Social Choice Theory and the Possibility of Democracy

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Outlook

Points I will want to make:

- There are many ways of making democratic decisions.
- Some of them can have paradoxical consequences.
- A systematic approach is required to understand this.
- This approach unveils certain fundamental limitations.
- <u>But:</u> not all hope is lost.

Two Germans, three Frenchmen, and four Dutchmen have to choose a drink for their lunch (the same drink for everyone).



Outcome?

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Outcome?

1. Just vote ... ?

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Outcome?

1. Plurality rule:



2. French rule ... ?

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Outcome?

1. Plurality rule:



- 2. French rule:
 - 1st round \sim 2 frontrunners
 - 2nd round: majority

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3. Borda rule ... ?



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The Majority Rule

Five members of parliament are looking for a compromise on their priorities w.r.t. public transport, culture and education.



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Condorcet Paradox

the majority rule sometimes is logically inconsistent



The Axiomatic Method

So there are many different ways of voting and some of them can yield highly counter-intuitive outcomes. *What now?* Formulate *axioms* saying what we expect from a voting rule (which in principle can be cast in precise mathematical terms):

- *Neutrality:* treat all alternatives the same!
- Anonymity: treat all voters the same!
- Unanimity: if everyone wants $A \succ B$, then accept $A \succ B$!
- *Independence:* if we just accepted $A \succ B$ and then one voter changes her mind on C, still accept $A \succ B$!

Is there a preference aggregation rule with these properties?

Arrow's Impossibility Theorem

In 1951, Kenneth J. Arrow published his famous theorem: All unanimous and independent preference aggregation rules for three or more alternatives are dictatorial.

Here '*dictatorial*' means that the rule always copies the ranking of the dictator. So this is very bad news!



Single-Peaked Preferences

Arrow's impossibility, in part, arises due to the requirement of being able to deal with *all* preference profiles ...

But often preferences come with some inherent structure.

For example, often preferences are *single-peaked* with respect to some natural left-to-right ordering of the alternatives:



Good News

When the preferences are *single-peaked*, then many of the difficulties we have encountered disappear.

For example, there now always exists an alternative that will win all pairwise majority contests against all other alternatives (so there is no more *Condorcet Paradox*):



The *median voter*'s top alternative wins all majority contests.

Summary and Discussion

Points I tried to make:

- Many voting rules: *beer-wine-milk* example.
- Paradoxes possible: *culture–education–transport* example.
- Systematic approach required: axiomatic method.
- Arrow's Theorem: impossible to get all good properties.
- Single-peakedness: allows for good solutions.

Possible points for the discussion:

- How realistic is the single-peakedness assumption?
- Can we achieve single-peakedness through deliberation?