

Coursework #4

Deadline: Tuesday, 29 April 2008, 11:00am**Question 1** (10 marks)

Recall the distributed resource allocation framework discussed in class, where agents negotiate a sequence of individually rational deals. Without restrictions on the structure of deals, such sequences are known to always converge to an optimal allocation (here, an allocation that maximises utilitarian social welfare), but with structural restrictions this may not be so. The purpose of this exercise is to investigate what happens when all kinds of (individually rational) deals are allowed, except those that involve the complete set of agents within a single deal. From a result cited in class we know that convergence will not hold anymore in this case. The question is whether convergence can be maintained if we restrict the range of possible utility functions. Check what happens if all agents have utility functions that are (a) supermodular, (b) submodular, or (c) super- and submodular.

Question 2 (10 marks)

How many cuts are required, in the worst case, when n players execute the Banach-Knaster last-diminisher procedure to fairly divide a cake? Justify your answer.

Question 3 (10 marks)

Describe a discrete minimal-cut procedure for dividing a cake between 4 players that guarantees that each player believes they received at least $\frac{1}{6}$ of the cake. (Moving knives are not allowed and “marks” count as cuts.)

(Adapted from J. Robertson and W. Webb, *Cake-Cutting Algorithms*, A.K. Peters, 1998.)