Computational Social Choice: Spring 2009

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Plan for Today

Multiagent resource allocation is the problem of dividing a set of resources amongst a group of agents, given certain criteria.

We will start with a *very* brief overview of the area (just one slide). See the "MARA Survey" for full details.

Then we will discuss the allocation of indivisible goods:

- *Complexity results* for achieving optimal allocations
- *Distributed MARA:* convergence and related issues

Y. Chevaleyre, P.E. Dunne, U. Endriss, J. Lang, M. Lemaître, N. Maudet, J. Padget, S. Phelps, J.A. Rodríguez-Aguilar and P. Sousa. Issues in Multiagent Resource Allocation. *Informatica*, 30:3–31, 2006.

The Problem

Consider a set of agents and a set of goods. Each agent has their own preferences regarding the allocation of goods to be selected.

▶ What constitutes a good allocation and how do we find it?

What goods? One or several goods? Available in single or multiple units? Divisible or indivisible? Can goods be shared? Static or changing properties (e.g., consumable or perishable goods)?

What preferences? Ordinal or cardinal preferences? Are monetary side payments possible, and how do they affect preferences?

Setting

Today we consider the case of allocating indivisible (non-sharable, single-unit, static) goods amongst agents with cardinal preferences.

We shall work in this framework:

- Set of *agents* $\mathcal{N} = \{1..n\}$ and finite set of indivisible *goods* $\mathcal{G}$.
- An *allocation* A is a partitioning of $\mathcal{G}$ amongst the agents in $\mathcal{N}$.
  Example: $A(i) = \{g_5, g_7\}$ — agent i owns goods g_5 and g_7
- Each agent $i \in \mathcal{N}$ has got a *valuation function* $v_i : 2^{\mathcal{G}} \to \mathbb{R}$.
  Example: $v_i(A) = v_i(A(i)) = 577.8$ — agent i is pretty happy

Later we will define utility functions over these valuations (to account for payments). For now think of valuation as utility.

An *allocation problem* is a triple $\langle \mathcal{N}, \mathcal{G}, \mathcal{V} \rangle$, where $\mathcal{V}$ is a set of valuation functions (+ possibly the *initial allocation* A_0).

Allocation Procedures

We can distinguish two approaches:

- In the *centralised approach*, we need to devise an optimisation algorithm to compute an allocation meeting our fairness and efficiency requirements.
  - Today: some complexity results
  - Later: combinatorial auctions
- In the *distributed approach*, allocations emerge as agents implement a sequence of local deals. What can we say about the properties of these emerging allocations?

Discussion: advantages and disadvantages of either approach (simplicity of protocols, trust towards the centre, ...)

Social Welfare

Recall that we have seen a number of criteria, most of them based on various social welfare orderings, that can be used to define what constitutes an optimal allocation.

Specifically, *utilitarian social welfare* is defined as follows:

$$sw_u(A) \quad = \quad \sum_{i \in \mathcal{N}} v_i(A)$$

Welfare Optimisation

How hard is it to find an allocation with maximal social welfare? Rephrase this *optimisation problem* as a *decision problem:*

WELFARE OPTIMISATION (WO)
Instance: $\langle \mathcal{N}, \mathcal{G}, \mathcal{V} \rangle$ and $K \in \mathbb{Q}$
Question: Is there an allocation A such that $sw_u(A) > K$?

Unfortunately, the problem is intractable:

Theorem 1 WELFARE OPTIMISATION *is NP-complete.*

The proof (following slides) uses a reduction from a standard reference problem (SET PACKING) known to be NP-complete.

In the context of MARA, this kind of result seems to have first been stated by Rothkopf *et al.* (1998).

M.H. Rothkopf, A. Pekeč, and R.M. Harstad. Computationally Manageable Combinational Auctions. *Management Science*, 44(8):1131–1147, 1998.

Proof of NP-hardness

We are going to reduce our problem to SET PACKING, one of the standard problems known to be NP-complete:

> SET PACKING
> **Instance:** Collection $\mathcal{C}$ of finite sets and $K \in \mathbb{Q}$
> **Question:** Is there a collection of disjoint sets $\mathcal{C}' \subseteq \mathcal{C}$ s.t. $|\mathcal{C}'| > K$?

Given an instance $\mathcal{C}$ of SET PACKING, consider this MARA setting:

- Goods: each item in one of the sets in $\mathcal{C}$ is a good
- Agents: one for each set in $\mathcal{C}$ + one other agent (called 0)
- Valuations: $v_C(S) = 1$ if $S = C$ and $v_C(S) = 0$ otherwise; $v_0(S) = 0$ for all bundles S

That is, every agent values "its" bundle at 1 and every other bundle at 0. Agent 0 values all bundles at 0.

