fact possible to show that the Borda rule is the *only* positional scoring rule (with a strictly decreasing scoring vector) that satisfies this property.

Question 3 (10 marks)

Give a polynomial-time algorithm that decides whether a given alternative will be the unique election winner for a given profile under every positional scoring rule with a strictly decreasing scoring vector. Briefly justify the correctness of your algorithm and explain why it is polynomial (a precise complexity analysis is not required).