

# OP-SF NET – Volume 28, Number 1 – January 15, 2021

The Electronic News Net of the  
SIAM Activity Group on Orthogonal Polynomials and Special Functions

<http://math.nist.gov/opsf>

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Topics:

1. Message from the Chair
2. Announcement: **Peter Clarkson** awarded the Senior Anne Bennett Prize
3. Announcement: *Introduction to  $q$ -analysis* by **Warren Johnson**
4. Obituary for **Mogens Flensted-Jensen** (1942–2020) by **Tom Koornwinder**
5. Obituary for **Dick Askey** (1933–2019) from the LMS Newsletter by **George Andrews**
6. Report on JMM 2021 Special Session on the Legacy of **Dick Askey**
7. Report on JMM 2021 Special Session on Continued Fractions
8. **Postponement**: *OPSF-S9 Summer School*, Radboud University, Nijmegen
9. Preprints in arXiv.org
10. Submitting contributions to OP-SF NET and SIAM-OPSF (OP-SF Talk)
11. Thought of the Month by **George Pólya**

**Calendar of Events:**

**June 7–11, 2021**

33<sup>rd</sup> International Colloquium on Group Theoretical Methods in Physics (Group33)  
Cotonou, Benin

<http://www.cipma.net/group33-cotonou-benin>

**June 20–26, 2021**

8<sup>th</sup> European Congress of Mathematics (8ECM)

Mini-symposium on Orthogonal Polynomials and Special Functions

Organized by Paco Marcellán, Juan J. Moreno-Balcázar and Galina Filipuk,  
Portorož, Slovenia

<https://www.8ecm.si/minisymposia>

## July 19–24, 2021

Mathematical Congress of the Americas (MCA 2021)

Special Session on *Special Functions and Orthogonal Polynomials*

Organized by Diego Dominici, Luis E. Garza, Jan Felipe van Diejen

Buenos Aires, Argentina—**Now virtual due to COVID-19 outbreak.**

<http://www.mca2021.org/en>

## January 10–14, 2022—**Updated new date due to COVID-19 outbreak.**

9<sup>th</sup> International Conference on Computational Methods and Function Theory (CMFT 2021)

Federico Santa María Technical University, Valparaíso, Chile

<http://cmft2021.inf.utfsm.cl/>

## July 11–15, 2022—**Tentative new dates due to COVID-19 outbreak.**

OPSFA-16

Centre de Recherches Mathématiques, Montreal, Canada

## August 2022—**Updated new date due to COVID-19 outbreak.**

OPSFA Summer School 2021

Radboud University, Nijmegen, The Netherlands

<https://www.ru.nl/radboudsummerschool/courses/2021/opsfa-summer-school/>

## Summer 2022—**Tentative new date due to COVID-19 outbreak.**

Functional Analysis, Approximation Theory and Numerical Analysis (FAATNA)

Matera, Italy

<http://web.unibas.it/faatna20/>

Topic #1 ——— OP – SF Net 28.1 ——— January 15, 2021

From: Peter Clarkson ([P.A.Clarkson@kent.ac.uk](mailto:P.A.Clarkson@kent.ac.uk))

Subject: Message from the Chair

Happy New Year! Let's hope that 2021 is better year than 2020, though the start of 2021 has been much the same as 2020 in many parts of the world.

There was some positive news for our Activity Group in 2020 with the renewal of our SIAM Charter for another two years. SIAM still have concerns about our SIAG, in particular the membership numbers. We have made some suggestions as to how SIAM might help.

It is disappointing that two major events for our Activity Group have been postponed to 2022. The 16<sup>th</sup> OPSFA International Symposium will now take place in July 2022 in Montreal, Canada and the next [OPSFA Summer School](#) will now take place in August 2022 in Nijmegen, The Netherlands.

Whilst there have been no physical conferences and workshops to attend in 2020, the OPSF community has been involved with various virtual events in 2020. There was an invited presentation by Andrei Martínez-Finkelshtein and mini-symposium organised by Andrei and Walter Van Assche at the SIAM Annual Meeting in July. There has been seminar series on “Orthogonal Polynomials, Special Functions, Operator Theory and Applications” ([OPSFOTA](#)) organised by Ana Loureiro, Thomas Bothner, Adria Olde Daalhuis, Walter Van Assche and Jani Virtanen, which is hosted by the ICMS in Edinburgh, Scotland. From a personal point of view, whilst much of my time in 2020 was taken up with teaching and admin related matters, much additional work due to the pandemic, it has been good to (virtually) attend a number of seminars on a virtual basis.

As reported elsewhere in this newsletter, there was a Special Session on the Legacy of Dick Askey at the JMM 2021 meeting earlier this month organized by George Andrews, Howard Cohl and

Mourad Ismail. I expect most conferences, minisymposia, special sessions and workshops will be virtual for several months to come.

I wish you a safe 2021 and hope to be able to see many or, virtually or physically, during the year.

Peter

## Topic #2 ——— OP – SF Net 28.1 ——— January 15, 2021

From: Walter Van Assche ([walter.vanassche@kuleuven.be](mailto:walter.vanassche@kuleuven.be))

Subject: Announcement: **Peter Clarkson** awarded the Senior Anne Bennett Prize

Peter Clarkson, Chair of the SIAM Activity Group on Orthogonal Polynomials and Special Functions, has been awarded the Senior Anne Bennett Prize in recognition of his tireless work to support gender equality in UK mathematics, and particularly for his leadership in developing good practice among departments of mathematical sciences. This prize is offered by the London Mathematical Society (LMS) every three years.

Clarkson, Professor of Mathematics at the University of Kent, School of Mathematics, Statistics and Actuarial Science, is an eminent mathematician with a long record of service to the profession. Most notable are his considerable efforts in the area of gender equality, in which he has had a tremendous impact as a Head of Department at Kent, as a member of the [Athena SWAN](#) advisory group at Equality Challenge Unit (ECU), and as one of the longest-serving and most active members of the LMS Women in Mathematics Committee. From joining the committee in 2007, he was a driving force behind its work, particularly in the establishment of the highly successful Good Practice Scheme (GPS) to support university Mathematics Departments. He was one of the founding members of the GPS Steering Group and chair from 2013–2018. Peter was one of the key GPS players (along with Cathy Hobbs and Gwyneth Stallard) involved in commissioning the first ever national benchmarking survey of good practice in UK mathematics departments.

As a highly regarded Head of Department, Peter provided a bridge between the [LMS WIM](#) Committee, ECU and Heads of Department of Mathematical Sciences (HoDoMS). His understanding of the issues and practicalities facing heads of departments was hugely important in shaping the GPS, ensuring that it is relevant and supportive to departments, and enabled him to provide valuable guidance and advice to ECU around the development of the Athena SWAN Charter process.

Peter's energy and commitment to improving gender diversity in the UK mathematics community has been substantial, inspiring and successful. He has a huge personal commitment and drive to ensure that women are able to flourish in their mathematical careers and has acted as mentor, both formally and informally, for many women at different levels of their careers. He has provided enormous moral and practical support in enabling the chairs of the WIM Committee to fulfill their roles successfully—in a world that is still predominantly male, the active support of a highly regarded male professor is of enormous benefit and importance.

More details about the 2020 LMS Prize Winners can be found at:

<https://www.lms.ac.uk/news-entry/26062020-1657/lms-prize-winners-2020>.

## Topic #3 ——— OP – SF Net 28.1 ——— January 15, 2021

From: OP–SF Net Editors

Subject: Announcement: *Introduction to  $q$ -analysis* by **Warren Johnson**

We would like to draw your attention to the following book “An Introduction to  $q$ -analysis,” by Warren P. Johnson: Connecticut College, New London, Connecticut. It is published by the American Mathematical Society, Providence, 2020, xv+519.

Link to AMS: <https://bookstore.ams.org/mbk-134>.