Proof of NP-hardness (cont.)

Observe that not every allocation immediately corresponds to a valid solution of Set Packing: the bundles owned by individual agents may not all be sets in $\mathcal{C}$.

But: for every given allocation there exists an(other) allocation with equal social welfare that does directly correspond to a valid solution for Set Packing — just assign any goods owned by an agent with valuation 0 to agent 0 (this reallocation does not affect social welfare). Note that social welfare is equal to $|\mathcal{C}'|$.

Hence, any algorithm for WO can also solve Set Packing problems; so WO must be at least NP-hard. ✓

Proof of Membership in NP

This part is in fact very easy ...

Recall that a problem belongs to NP if it is possible to verify the correctness of a candidate solution in polynomial time.

This is clearly the case here: Given an allocation A, we can compute $sw_u(A)$ in polynomial time. And A constitutes a correct solution iff $sw_u(A) > K$. ✓

Remarks

- To be precise, we have proved NP-hardness wrt. *the number of pairs of agents and bundles with non-zero value*, corresponding to the number of sets involved in SET PACKING.
- Observe that this number itself may already be very high (exponential in the number of goods).
- In other words, we have proved NP-completeness wrt. the *explicit form* of representing valuation (utility) functions.

Representation Issues

- As for all complexity results, the *representation* of the input problem is crucial: if the input is represented inefficiently (e.g., using exponential space when this is not required), then complexity results (expressed with respect to the size of the input) may seem much more favourable than they really are.
- NP-completeness of WELFARE OPTIMISATION has been shown with respect to several *preference representation languages* (such as the k-additive form).
- In the sequel, the focus is on demonstrating *what questions* people have been asking rather than on exact results.

  Therefore, we do not give details regarding the representation (but most results apply to a variety of languages).

Welfare Improvement

The following problem is also NP-complete:

WELFARE IMPROVEMENT (WI)
Instance: $\langle \mathcal{N}, \mathcal{G}, \mathcal{V} \rangle$ and allocation A
Question: Is there an allocation A' such that $sw_u(A) < sw_u(A')$?

Given the close connection to WELFARE OPTIMISATION, this is not very surprising.

Pareto Optimality

A decision problem is said to be in coNP iff its complementary problem ("is it *not* the case that …") is in NP.

Checking whether a given allocation is Pareto optimal is an example for a coNP-complete decision problem:

Pareto Optimality (PO)
Instance: $\langle \mathcal{N}, \mathcal{G}, \mathcal{V} \rangle$ and allocation A
Question: Is A Pareto optimal?

Envy-Freeness

Checking whether a given setting admits an envy-free allocation (assuming all goods need to be allocated) is again NP-complete:

ENVY-FREENESS (EF)
Instance: $\langle \mathcal{N}, \mathcal{G}, \mathcal{V} \rangle$
Question: Is there a (complete) allocation A that is envy-free?

Checking whether there is an allocation that is both Pareto optimal and envy-free is even harder: Σ_2^p-complete (NP with NP oracle).

S. Bouveret and J. Lang. Efficiency and Envy-freeness in Fair Division of Indivisible Goods: Logical Representation and Complexity. *Journal of Artificial Intelligence Research*, 32:525–564, 2008.

Distributed Approach

Instead of devising algorithms for computing a socially optimal allocation in a centralised manner, we now want agents to be able to do this in a distributed way by contracting deals locally.

- A *deal* $\delta = (A, A')$ is a pair of allocations (before/after).
- A deal may come with a number of side payments to compensate some of the agents for a loss in valuation.

  A *payment function* is a function $p : \mathcal{N} \to \mathbb{R}$ with $\sum_{i \in \mathcal{N}} p(i) = 0$.

  Example: $p(i) = 5$ and $p(j) = -5$ means that agent i *pays* €5, while agent j *receives* €5.
- Agents have *quasi-linear utility functions:*

  utility = valuation for the bundle held − sum of payments

Negotiating Socially Optimal Allocations

We are not going to talk about designing a concrete negotiation protocol, but rather study the framework from an abstract point of view. The main question concerns the relationship between

- the *local view:* what deals will agents make in response to their individual preferences?; and
- the *global view:* how will the overall allocation of resources evolve in terms of social welfare?

U. Endriss, N. Maudet, F. Sadri and F. Toni. Negotiating Socially Optimal Allocations of Resources. *Journal of AI Research*, 25:315–348, 2006.

The Local/Individual Perspective

A rational agent (who does not plan ahead) will only accept deals that improve its individual welfare:

- A deal $\delta = (A, A')$ is called *individually rational* (IR) iff there exists a payment function p such that $v_i(A') - v_i(A) > p(i)$ for all agents $i \in \mathcal{N}$, except possibly $p(i) = 0$ for agents i that are not involved in the deal (those with $A(i) = A'(i)$).

