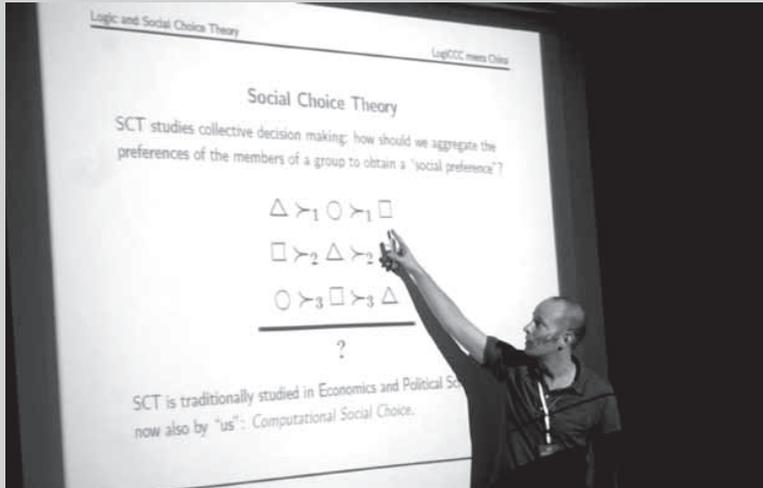


# Automated Theorem Proving and Social Choice Theory



During my final year as an undergraduate at the University of Karlsruhe, I picked an elective course with the mysteriously sounding name “Automated Theorem Proving”. I expected some sort of black magic: how marvelous would it be if you could get a computer to automatically prove things for you! Naturally, I was thinking of things such as Fermat’s Last Theorem (saying that the equation  $x^n + y^n = z^n$  has no solutions in positive integers for any  $n$  greater than 2), which at the time had just been proven (over 350 years after it had first been claimed to be true). Fame and fortune seemed but a small step away.

It didn’t quite turn out to be like that, but it was still pretty amazing. In fact, some of the best work in Automated Theorem Proving at that time came out of Peter Schmitt’s group, who was teaching the course. A very nice example is a paper by Bernhard Beckert and Joachim Posegga that shows how a simple computer program with just five lines of code can implement a complete and efficient theorem prover. Not everyone will find the paper immediately appealing (the abstract basically consists of that

five-line program!), but the basic idea that we can use a very simple formal language to describe a vast range of different problems and that we can often succeed in solving those problems using an entirely mechanical method I still find deeply inspiring today—and that paper is just a particularly compact expression of that very idea.

A few years later I started working as a postdoc at Imperial College in London, in a research group looking for new applications of computational logic to multiagent systems, a.k.a. “societies of agents”. It soon became clear that if we wanted to make a real contribution we would have to find out what economists and political scientists had to say on the matter. So, one day, I went to the library at Imperial and just took out a pile of books that had the word “social” in the title, but that still looked a bit technical. I started reading the smallest amongst them first: “Social Choice and Individual Values” by one Kenneth Arrow.

I had no idea that what I held in my hands was one of the most influential works in the social sciences of the 20th century, a book

that single-handedly started Social Choice Theory (the formal study of how groups of people should and do make collective decisions), and the debut of someone who, in 1972, had become the youngest ever Nobel Prize winner in Economics. But even without all of that background knowledge, I soon realized that what I was reading had to be important stuff. The technical core of Arrow’s contribution may be more easily accessible through modern expositions, but his strong plea for the use of formal methods in a field where you might not immediately expect that they could play a role still makes for highly recommended reading for everyone.

Nowadays I’m trying to combine both my early and this somewhat more recent fascination. For example, with Umberto Grandi, who is writing a PhD thesis in Computational Social Choice, I’ve been working on modeling the Arrowian framework of preference aggregation in classical first-order logic, which is interesting in its own right but which also might, one day, provide the foundations for using first-order theorem provers as a tool in Social Choice Theory. And with Christian Geist, a recent Master of Logic graduate, I’ve been working on using satisfiability solvers to automatically search for new theorems in an area of Social Choice Theory concerned with the problem of ranking sets of objects.

## References

- B. Beckert and J. Posegga. leanTAP: Lean Tableau-based Deduction. *Journal of Automated Reasoning*, 15(3):339–358, 1995.
- K.J. Arrow. *Social Choice and Individual Values*. 2nd edition, Cowles Foundation Monographs Series, Yale University Press, New Haven, 1963.