Description: Starting from simple generalizations of factorials and binomial coefficients, this book gives a friendly and accessible introduction to  $q$ -analysis, a subject consisting primarily of identities between certain kinds of series and products. Many applications of these identities to combinatorics and number theory are developed in detail. There are numerous exercises to help students appreciate the beauty and power of the ideas, and the history of the subject is kept consistently in view.

The book has few prerequisites beyond calculus. It is well suited to a capstone course, or for self-study in combinatorics or classical analysis. Ph.D. students and research mathematicians will also find it useful as a reference.

Readership: Undergraduate students interested in  $q$ -analysis, combinatorics, and number theory.

### Table of Contents:

Title page	iv
An Introduction to $q$ -analysis	xi
Chapter 1. Inversions	1
1.1. Stern’s problem	18
1.2. The $q$ -factorial	24
1.3. $q$ -binomial coefficients	31
1.4. Some identities for $q$ -binomial coefficients	37
1.5. Another property of $q$ -binomial coefficients	42
1.6. $q$ -multinomial coefficients	46
1.7. The Z-identity	50
Chapter 2. $q$ -binomial Theorems	56
2.1. A noncommutative $q$ -binomial Theorem	56
2.2. Potter’s proof	62
2.3. Rothe’s $q$ -binomial theorem	66
2.4. The $q$ -derivative	74
2.5. Two $q$ -binomial theorems of Gauss	78
2.6. Jacobi’s $q$ -binomial theorem	88
2.7. MacMahon’s $q$ -binomial theorem	91
2.8. A partial fraction decomposition	96
2.9. A curious $q$ -identity of Euler, and some extensions	99
2.10. The Chen-Chu-Gu identity	105
Chapter 3. Partitions I: Elementary Theory	110
3.1. Partitions with distinct parts	110
3.2. Partitions with repeated parts	115
3.3. Ferrers diagrams	123
3.4. $q$ -binomial coefficients and partitions	133
3.5. An identity of Euler, and its “finite” form	137
3.6. Another identity of Euler, and its finite form	145
3.7. The Cauchy/Crelle $q$ -binomial series	149
3.8. $q$ -exponential functions	158

Chapter 4. Partitions II: Geometric Theory	166
4.1. Euler’s pentagonal number theorem	166
4.2. Durfee squares	174
4.3. Euler’s pentagonal number theorem: Franklin’s proof	181
4.4. Divisor sums	184
4.5. Sylvester’s fishhook bijection	197
Chapter 5. More $q$ -identities: Jacobi, Gauss, and Heine	208
5.1. Jacobi’s triple product	208
5.2. Other proofs and related results	218
5.3. The quintuple product identity	231
5.4. Lebesgue’s identity	238
5.5. Basic hypergeometric series	244
5.6. More ${}_2\phi_1$ identities	250
5.7. The $q$ -Pfaff–Saalschütz identity	256
Chapter 6. Ramanujan’s ${}_1\psi_1$ Summation Formula	264
6.1. Ramanujan’s formula	264
6.2. Four proofs	267
6.3. From the $q$ -Pfaff–Saalschütz sum to Ramanujan’s ${}_1\psi_1$ summation	273
6.4. Another identity of Cauchy, and its finite form	276
6.5. Cauchy’s “mistaken identity”	280
6.6. Ramanujan’s formula again	283
Chapter 7. Sums of Squares	288
7.1. Cauchy’s formula	288
7.2. Sums of two squares	293
7.3. Sums of four squares	298
Chapter 8. Ramanujan’s Congruences	306
8.1. Ramanujan’s congruences	306
8.2. Ramanujan’s “most beautiful” identity	309
8.3. Ramanujan’s congruences again	317
Chapter 9. Some Combinatorial Results	322
9.1. Revisiting the $q$ -factorial	322
9.2. Revisiting the $q$ -binomial coefficients	328
9.3. Foata’s bijection for $q$ -multinomial coefficients	333
9.4. MacMahon’s proof	336
9.5. $q$ -derangement numbers	340
9.6. $q$ -Eulerian numbers and polynomials	348
9.7. $q$ -trigonometric functions	355
9.8. Combinatorics of $q$ -tangents and secants	360
Chapter 10. The Rogers–Ramanujan Identities I: Schur	368
10.1. Schur’s extension of Franklin’s argument	368
10.2. The Bressoud–Chapman proof	374
10.3. The AKP and GIS identities	380
10.4. Schur’s second partition theorem	382
Chapter 11. The Rogers–Ramanujan Identities II: Rogers	394
11.1. Ramanujan’s proof	394
11.2. The Rogers–Ramanujan identities and partitions	400
11.3. Rogers’s second proof	405
11.4. More identities of Rogers	411
11.5. Rogers’s identities and partitions	416

11.6. The Göllnitz–Gordon identities	420
11.7. The Göllnitz–Gordon identities and partitions	429
Chapter 12. The Rogers–Selberg Function	434
12.1. The Rogers–Selberg function	434
12.2. Some applications	437
12.3. The Selberg coefficients	440
12.4. The case $k = 3$	444
12.5. Explicit formulas for the $Q$ functions	446
12.6. Explicit formulas for $S_{3,i}(x)$	447
12.7. The payoff for $k = 3$	449
12.8. Gordon’s theorem	451
Chapter 13. Bailey’s ${}_6\psi_6$ Sum	454
13.1. Bailey’s formula	454
13.2. Another proof of Ramanujan’s “most beautiful” identity	459
13.3. Sums of eight squares and of eight triangular numbers	461
13.4. Bailey’s ${}_6\psi_6$ summation formula	466
13.5. Askey’s proof: Phase 1	471
13.6. Askey’s proof: Phase 2	474
13.7. Askey’s proof: Phase 3	477
13.8. An integral	482
13.9. Bailey’s lemma	488
13.10. Watson’s transformation	492
Appendix A. A Brief Guide to Notation	500
Appendix B. Infinite Products	504
Appendix C. Tannery’s Theorem	512
Bibliography	518
Index of Names	530
Index of Topics	534

Topic #4 ——— OP – SF Net 28.1 ——— January 15, 2021

From: Tom Koornwinder ([thkmath@xs4all.nl](mailto:thkmath@xs4all.nl))

Subject: Obituary for **Mogens Flensted–Jensen** (1942–2020) by **Tom Koornwinder**

This was prepared by Tom Koornwinder (with thanks to Henrik Schlichtkrull).

The Danish mathematician Mogens Flensted–Jensen, Professor Emeritus of the University of Copenhagen, passed away in December 2020. See an extensive obituary on <https://www.math.ku.dk/english/about/news/obituary-of-professor-mogens-flensted-jensen/>.

I got acquainted with Mogens when we both participated as graduate students in a special year on noncommutative harmonic analysis at the Mittag–Leffler Institute in Djursholm, Sweden during the academic year 1970–71. We have kept friendly contacts during the fifty years since then. Precisely in the middle of that half century, in the fall of 1995, we were again together for a longer period at the Mittag–Leffler Institute on the same theme as in 1970 (see the photo).

In 1970–71 Mogens worked on harmonic analysis for Jacobi functions  $\phi_\lambda^{(\alpha,\beta)}$ . His motivation came from the occurrence of these functions as spherical functions on noncompact Riemannian symmetric spaces of rank one, for instance on the hyperbolic space  $SO_0(p,1)/SO(p)$ . In joint work [1] we obtained the positive convolution structure for Jacobi function expansions. In the late seventies we had two further joint papers on Jacobi functions.



Mogens Flensted-Jensen (right) and Tom Koornwinder in Djursholm, 1995.

Mogens also studied the case of parameter values for which the Fourier-Jacobi transform has a Plancherel formula with (finitely many) discrete terms for certain imaginary values of  $\lambda$ . Such Jacobi functions occur in several group settings. They can be generalized spherical functions with respect to one-dimensional  $K$ -types, notably for  $G = \mathrm{SU}(p, 1)$ ,  $K = \mathrm{S}(\mathrm{U}(p) \times \mathrm{U}(1))$ . They also live as  $K$ -invariant functions on semisimple symmetric spaces  $G/H$  of rank one. For instance,  $G = \mathrm{SO}_0(p, q)$ ,  $H = \mathrm{SO}_0(p, q - 1)$ ,  $K = \mathrm{SO}(p) \times \mathrm{SO}(q)$ . The discrete spectrum for Jacobi functions living there gives rise to subspaces of  $L^2(G/H)$  which are invariant and irreducible under  $G$  and contain a nonzero  $K$ -fixed vector ( $K$ -spherical discrete series for  $G/H$ ).



