

Homework #2

Deadline: Tuesday, 19 October 2010, 11:00

Question 1 (10 marks)

Young's voting procedure works as follows: ballots are strict rankings of the alternatives; the score of an alternative x is the *number of ballots* that need to be removed from the ballot profile for x to become a Condorcet winner; and the alternative(s) with the lowest score win(s). Now consider the following procedure, let's call it Young*: ballots are strict rankings of the alternatives; the score of an alternative x is the *number of alternatives* that need to be removed from the ballot profile for x to become a Condorcet winner; and the alternative(s) with the lowest score win(s). Find three interesting properties (good or bad ones; axiomatic, computational, or otherwise) of the Young* procedure and discuss them, at an appropriate level of detail.

Question 2 (10 marks)

This question concerns two alternative definitions of the property of *strong monotonicity* of a *resolute* voting procedure $F : \mathcal{L}(\mathcal{X})^{\mathcal{N}} \rightarrow \mathcal{X}$. Recall the definition given in class:

- (a) F is called strongly monotonic if $F(\underline{b}) = x$ implies $F(\underline{b}') = x$ for any alternative x and any two ballot profiles \underline{b} and \underline{b}' satisfying $\underline{b}(x \succ y) \subseteq \underline{b}'(x \succ y)$ for all alternatives y .

An alternative definition to be found in the literature is the following:

- (b) F is called strongly monotonic if $F(\underline{b}') = F(\underline{b})$ or $F(\underline{b}') = x$ for any alternative x and any two ballot profiles \underline{b} and \underline{b}' satisfying $\underline{b}(x \succ y) \subseteq \underline{b}'(x \succ y)$ and $\underline{b}(y \succ z) = \underline{b}'(y \succ z)$ for all alternatives y and z different from x .

Explain each definition in plain English and briefly argue why it is a reasonable definition. Then check whether the two definitions are equivalent (proof or counterexample).

Notation: Recall that $\underline{b}(x \succ y)$ is the set of voters who rank alternative x above alternative y in the ballot profile \underline{b} .

Question 3 (10 marks)

Recall the definition of the *veto rule*: the voters rank the alternatives, and the alternative(s) ranked last the least often win(s). The purpose of this question is to find a number of different characterisations of the veto rule.

- (a) Find a consensus criterion such that the veto rule is characterised by that criterion and the *discrete distance*.
- (b) Find a way of measuring distances such that the veto rule is characterised by the *unanimous winner* consensus criterion and that distance.
- (c) Find a noise model such that the corresponding maximum likelihood estimator is equivalent to the veto rule.