So: an agent will only accept a deal *iff* it results in a gain in value (or money) that strictly outweighs any loss in money (or value).

The Global/Social Perspective

Suppose that as system designers we are interested in maximising *utilitarian social welfare:*

$$sw_u(A) \quad = \quad \sum_{i \in \mathcal{N}} v_i(A)$$

Observe that there is no need to include the agents' monetary balances into this definition, because they'd always add up to 0.

While the local perspective is driving the negotiation process, we use the global perspective to assess how well we are doing.

▶ How well will this work?

Example

Let $\mathcal{N} = \{ann, bob\}$ and $\mathcal{G} = \{chair, table\}$ and suppose our agents use the following valuation functions:

$$\begin{array}{rclrcl} v_{ann}(\{\,\}) & = & 0 & v_{bob}(\{\,\}) & = & 0 \\ v_{ann}(\{chair\}) & = & 2 & v_{bob}(\{chair\}) & = & 3 \\ v_{ann}(\{table\}) & = & 3 & v_{bob}(\{table\}) & = & 3 \\ v_{ann}(\{chair, table\}) & = & 7 & v_{bob}(\{chair, table\}) & = & 8 \end{array}$$

Furthermore, suppose the initial allocation of goods is A_0 with $A_0(ann) = \{chair, table\}$ and $A_0(bob) = \{\,\}$.

Social welfare for allocation A_0 is 7, but it could be 8. By moving only a *single* good from agent *ann* to agent *bob*, the former would lose more than the latter would gain (not individually rational).

The only possible deal is to move the entire *set* $\{chair, table\}$.

Linking the Local and the Global Perspectives

It turns out that individually rational deals are exactly those deals that increase social welfare:

Lemma 1 (Rationality and social welfare) *A deal $\delta = (A, A')$ with side payments is individually rational iff $sw_u(A) < sw_u(A')$.*

<u>Proof:</u> "$\Rightarrow$": Rationality means that overall utility gains outweigh overall payments (which are $= 0$).

"$\Leftarrow$": The social surplus can be divided amongst all agents by using, say, the following payment function:

$$p(i) \;=\; v_i(A') \;-\; v_i(A) \;-\; \underbrace{\frac{sw_u(A') - sw_u(A)}{|\mathcal{N}|}}_{>0} \qquad \checkmark$$

<u>Discussion:</u> The lemma confirms that individually rational behaviour is "appropriate" in utilitarian societies.

Termination

We can now prove a first result on negotiation processes:

Lemma 2 (Termination) *There can be no infinite sequence of IR deals; that is, negotiation must always terminate.*

Proof: Follows from the first lemma and the observation that the space of distinct allocations is finite. ✓

Convergence

It is now easy to prove the following *convergence* result (originally stated by Sandholm in the context of distributed task allocation):

Theorem 2 (Sandholm, 1998) *Any sequence of IR deals will eventually result in an allocation with maximal social welfare.*

Proof: Termination has been shown in the previous lemma. So let A be the terminal allocation. Assume A is *not* optimal, i.e., there exists an allocation A' with $sw_u(A) < sw_u(A')$. Then, by our first lemma, $\delta = (A, A')$ is individually rational $\Rightarrow$ contradiction. ✓

Discussion: Agents can act *locally* and need not be aware of the global picture (convergence is guaranteed by the theorem).

T. Sandholm. Contract Types for Satisficing Task Allocation: I Theoretical Results. Proc. AAAI Spring Symposium 1998.

Multilateral Negotiation

On the downside, outcomes that maximise utilitarian social welfare can only be guaranteed if the negotiation protocol allows for deals involving *any number of agents* and *goods*:

Theorem 3 (Necessity of complex deals) *Any deal $\delta = (A, A')$ may be necessary: there are valuation functions and an initial allocation such that any sequence of individually rational deals leading to an allocation with maximal utilitarian social welfare would have to include δ (unless δ is "independently decomposable").*

(Independently decomposable deals are deals that can be split into two subdeals involving distinct agents.)

The proof involves the systematic definition of valuation functions such that A' is optimal and A is the second best allocation.

The theorem holds even when valuation functions are restricted to be monotonic or dichotomous.

Modular Domains

A valuation function v_i is called *modular* iff it satisfies the following condition for all bundles $B_1, B_2 \subseteq \mathcal{G}$:

$$v_i(B_1 \cup B_2) \quad = \quad v_i(B_1) + v_i(B_2) - v_i(B_1 \cap B_2)$$

That is, in a modular domain there are no synergies between items; you can get the value of a bundle by adding up the values of its elements.