Mogens Flensted-Jensen in 2013

Also Mogens' most important work dealt with the discrete series on  $G/H$ , but now of general rank and the discrete series being not necessarily  $K$ -spherical. His paper [2] in 1980 in *Annals of Mathematics* gave a simple sufficient condition in terms of  $G$  and  $H$  for existence of infinitely many discrete series representations on  $G/H$ . In 1984 Oshima and Matsuki showed the condition also to be necessary. The problem is still open to find all discrete series representations for each  $G/H$ . Recently Mogens was considering this on some special  $G/H$ . The  $K$ -spherical discrete series for general rank is relevant here. This means to find the discrete mass points in

the Plancherel formula for Opdam's Jacobi functions associated with root systems, which may occur if there are negative root multiplicities. Right now, but too late for Mogens to watch it, some progress has been reported on this.

During 1979–2007 Mogens was a professor at the Royal Veterinary and Agricultural University in Copenhagen. His tasks of teaching and administration there were quite a change from doing analysis on Lie groups, but he performed them with great enthusiasm and success.

[1] M. Flensted-Jensen and T. Koornwinder, The convolution structure for Jacobi function expansions, *Ark. Mat.* 11 (1973), 245–262.

[2] M. Flensted-Jensen, Discrete series for semisimple symmetric spaces, *Ann. of Math. (2)* 111 (1980), 253–311.

## Topic #5 ——— OP – SF Net 28.1 ——— January 15, 2021

From: George Andrews ([gea1@psu.edu](mailto:gea1@psu.edu))

Subject: Obituary for **Dick Askey** (1933–2019) from the LMS Newsletter by **George Andrews**

The following obituary, Richard Askey (1933–2019), written by George Andrews, appeared in the London Mathematical Society Newsletter, *Obituaries of Members*, Issue 487, March 2020, page 51.

\* \* \*

Richard Allen (Dick) Askey was born on June 4, 1933, in St. Louis, Missouri, to Philip and Bessie Askey. He received his B.A. from Washington University in 1955, an M.A. from Harvard in 1956 and from there went to Princeton University where he received his Ph.D. in 1961 under the direction of Salomon Bochner. His thesis was titled “Mean Convergence of Orthogonal Series and Conjugate Series”, foreshadowing extensive and important contributions to the study of orthogonal polynomials.

After instructorships at Washington University and the University of Chicago, Dick joined the faculty of the University of Wisconsin, Madison in 1963 where he remained for 40 years until retirement in 2003. At Madison, he was named Gábor Szegő Professor of Mathematics in 1986 and in 1995 was awarded a John Bascom Professorship.

His many honors included Fellowships in the American Academy of Arts and Sciences (1993), the National Academy of Sciences (1999), the Society of Industrial and Applied Mathematics (2009), and the American Mathematical Society (2012). Also, he was appointed an Honorary Fellow of the Indian Academy of Sciences and received an honorary doctorate from SASTA University in India (2012). He was an invited speaker at the ICM in Warszawa (1983).

Dick's impact and influence in the world of special functions and orthogonal polynomials cannot be overstated. His early work with George Gasper yielded an inequality that was central to Louis deBrange's proof of the Bieberbach conjecture. In the mid-1980's, building on long-neglected work of L.J. Rogers, Dick wrote two AMS memoirs, the first with Mourad Ismail and the second with Jim Wilson. The latter memoir not only introduced the now widely used Askey–Wilson polynomials but also included the famous Askey–scheme which provided a hierarchical classification of the classical orthogonal polynomials. The influence of L. J. Rogers is also especially evident in the series of papers, “Sieved Orthogonal Polynomials,” a topic initiated by Askey and extended by Ismail and others. This latter series pointed to a new connection with number theory which is in its infancy.

Dick was not just a powerful researcher. He was a charismatic force in the mathematics community generally. Early in his career, he determined that the three-volume work, *Higher Transcendental Functions*, (a.k.a. the Bateman Project) needed to be extensively updated. His vision and persistence inspired many, and it is not unreasonable to suggest that the Digital Library of Mathematical Functions owes much to the influence of Richard Askey.

Dick was also passionate in his efforts to improve mathematics education at all levels. He was extremely skeptical of many fads that promised instant improvement and delivered little.

All of the above points to the importance of Askey's contributions; however, the charm, the kindness and the mischievous sense of humor are perhaps impossible to convey here. I will close with a short anecdote that at least hints at these latter qualities:

In the early 1990s, Paul Halmos wrote an article for the MAA Focus entitled *The Calculus Turmoil*. I thought it was a wonderful defense of the traditional teaching of calculus and wrote to tell him so. Halmos wrote back an equally warm response. I then called Dick to discuss the article and to tell him of my exchange with Halmos. Dick's first words were, "I hated that article". Adrenaline surged in my system. It seemed to me there were only two possible reasons for Dick's words. Either I had completely misread the article (unlikely), or this was going to be my first real educational clash with Dick. Having seen how effective he was in debate, I was not looking forward to the rest of the conversation. I managed to stammer out: "What makes you say that?" He replied, "I HATED it! I cannot stand to agree with a single word Paul Halmos writes, and I agreed with every word in that article."

## Topic #6 ——— OP – SF Net 28.1 ——— January 15, 2021

From: George Andrews ([gea1@psu.edu](mailto:gea1@psu.edu)), Howard Cohl ([howard.cohl@nist.gov](mailto:howard.cohl@nist.gov))  
and Mourad Ismail ([mourad.eh.ismail@gmail.com](mailto:mourad.eh.ismail@gmail.com))  
Subject: Report on JMM 2021 Special Session on the Legacy of **Dick Askey**

A special session on the Legacy of Dick Askey at JMM 2021 was organized by George Andrews, Howard Cohl and Mourad Ismail. The special session was composed of two different sessions. In the first session on Thursday January 7, there were 8 speakers (Diaconis, Koornwinder, Driver, Koelink, Christiansen, Rosengren, Howe and Cuoco) and in the second afternoon session there were 10 speakers (Andrews, Ono, Vinet, Zeilberger, Stanton, Johnson, Post, Berndt, Cooper, Ismail).

The first morning session started with a lecture by **Persi Diaconis** (joint work with Chenyang Zhong) entitled, *Orthogonal polynomials and the Burnside Process*. Diaconis' talk began by expressing how important it was to Askey—that theories should be not only elegant and complete, but also useful to scientists working in other fields. Diaconis gave an example of thinking via the Askey scheme which is a natural problem in enumerative combinatorics where Hahn polynomials allow for very sharp answers. Consider the set  $\mathcal{X}$  of binary  $n$ -tuples. The symmetric group  $S(n)$  acts on  $\mathcal{X}$  by permuting coordinates. The Burnside process, specialized to this setting, allows a choice of a uniformly random orbit. From  $x \in \mathcal{X}$ , choose a permutation  $w$  fixing  $x$  (uniformly at random). From  $w$  choose a binary  $n$ -tuple  $y$  fixed by  $w$ . This is one step of a Markov chain, going from  $x$  to  $y$ . This chain has discrete Chebyshev polynomial eigenfunctions and this permits us to prove that the chain 'gets random' in a few steps, no matter how large  $n$ . The Discrete Chebyshev polynomials are a special case ( $\alpha = \beta = 1$ ) of the Hahn polynomials. We managed to deform the chain to give a natural chain with general (symmetric) Hahn eigenfunctions. This deformation gives an abstract generalization for all applications of the Burnside process. The next talk was by **Tom Koornwinder** entitled, *Charting the Askey and  $q$ -Askey scheme* which discussed the

