

### Appendix 8: Pictures of prime numbers and ideals for real fields of class number 3

The pictures show the quadratic character and a picture of **prime numbers**, **units** and two mutually conjugate classes of **non-principal prime ideals**, **one class red**, and **the other class green** for some real quadratic fields of class number 3, namely

the fields of discriminant congruent 0 modulo 4:

$$Q(\sqrt{79}), Q(\sqrt{142}), Q(\sqrt{223}), Q(\sqrt{254}), Q(\sqrt{326}), Q(\sqrt{359})$$

and the fields of discriminant congruent 1 modulo 4:

$$Q(\sqrt{229}), Q(\sqrt{257}), Q(\sqrt{321}).$$

The pictures display the prime numbers, which generate the principal prime ideals, but not those irreducible numbers which are not prime.

Moreover, the non-principal prime ideals are displayed as follows.

The non-principal ideals are obtained by dividing principal ideals by a certain non-principal prime ideal,  $I$ , or its conjugate, where  $I := [\text{norm}, \zeta]$ ,  $\zeta := \text{shift} + (d \bmod 4 + \sqrt{d}) / 2$ , i.e.  $I$  is generated by ‘norm’ being its norm, and the integer  $\zeta$  of  $Q(\sqrt{r})$ .

In the picture, the non-principal prime ideals then are represented by those numbers whose norm is equal to a prime norm times the norm of  $I$ . This norm of  $I$  and shift are mentioned at the top of the picture, shift being needed to distinguish between the two mutually conjugate classes of non-principal ideals.



