Coursework #3

Deadline: Wednesday, 1 April 2009, 15:00

Question 1 (10 marks)

For this question, we restrict attention to judgement aggregation problems with the agenda $\{A, \neg A, B, \neg B, A \wedge B, \neg(A \wedge B)\}$. We know that there exists no judgement aggregation rule for this agenda that is consistent, complete, universal, anonymous, neutral, and independent.

(a) Show that the 2/3-supermajority rule (which accepts a proposition from the agenda if and only if strictly more than 2/3 of the individuals accept it) satisfies all of these axioms, except for completeness. (*Hint:* The difficult part is to prove consistency.)

(b) Show that no supermajority rule with a quota of less than 2/3 will always produce a consistent collective judgement set.

Question 2 (10 marks)

Suppose three towns, A, B and C, are located in the plane $\mathbb{R}^2$. We have to decide where to build a new hospital H. Any point in the plane is feasible. The *dis*utility of a town is the distance of that town to H.

(a) Show that the Pareto optimal locations for H are precisely those that are lying within the triangle ΔABC.

(b) Show that we have an equality-efficiency dilemma iff that triangle is obtuse angled (that is, iff it has an angle of more than 90 degrees).

(c) Give a geometric characterisation of the optimum of the egalitarian CUF in case the triangle is obtuse angled.

(Adapted from H. Moulin, *Axioms of Cooperative Decision Making*, CUP, 1988.)

Question 3 (10 marks)

Suppose there are n agents located anywhere on the interval $[0, 1]$. We have to decide where to build an amusement park A, also anywhere on the same interval. The *dis*utility of an agent is its distance to A.

(a) What is the solution selected by the egalitarian CUF?

(b) What is the solution selected by the elitist (n-rank dictator) CUF?

(c) For arbitrary $k \leq n$, give a general algorithm to compute a solution that is optimal with respect to the k-rank dictator CUF. What is the complexity of your algorithm?