preprint by Luis Verde-Star, [arXiv:2002.07932](https://arxiv.org/abs/2002.07932), which showed that the  $q$ -hypergeometric representations of the OPs in the  $q$ -Askey scheme can be described by  $3 + 3 + 5$  parameters under three constraints and four invariances. Koornwinder presented a  $q$ -Askey scheme where each arrow implies the vanishing of a parameter. Koornwinder also gave the dependence on these parameters of the structure constants in the Zhedanov algebra and gave a  $q$ -Zhedanov scheme where each arrow implies the vanishing of a structure constant. Then **Kathy Driver** presented a talk entitled *Zeros of Jacobi polynomials*. Driver explained that it is known that the zeros of the Jacobi polynomials  $P_n^{(\alpha, \beta)}(x)$  and  $P_{n-1}^{(\alpha+t, \beta+k)}(x)$  interlace when  $\alpha, \beta > -1$ ,  $0 \leq t, k \leq 2$ . Driver explained how to prove that partial, but in general not full, interlacing holds between the zeros of  $P_n^{(\alpha, \beta)}(x)$  and  $P_{n+1}^{(\alpha+t, \beta+1)}(x)$  for  $t \in \{0, 1\}$ . Then **Erik Koelink** (joint work with Maarten van Puijssen and Pablo Román) presented a talk entitled *Multivariable matrix valued orthogonal polynomials from representation theory*. Koelink explained how by using group representations one can derive multivariable matrix valued orthogonal polynomials for the root system of type  $A$ . This is done by viewing the corresponding compact group as a symmetric space, and studying matrix spherical functions. Then **Jacob Christiansen** (joint work with Barry Simon and Maxim Zinchenko) presented a talk entitled *Residual polynomials*. Christiansen started by explaining how Dick Askey was a great fan of Gabor Szegő as seen by his wonderful, readable notes on Szegő's papers in Szegő's complete works, which Dick edited as a clear labor of love. In particular, it is clear that Askey was fond of Szegő asymptotics so he dedicated an extension of such asymptotics for Residual Polynomials to Dick's memory. Then **Hjalmar Rosengren** presented a talk entitled *On the Kanade-Russell identities*. Rosengren explained that the Kanade-Russell identities are a collection of Rogers-Ramanujan-type identities for triple series, which are related to affine Lie algebras and to partition theory. Rosengren discussed how to prove some of these conjectures, using quadratic transformations for Askey-Wilson polynomials. This work resulted in some new summation identities for  ${}_2\phi_1$  functions. Then **Roger Howe** presented a talk entitled *Dick Askey and Mathematics Education*. In that talk Howe gave several examples of the many contributions of Richard Askey to the national conversation on how to improve mathematics education in the U.S. One major contribution was the founding and hosting for over 20 years of the discussion group MathEd. However, his activities also included writing reviews, advising projects, and speaking at and organizing conferences. Then **Al Cuoco** presented a talk entitled *Adventures with Dick in Mathematics Education*. There, Cuoco discussed Dick's important help as a critical reviewer for the CME high school mathematics program and Dick's approach to elementary trigonometry. The talk ended with a [video clip](#) from a talk Dick gave in Santiago, Chile in 2010. See also the following [link](#) which points to a set of Dick's lecture notes on trigonometry edited by Al Cuoco and Shaun Cooper.

**George Andrews** started out the afternoon session by presenting a talk entitled *Chebyshev polynomials and Compositions*. The object of the presentation was to reveal the relationship of the Chebyshev polynomials to the theory of integer compositions. Numerous classical identities for Chebyshev polynomials were given combinatorial interpretations and proofs. Then **Ken Ono** presented a talk entitled *Variants of Lehmer's Conjecture on Ramanujan's tau-function*. Ono explained how Dick was a key figure in his career, and how grateful he was for the opportunity to share some of his recent work with friends on Ramanujan's tau-function. Ono's lecture was on variations of Lehmer's Conjecture. He explained how to rigorously determine whether any given non-zero integer alpha appears as a value of Ramanujan's tau function. The method makes use of the theory of Lucas sequences and modular forms which reduce the problem to a search for special rational points on specific curves of hyperelliptic and Thue type. Among the results, he showed that the odd primes less than 100 are never absolute values of tau-values. The next talk by **Luc Vinet** was entitled *A unified algebraic underpinning for the Hahn polynomials and rational functions*. Vinet explained that at present, we have a useful description of the polynomials of the Askey scheme based on the representation theory of algebras said to be of the Askey-Wilson type. Vinet's presentation explored the possibility of accounting similarly for the bispectral prop-

erties of biorthogonal rational functions. He presented such a picture for functions of the Hahn class based on an algebra called meta-Hahn. The study leads one to believe that there should exist other meta algebras that could offer a unified description of the Askey scheme augmented with associated biorthogonal rational functions. Then **Doron Zeilberger** presented a talk entitled *How Richard Askey Inspired the Happy Marriage of Special Functions and Combinatorics*. In that talk Zeilberger explained that While Dick Askey was not a combinatorist himself (he only had one paper, joint with Tom Koornwinder and Mourad Ismail) that may be called combinatorial proper), he inspired lots of seminal combinatorics. He first popularized that first revolution of combinatorial special functions initiated by Joseph Gillis, and through his challenges, inspired the second revolution initiated by Dominique Foata. Zeilberger discussed first revolution, that implied that the EXACT number of ways of deranging a standard deck of cards (ignoring suits) is EXACTLY:

1493804444499093354916284290188948031229880469556.

The next talk by **Dennis Stanton** was entitled *Combinatorics of type  $R_I$  orthogonal polynomials*. In 1995 Ismail and Masson defined a new general family of orthogonal polynomials by perturbing the 3-term recurrence relation. Stanton described the combinatorial theory for these polynomials which corresponds to Viennot's theory. He included path models for the polynomials as well as moments, continued fractions and Hankel determinants. Examples are given in the Askey scheme, including Askey-Wilson and  $q$ -Racah polynomials. Then **Warren Johnson** presented a talk entitled *Functional Equations in the Research and Teaching of Dick Askey*. Johnson went through several functional equation arguments that Dick gave either in his papers or in his Special Functions class. One argument evaluated the beta function assuming no previous knowledge of the Gamma function, one derives Ramanujan's  ${}_1\psi_1$  summation formula, and the other worked out a generalization of the Askey-Wilson integral. Johnson also evaluated a determinant that generalizes one of Sylvester that Dick was interested in. Then **Sarah Post** (joint work with Ian Marquette and Lisa Ritter) presented a talk entitled *Exceptional Orthogonal Polynomials and rational solutions of Painlevé Equations*. In that talk Post described how rational solutions of Painlevé IV, expressed in terms of Hermite polynomials, can be obtained from considering superintegrable Hamiltonian systems connected with exceptional orthogonal polynomials. She then described how this method can be extended to Jacobi polynomials and their connection to rational solutions of Painlevé VI. Then **Bruce Berndt** presented a talk entitled *Ramanujan's Beautiful Integrals*. In that talk Berndt offered a selection of some of Ramanujan's most fascinating integrals. Representatives were taken from his published papers, (earlier) notebooks, lost notebook, and questions he posed in the Journal of the Indian Mathematical Society. The lecture concluded with two entries on Gaussian quadrature and orthogonal polynomials from the unorganized pages in Ramanujan's second notebook that were proved by Richard Askey. Then **Shaun Cooper** presented a talk entitled *Some elliptic integrals in Ramanujan's lost notebook*. He outlined how an integral recorded by Ramanujan can be incorporated into a broad theory that can be used to create series for  $1/\pi$ . He gave a new integral of a similar type that was not recorded by Ramanujan, and described how it and several other functions studied by Ramanujan, e.g., the Rogers-Ramanujan continued fraction, fit within the theory. Then **Mourad Ismail** ended the session by presenting a talk entitled *Mathematical Reminiscence about Dick Askey*. Ismail talked about his association and friendship with Dick Askey and Dick's influence on his research and his outlook on mathematics. He was Dick's post doctorate assistant and their collaboration continued for many years. They worked on combinatorial problems involving orthogonal polynomials, moment problems, specific systems of orthogonal polynomials, and positivity.