▶ Negotiation in modular domains *is* feasible:

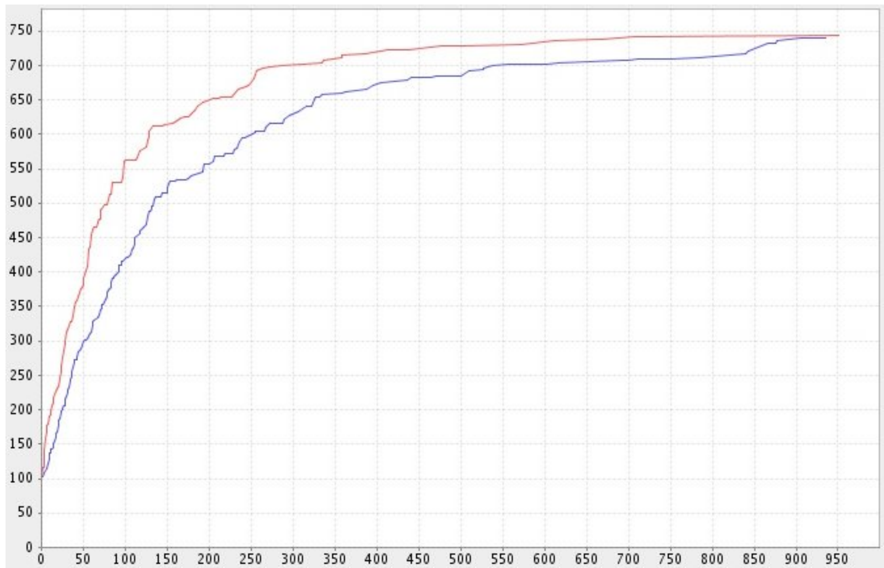
Theorem 4 (Modular domains) *If all valuation functions are modular, then individually rational 1-deals (each involving just one item) suffice to guarantee outcomes with maximal utilitarian social welfare.*

We also know that the class of modular valuation functions is *maximal:* no larger class could still guarantee the same convergence property.

Y. Chevaleyre, U. Endriss, and N. Maudet. Simple Negotiation Schemes for Agents with Simple Preferences: Sufficiency, Necessity and Maximality. *Journal of Autonomous Agents and Multiagent Systems*, 2009. In press.

Comparing Negotiation Policies

While we know from Theorem 4 that 1-deals (blue) guarantee an optimal result, an experiment (20 agents, 200 goods, modular utilities) suggests that general bilateral deals (red) achieve the same goal in fewer steps:



The graph shows how utilitarian social welfare (y-axis) develops as agents attempt to contract more an more deals (x-axis) amongst themselves. Graph generated using the MADRAS platform of Buisman *et al.* (2007).

H. Buisman, G. Kruitbosch, N. Peek, and U. Endriss. *Simulation of Negotiation Policies in Distributed Multiagent Resource Allocation*. ESAW-2007.

Other Topics

For most of the following topics there are some results available, but none of them has been treated exhaustively:

- Besides modularity, can *simple preferences* guarantee convergence by means of *simple deals*?
- What about convergence for *other social optimality criteria*?
- What about other types of models (such as *sharable goods* or *agents on a graph*)?
- Can we give bounds on the number of deals required to reach the optimum? ($\leadsto$ *communication complexity*)
- How well can we *approximate* the optimum if full convergence cannot be guaranteed?
- What are suitable logics for modelling MARA mechanisms and verifying, say, convergence results? ($\leadsto$ *social software*)

Summary

We have discussed two aspects of multiagent resource allocation:

- Computational complexity of computing an optimal allocation, for different interpretations of optimality
- Convergence to an optimal allocation in a distributed negotiation setting

Some remarks in relation to earlier lectures:

- MARA with indivisible goods is an example for social choice in combinatorial domains (like e.g. multiple referenda)
- MARA is more specific a problem than voting: agents are indifferent between any alternatives awarding them the same bundle ("no externalities" assumption)

References

Read the MARA Survey for a tentative overview of the field (only sketched in this class; earlier classes have treated some aspects):

- Y. Chevaleyre, P.E. Dunne, U. Endriss, J. Lang, M. Lemaître, N. Maudet, J. Padget, S. Phelps, J.A. Rodríguez-Aguilar and P. Sousa. Issues in Multiagent Resource Allocation. *Informatica*, 30:3–31, 2006.

What next?

Finding an allocation that maximises utilitarian social welfare is equivalent to determining the winners in a *combinatorial auction*.

Next we will discuss MARA from this perspective:

- *Bidding languages* for combinatorial auctions: another family of preference representation languages
- *Algorithms* for determining the winners of an auction
- Game-theoretical considerations: *mechanism design*