The special session was recorded and the videos will be available for meeting participants on the Meeting Platform within a couple weeks.

From: James Mc Laughlin ([jmclaughlin@wcupa.edu](mailto:jmclaughlin@wcupa.edu))

Subject: Report on JMM 2021 Special Session on Continued Fractions

An AMS Special Session on Continued Fractions was held at the Joint Mathematics Meetings on January 8th 2021.

This special session was co-organized by **James G. Mc Laughlin**, West Chester University, **Geremías Polanco Encarnación**, Hampshire College, **Barry Smith**, Lebanon Valley College and **Nancy Wyshinski**, Trinity College.

This special session is the latest in a series of special sessions on continued fractions, with the first (organized by Jimmy and Nancy) taking place at JMM 2004 in Phoenix. Previous special sessions were held at the:

- 2019 Joint Meetings in Baltimore
- 2017 Joint Meetings in Atlanta
- 2015 Joint Meetings in San Antonio
- 2013 Joint Meetings in San Diego
- 2011 Joint Meetings in New Orleans
- 2009 Joint Meetings in Washington, D.C.
- 2006 Joint Meetings in San Antonio
- 2004 Joint Meetings in Phoenix

This year the special session was held remotely due to the coronavirus (as was the entire JMM), and this was likely the reason that there were fewer presentations than in previous years.

The schedule of talks was as follows:

- 8:00 – 8:45 AM Subgroups of  $SL_2(\mathbb{Z})$  characterized by certain continued fraction representations. Johann Thiel, Sandie Han, Ariane Masuda and Satyanand Singh, New York City College of Technology – CUNY.
- 9:00 – 9:20 AM On Hurwitz stability of Composite Polynomials. Saroj Aryal and Sarita Nemani, Georgian Court University.
- 9:30 – 10:15 AM Random knots obtained from finite continued fractions in +1 and -1. Moshe Cohen and Keith Grover, State University of New York at New Paltz.
- 1:00 – 1:45 PM Approximation on Affine Subspaces: A Khintchine Type result. Daniel Alvey, Wesleyan University.

The topic of the first talk, by **Johann Thiel**, may be described as follows. For positive integers  $u$  and  $v$ , let

$$L_u = \begin{bmatrix} 1 & 0 \\ u & 1 \end{bmatrix} \text{ and } R_v = \begin{bmatrix} 1 & v \\ 0 & 1 \end{bmatrix}.$$

Let  $S_{u,v}$  be the monoid generated by  $L_u$  and  $R_v$ , and  $G_{u,v}$  be the group generated by  $L_u$  and  $R_v$ . In the talk the author describes an expansion of a characterization of matrices

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

in  $S_{k,k}$  and  $G_{k,k}$  when  $k \geq 2$  given by Esbelin and Gutan to  $S_{u,v}$  when  $u, v \geq 2$  and  $G_{u,v}$  when  $u, v \geq 3$ . He gives a simple algorithmic way of determining if  $M$  is in  $G_{u,v}$  using a recursive function and the short continued fraction representation of  $b/d$ .

In the second talk, by **Saroj Aryal**, a conjecture on the robustness of stable transfer polynomials has been settled. In this talk, the author formulates a characterization of stable transfer polynomials, which are used to identify a class of polynomials satisfying the conjecture. Further results are derived as variations of the conjecture.

In the third talk, by **Moshe Cohen**, is random knots obtained from finite continued fractions in  $+1$  and  $-1$ . A knot is a circle embedded in 3-space. A 2-bridge knot can be described by a finite sequence of nonzero integers counting crossings in alternating twist regions; such a knot is also called a rational knot. A Chebyshev knot diagram has in its finite continued fraction only  $+1$  and  $-1$ , giving way to fewer knot diagrams for the same knot. Schubert translated results on continued fractions into results on 2-bridge knots, and Koseleff and Pecker, in formalizing Chebyshev knots, built on these results for continued fractions in  $+1$  and  $-1$ . Together with Sunder Ram Krishnan and then Chaim Even-Zohar, the speaker developed this into a model for random knots. In this talk the speaker presents extensions of this work, including new work with the second author, an undergraduate student.

The topic of the fourth talk, by **Daniel Alvey**, is motivated by a conjecture of Mahler, which led to a rich tradition of investigating the Diophantine properties of subspaces of Euclidean space. Mahler's original conjecture was concerned with the extremality of a specific manifold. Another such property is that of Khintchine type, or whether the zero-one law of Khintchine's Theorem is inherited by the measure on a subspace. The author presented a Khintchine type for divergence result for affine subspaces which satisfy a certain multiplicative Diophantine condition.

The special session was recorded and the videos will be available for meeting participants on the Meeting Platform within a couple weeks.

## Topic #8 ——— OP – SF Net 28.1 ——— January 15, 2021

From: Erik Koelink ([e.koelink@math.ru.nl](mailto:e.koelink@math.ru.nl))

Subject: **Postponement**: OPSF-S9 Summer School, Radboud University, Nijmegen

Due to the current developments in the COVID crisis, we have decided to reschedule the OPSFA Summer School at the Radboud University, Nijmegen, the Netherlands (originally planned for August 2020, and initially postponed to August 2021) to **August 2022**. Precise dates will follow later.

Please see the [Calendar of Events](#) for a complete list of postponements and updates.

## Topic #9 ——— OP – SF Net 28.1 ——— January 15, 2021

From: OP-SF Net Editors

Subject: Preprints in arXiv.org

The following preprints related to the fields of orthogonal polynomials and special functions were posted or cross-listed to one of the subcategories of arXiv.org during November and December

2020. This list has been separated into two categories.

## OP–SF Net Subscriber E–Prints

<http://arxiv.org/abs/2011.00045>

Computing Equilibrium Measures with Power Law Kernels  
Timon S. Gutleb, José A. Carrillo, Sheehan Olver

<http://arxiv.org/abs/2011.00255>

On difference equations of Kravchuk–Sobolev type polynomials of higher order  
Roberto S. Costas–Santos, Anier Soria–Lorente

<http://arxiv.org/abs/2011.02187>

Strichartz Estimates with Broken Symmetries  
Felipe Gonçalves, Don Zagier

<http://arxiv.org/abs/2011.02463>

A compact presentation for the alternating central extension of the positive part of  $U_q(\widehat{\mathfrak{sl}}_2)$   
Paul Terwilliger

<http://arxiv.org/abs/2011.02710>

On positivity of orthogonal series and its applications in probability  
Paweł J. Szablowski

<http://arxiv.org/abs/2011.03171>

An application of the Goulden–Jackson cluster theorem  
Ira M. Gessel

<http://arxiv.org/abs/2011.04630>

On the Logarithmic Energy of Points on  $S^2$   
Stefan Steinerberger

<http://arxiv.org/abs/2011.05156>

Asymptotics of some generalised sine–integrals  
R. B. Paris

<http://arxiv.org/abs/2011.05261>

Reflectionless canonical systems, I. Arov gauge and right limits  
Roman Bessonov, Milivoje Lukić, Peter Yuditskii

<http://arxiv.org/abs/2011.05266>

Reflectionless canonical systems, II. Almost periodicity and character–automorphic Fourier transforms  
Roman Bessonov, Milivoje Lukić, Peter Yuditskii

<http://arxiv.org/abs/2011.05709>

On the domain of convergence of spherical harmonic expansions  
O. Costin, R. D. Costin, C. Ogle, M. Bevis

<http://arxiv.org/abs/2011.05848>

A note on generalized  $q$ –difference equations for general Al–Salam–Carlitz polynomials  
Jian Cao, Binbin Xu, Sama Arjika

<http://arxiv.org/abs/2011.05886>

Nonsymmetric Macdonald Superpolynomials  
Charles F. Dunkl

<http://arxiv.org/abs/2011.05964>

A 2-component Camassa–Holm equation, Euler–Bernoulli Beam Problem and Non-Commutative Continued Fractions  
Richard Beals, Jacek Szmigielski

<http://arxiv.org/abs/2011.07625>

Two Quick Proofs of a Catalan Lemma Needed by Lisa Sauermann and Yuval Wigderson  
Shalosh B. Ekhad, Doron Zeilberger

<http://arxiv.org/abs/2011.07856>

Relations between moments for the Jacobi and Cauchy random matrix ensembles  
Peter J. Forrester, Anas A. Rahman

<http://arxiv.org/abs/2011.07883>

Note on the Equilibrium Measures of Julia sets of Exceptional Jacobi Polynomials  
Á. P. Horváth

<http://arxiv.org/abs/2011.08040>

Discrete index transformations with Bessel and Lommel functions  
Semyon Yakubovich

<http://arxiv.org/abs/2011.08439>

A variational characterisation of projective spherical designs over the quaternions  
Shayne Waldron

<http://arxiv.org/abs/2011.08535>

A note on degenerate derangement polynomials and numbers  
Taekyun Kim, Dae san Kim, Hyunseok Lee, Lee-Chae Jang

<http://arxiv.org/abs/2011.08775>

Representation of hypergeometric products of higher nesting depths in difference rings  
Evans Doe Ocansey, Carsten Schneider

<http://arxiv.org/abs/2011.08874>

Fractional partitions and conjectures of Chern–Fu–Tang and Heim–Neuhauser  
Kathrin Bringmann, Ben Kane, Larry Rolin, Zack Tripp

<http://arxiv.org/abs/2011.09910>

Weighted uniform convergence of entire Grünwald operators on the real line  
Friedrich Littmann, Mark Spanier

<http://arxiv.org/abs/2011.10673>

Orthogonality of the Dickson polynomials of the  $(k + 1)$ -th kind  
Diego Dominici

<http://arxiv.org/abs/2011.10884>

Orthogonal polynomials on planar cubic curves  
Marco Fasoldini, Sheehan Olver, Yuan Xu

<http://arxiv.org/abs/2011.11302>

A combinatorial bijection on di-sk trees  
Shishuo Fu, Zhicong Lin, Yaling Wang

<http://arxiv.org/abs/2011.11457>

The monogenic Hua–Radon transform and its inverse  
Denis Constales, Hendrik De Bie, Teppo Mertens, Frank Sommen

<http://arxiv.org/abs/2011.11659>

Heun operator of Lie type and the modified algebraic Bethe ansatz  
Pierre–Antoine Bernard, Nicolas Crampe, Dounia Shaaban Kabakibo, Luc Vinet

<http://arxiv.org/abs/2011.11783>

Global and local scaling limits for the  $\beta = 2$  Stieltjes–Wigert random matrix ensemble  
Peter J. Forrester

<http://arxiv.org/abs/2011.12084>

$(q-)$ Supercongruences hit again  
Wadim Zudilin

<http://arxiv.org/abs/2011.12828>

Cylindric partitions and some new  $A_2$  Rogers–Ramanujan identities  
Sylvie Corteel, Jehanne Dousse, Ali K. Uncu

<http://arxiv.org/abs/2011.13343>

Absorbing–reflecting factorizations for birth–death chains on the integers and their Darboux transformations  
Manuel D. de la Iglesia, Claudia Juarez

<http://arxiv.org/abs/2011.13373>

Quadrant Walks Starting Outside the Quadrant  
Manfred Buchacher, Manuel Kauers, Amelie Trotignon

<http://arxiv.org/abs/2011.13701>

Construction of a generalization of the Leibnitz numbers and their properties  
Yilmaz Simsek

<http://arxiv.org/abs/2011.13808>

Asymptotic behavior and zeros of the Bernoulli polynomials of the second kind  
František Štampach

<http://arxiv.org/abs/2011.14180>

Approximation and localized polynomial frame on cones and hyperboloids  
Yuan Xu

<http://arxiv.org/abs/2011.14248>

The  $\mathbb{F}_p$ –Selberg Integral  
Richard Rimanyi, Alexander Varchenko

<http://arxiv.org/abs/2011.14455>

On the Density arising from the Domain of Attraction between Sum and Supremum: the  $\alpha$ –Sun operator  
N. S. Witte, P. E. Greenwood

<http://arxiv.org/abs/2012.00816>

The generating function of Kreweras walks with interacting boundaries is not algebraic  
Alin Bostan, Manuel Kauers, Thibaut Verron

<http://arxiv.org/abs/2012.01391>

The  $\mathbb{F}_p$ -Selberg integral of type  $A_n$   
Richard Rimanyi, Alexander Varchenko

<http://arxiv.org/abs/2012.01423>

On some Féjer-type trigonometric sums  
R. B. Paris

<http://arxiv.org/abs/2012.01670>

Kronecker theta function and a decomposition theorem for theta functions I  
Zhi-Guo Liu

<http://arxiv.org/abs/2012.01628>

Multivariate Difference Gončarov Polynomials  
Ayomikun Adeniran, Lauren Snider, Catherine Yan

<http://arxiv.org/abs/2012.01769>

Moments of  $q$ -Jacobi Polynomials and  $q$ -Zeta Values  
Frédéric Chapoton, Christian Krattenthaler, Jiang Zeng

<http://arxiv.org/abs/2012.02354>

Spectral Theory of Exceptional Hermite Polynomials  
David Gomez-Ullate, Yves Grandati, Robert Milson

<http://arxiv.org/abs/2012.02376>

A vertex model for LLT polynomials  
Sylvie Corteel, Andrew Gitlin, David Keating, Jeremy Meza

<http://arxiv.org/abs/2012.02808>

Persistent Laplacians: properties, algorithms and implications  
Facundo Mémoli, Zhengchao Wan, Yusu Wang

<http://arxiv.org/abs/2012.03878>

Bounded Dyck paths, bounded alternating sequences, orthogonal polynomials, and reciprocity  
Johann Cigler, Christian Krattenthaler

<http://arxiv.org/abs/2012.04019>

Guessing about Guessing: Practical Strategies for Card Guessing with Feedback  
Persi Diaconis, Ron Graham, Sam Spiro

<http://arxiv.org/abs/2012.04408>

Indeterminate moment problem associated with continuous dual  $q$ -Hahn polynomials  
Kerstin Jordaan, Maurice Kenfack Nangho

<http://arxiv.org/abs/2012.04625>

Finding Structure in Sequences of Real Numbers via Graph Theory: a Problem List  
Dana G. Korssjoen, Biyao Li, Stefan Steinerberger, Raghavendra Tripathi, Ruimin Zhang

<http://arxiv.org/abs/2012.04871>

A note on truncated degenerate Bell polynomials  
Taekyun Kim, Dae san Kim

<http://arxiv.org/abs/2012.05040>

Filter integrals for orthogonal polynomials  
Tewodros Amdeberhan, Adriana Duncan, Victor H. Moll, Vaishavi Sharma

<http://arxiv.org/abs/2012.05102>

A heuristic guide to evaluating triple-sums  
Eric T. Mortenson

<http://arxiv.org/abs/2012.05664>

Construction of eigenfunctions for the elliptic Ruijsenaars difference operators  
Edwin Langmann, Masatoshi Noumi, Junichi Shiraishi

<http://arxiv.org/abs/2012.06515>

Representations of degenerate poly-Bernoulli polynomials  
Taekyun Kim, Dae San Kim, Jongkyum Kwon, Hyunseok Lee

<http://arxiv.org/abs/2012.06676>

A new approach to the Dyson rank conjectures  
Frank Garvan

<http://arxiv.org/abs/2012.07068>

Application of the Efron theorem to the function represented by the inverse Laplace transform of  $s^{-\mu} \exp(-s^\nu)$   
Alexander Apelblat, Francesco Mainardi

<http://arxiv.org/abs/2012.07400>

A Differential Analogue of Favard's Theorem  
Arieh Iserles, Marcus Webb

<http://arxiv.org/abs/2012.07493>

$J$ -matrix method of scattering for inverse-square singular potential with supercritical coupling  
I. Theory  
Abdulaziz D. Alhaidari, Hocine Bahlouli, Carlos P. Aparicio, Saeed M. Al-Marzoug

<http://arxiv.org/abs/2012.07549>

$q$ -Fractional Askey-Wilson Integrals and Related Semigroups of Operators  
Mourad E. H. Ismail, Ruiming Zhang, Keru Zhou

<http://arxiv.org/abs/2012.07618>

Bispectral Jacobi type polynomials  
Antonio J. Durán, Manuel D. de la Iglesia

<http://arxiv.org/abs/2012.08465>

Neural Collapse with Cross-Entropy Loss  
Jianfeng Lu, Stefan Steinerberger

<http://arxiv.org/abs/2012.08618>

Joint moments of a characteristic polynomial and its derivative for the circular  $\beta$ -ensemble  
Peter J. Forrester

<http://arxiv.org/abs/2012.09661>

Geometric Brownian motion with affine drift and its time-integral  
Runhuan Feng, Pingping Jiang, Hans Volkmer

<http://arxiv.org/abs/2012.09789>

A new asymptotic representation and inversion method for the Student's  $t$  distribution  
Amparo Gil, Javier Segura, Nico M. Temme

<http://arxiv.org/abs/2012.10265>

Rational hypergeometric identities  
Gor A. Sarkissian, Vyacheslav P. Spiridonov

<http://arxiv.org/abs/2012.11010>

The Sigma Form for the Second Painlevé Hierarchy  
Irina Bobrova, Marta Mazzocco

<http://arxiv.org/abs/2012.11493>

Sparse spectral methods for partial differential equations on spherical caps  
Ben Snowball, Sheehan Olver

<http://arxiv.org/abs/2012.11947>

Momentum approach to the  $1/r^2$  potential as a toy model of the Wilsonian renormalization  
Jan Dereziński, Oskar Grocholski

<http://arxiv.org/abs/2012.11993>

Cyclic Pólya Ensembles on the Unitary Matrices and their Spectral Statistics  
Mario Kieburg, Shi-Hao Li, Jiyuan Zhang, Peter J. Forrester

<http://arxiv.org/abs/2012.12022>

Sharp Estimates of Radial Dunkl and Heat Kernels in the Complex Case  $A_n$   
P. Graczyk, P. Sawyer

<http://arxiv.org/abs/2012.12064>

A generalized modified Bessel function and explicit transformations of certain Lambert series  
Atul Dixit, Aashita Kesarwani, Rahul Kumar

<http://arxiv.org/abs/2012.12168>

Hahn polynomials for hypergeometric distribution  
Plamen Iliev, Yuan Xu

<http://arxiv.org/abs/2012.12889>

Stahl-Totik Regularity for Dirac Operators  
Benjamin Eichinger, Ethan Gwaltney, Milivoje Lukić

<http://arxiv.org/abs/2012.12980>

Spectral density functions of bivariable stable polynomials  
Jeffrey S. Geronimo, Hugo J. Woerdeman, Chung Y. Wong

<http://arxiv.org/abs/2012.13279>

Generalised Airy Polynomials  
Peter A. Clarkson, Kerstin Jordaan

<http://arxiv.org/abs/2012.13570>

Asymptotics and statistics on Fishburn Matrices: dimension distribution and a conjecture of Stoimenow

Hsien-Kuei Hwang, Emma Yu Jin, Michael J. Schlosser

<http://arxiv.org/abs/2012.13672>

An extension of a supercongruence of Long and Ramakrishna

Victor J. W. Guo, Ji-Cai Liu, Michael J. Schlosser

<http://arxiv.org/abs/2012.13822>

Relations for a class of terminating  ${}_4F_3(4)$  hypergeometric series

Ilija D. Mishev

<http://arxiv.org/abs/2012.13829>

Hahn polynomials and the Burnside process

Persi Diaconis, Chenyang Zhong

<http://arxiv.org/abs/2012.13913>

Multiple orthogonal polynomials with respect to Gauss' hypergeometric function

Helder Lima, Ana Loureiro

<http://arxiv.org/abs/2012.15055>

An asymptotic expansion for the expected number of real zeros of Kac-Geronimus polynomials

Hanan Aljubran, Maxim L. Yattselev

<http://arxiv.org/abs/2012.15794>

Elliptic solutions of dynamical Lucas sequences

Michael J. Schlosser, Meesue Yoo

## Other Relevant OP-SF E-Prints

<http://arxiv.org/abs/2011.00142>

Analytic continuation of multiple polylogarithms in positive characteristic

Hidekazu Furusho

<http://arxiv.org/abs/2011.00772>

Some extended hypergeometric matrix functions and their fractional calculus

Ashish Verma, Ravi Dwivedi, Vivek Sahai

<http://arxiv.org/abs/2011.01110>

Resurgence Analysis of Meromorphic Transforms

Jørgen Ellegaard Andersen

<http://arxiv.org/abs/2011.01432>

CDF of non-central  $\chi^2$  distribution revisited. Incomplete hypergeometric type functions approach

Dragana Jankov Maširević, Tibor K. Pogány

<http://arxiv.org/abs/2011.01487>

Close-to-Convexity properties of Clausen's Hypergeometric Function  ${}_3F_2(a, b, c; d, e; z)$

K. Chandrasekran, D. J. Prabhakaran

<http://arxiv.org/abs/2011.01642>

Equiconvergence for perturbed Jacobi polynomial expansions  
K. Jotsaroop, Giacomo Gigante

<http://arxiv.org/abs/2011.01717>

On the algebraic dependence of holonomic functions  
Julien Roques, Michael F. Singer

<http://arxiv.org/abs/2011.01972>

Determinant formulas for the five-vertex model  
Ivan N. Burennev, Andrei G. Pronko

<http://arxiv.org/abs/2011.02295>

Exponential of tridiagonal Toeplitz matrices: applications and generalization  
Mehdi Tatari, Majed Hamadi

<http://arxiv.org/abs/2011.02334>

A Gelfand–Tsetlin type base for the algebra  $\mathfrak{sp}_4$  and hypergeometric functions  
Dmitry Artamonov

<http://arxiv.org/abs/2011.02393>

Weighted Sums of Euler Sums and Other Variants of Multiple Zeta Values  
Sasha Berger, Aarav Chandra, Jasper Jain, Daniel Xu, Ce Xu, J. Zhao

<http://arxiv.org/abs/2011.02755>

Lauricella hypergeometric series  $F_A^{(n)}$  over finite fields  
Arjun Singh Chetry, Gautam Kalita

<http://arxiv.org/abs/2011.02762>

Proof of a supercongruence conjecture of (F.3) of Swisher using the WZ-method  
Arijit Jana, Gautam Kalita

<http://arxiv.org/abs/2011.02847>

A positivity conjecture related to the Riemann zeta function  
Hugues Bellemare, Yves Langlois, Thomas Ransford

<http://arxiv.org/abs/2011.03101>

New identities with Stirling, hyperharmonic, and derangement numbers, Bernoulli and Euler polynomials, powers, and factorials  
Khristo N. Boyadzhiev

<http://arxiv.org/abs/2011.03672>

Elliptic umbilic representations connected with the caustic  
E. G. Abramochkin, E. V. Razueva

<http://arxiv.org/abs/2011.04060>

Closed-form Tight Bounds and Approximations for the Median of a Gamma Distribution  
Richard F. Lyon

<http://arxiv.org/abs/2011.04082>

Jacobi Ensemble, Hurwitz Numbers and Wilson Polynomials  
Massimo Gissoni, Tamara Grava, Giulio Ruzza

<http://arxiv.org/abs/2011.04220>

Generating functions for sums of polynomial multiple zeta values  
Minoru Hirose, Hideki Murahara, Shingo Saito

<http://arxiv.org/abs/2011.04498>

Degenerate Riemann–Hilbert–Birkhoff problems, semisimplicity, and convergence of WDVV–potentials  
Giordano Cotti

<http://arxiv.org/abs/2011.04563>

On random convex chains, orthogonal polynomials, PF sequences and probabilistic limit theorems  
Anna Gusakova, Christoph Thäle

<http://arxiv.org/abs/2011.05184>

Some properties of Wigner  $3j$  coefficients: non-trivial zeros and connections to hypergeometric functions  
Jean-Christophe Pain

<http://arxiv.org/abs/2011.05635>

Exponential–Weierstrass type, exponential–Jacobi type and solitary type solutions to some conformable fractional equations  
Sirendaoreji

<http://arxiv.org/abs/2011.06117>

Stationary probabilities of the multispecies TAZRP and modified Macdonald polynomials: I  
Arvind Ayyer, Olya Mandelshtam, James B. Martin

<http://arxiv.org/abs/2011.06178>

Multiple Fourier series and lattice point problems  
Shigehiko Kuratsubo, Eiichi Nakai

<http://arxiv.org/abs/2011.06270>

Unique positive solutions to  $q$ -discrete equations associated with orthogonal polynomials  
Tomas Lasic Latimer

<http://arxiv.org/abs/2011.06410>

Fonctions spéciales et polynômes orthogonaux: cours et exercices corrigés  
Benaoumeur Bakhti

<http://arxiv.org/abs/2011.06889>

Gaps in the spectrum of two-dimensional square packing of stiff disks  
L. D'Elia, S. A. Nazarov

<http://arxiv.org/abs/2011.07498>

A correlation function for the classical orthogonal polynomials  
Enno Diekema

<http://arxiv.org/abs/2011.07796>

Mock Eisenstein Series  
Ranveer Kumar Singh

<http://arxiv.org/abs/2011.07839>

On families of constrictions in model of overdamped Josephson junction and Painlevé 3 equation  
Yulia Bibilo, Alexey Glutsyuk

<http://arxiv.org/abs/2011.07877>

The family of confluent Virasoro fusion kernels and a non-polynomial  $q$ -Askey scheme  
Jonatan Lenells, Julien Roussillon

<http://arxiv.org/abs/2011.07890>

Muttalib-Borodin plane partitions and the hard edge of random matrix ensembles  
Dan Betea, Alessandra Occelli

<http://arxiv.org/abs/2011.07940>

Solutions of Heun's general equation and elliptic Darboux equation  
Bartolomeu D. B. Figueiredo

<http://arxiv.org/abs/2011.08079>

A Note on harmonic maps  
G. Polychrou

<http://arxiv.org/abs/2011.08104>

A New Product Formula Involving Bessel Functions  
Mohamed Amine Boubatra, Selma Negzaoui, Mohamed Sifi

<http://arxiv.org/abs/2011.08392>

Laplace Green's Functions for Infinite Ground Planes with Local Roughness  
Nail Gumerov, Ramani Duraiswami

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Functional equations for Selberg zeta functions with Tate motives  
Shin-ya Koyama, Nobushige Kurokawa

<http://arxiv.org/abs/2011.08667>

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Shinpei Sakane, Miho Aoki

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Pfaffian control of some polynomials involving the  $j$ -function and Weierstrass elliptic functions  
John Armitage

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Decorated Dyck paths, polyominoes, and the Delta conjecture  
Michele D'Adderio, Alessandro Iraci, Anna Vanden Wyngaerd

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Keke Zhang

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Lahoucine Elaissaoui

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Higher moments of distribution of zeta zeros  
Farzad Aryan

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Existence and orbital stability of standing waves to a nonlinear Schrödinger equation with inverse square potential on the half-line  
Elek Csobo

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Ricardo Pérez-Marco

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A. Torres-Hernandez, F. Brambila-Paz

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Another look at Zagier's formula for multiple zeta values involving Hoffman elements  
Cezar Lupu

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Completion of local zeta functions associated with a certain class of homogeneous cones  
Hideto Nakashima

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The shifted harmonic oscillator and the hypoelliptic Laplacian on the circle  
Boris Mityagin, Petr Siegl, Joe Viola

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Notes on the historical bibliography of the gamma function  
Ricardo Pérez-Marco

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Schottky uniformization and Bloch-Wigner dilogarithm on higher genus curves  
Ilyas Bayramov

<http://arxiv.org/abs/2011.13680>

Riemannian Gaussian distributions, random matrix ensembles and diffusion kernels  
Leonardo Santilli, Miguel Tierz

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A Primer on Zeta Functions and Decomposition Spaces  
Andrew Kobin

<http://arxiv.org/abs/2011.14140>

Calculating the Mandel parameter for an oscillator-like system generated by generalized Chebyshev polynomials  
V. V. Borzov, E. V. Damaskinsky

<http://arxiv.org/abs/2011.14162>

A note on the Grover walk and the generalized Ihara zeta function of the one-dimensional integer lattice

Takashi Komatsu, Norio Konno, Iwao Sato

<http://arxiv.org/abs/2011.14338>

A generalized regularization theorem and Kawashima's relation for multiple zeta values

Masanobu Kaneko, Ce Xu, Shuji Yamamoto

<http://arxiv.org/abs/2011.14987>

Universal relations in asymptotic formulas for orthogonal polynomials

D. R. Yafaev

<http://arxiv.org/abs/2011.15106>

On local zeta-integrals for  $GSp(4)$  and  $GSp(4) \times GL(2)$

David Loeffler

<http://arxiv.org/abs/2012.00285>

Connectors of the Ohno relation for parametrized multiple zeta series

Hideki Murahara, Tomokazu Onozuka

<http://arxiv.org/abs/2012.00539>

Zagier's weight  $3/2$  mock modular form

Ajit Bhand, Ranveer Kumar Singh

<http://arxiv.org/abs/2012.00550>

Multiplier Theorem for Fourier Series in continuous-discrete Sobolev orthogonal polynomials

B. P. Osilenker

<http://arxiv.org/abs/2012.00603>

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Jibran Iqbal Shah

<http://arxiv.org/abs/2012.01161>

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Martin Nicholson

<http://arxiv.org/abs/2012.01627>

A combinatorial formula for the nabla operator

Erik Carlsson, Anton Mellit

<http://arxiv.org/abs/2012.01712>

On the behavior of multiple zeta-functions with identical arguments on the real line I

Kohji Matsumoto, Ilija Tanackov

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On the behavior of multiple zeta-functions with identical arguments on the real line

Kohji Matsumoto, Toshiki Matsusaka, Ilija Tanackov

<http://arxiv.org/abs/2012.01809>

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Martin Ortiz Ramirez

<http://arxiv.org/abs/2012.01943>

Finite-Part Integration in the Presence of Competing Singularities: Transformation Equations for the hypergeometric functions arising from Finite-Part Integration

Lloyd Villanueva, Eric A. Galapon

<http://arxiv.org/abs/2012.01969>

A new generalization of the Genocchi numbers and its consequence on the Bernoulli polynomials

Bakir Farhi

<http://arxiv.org/abs/2012.02588>

Explicit Relations between Multiple Zeta Values and Related Variants

Ce Xu

<http://arxiv.org/abs/2012.02622>

Perturbative and Geometric Analysis of the Quartic Kontsevich Model

Johannes Branahl, Alexander Hock, Raimar Wulkenhaar

<http://arxiv.org/abs/2012.02793>

A focus on the Riemann's Hypothesis

Jean Max Coranson Beaudu

<http://arxiv.org/abs/2012.02887>

Integral formula for the Bessel function of the first kind

Enrico De Micheli

<http://arxiv.org/abs/2012.03068>

Modules of Zeta Integrals for  $GL(1)$

Gal Dor

<http://arxiv.org/abs/2012.03530>

An Inequality for Coefficients of the Real-rooted Polynomials

J. J. F. Guo

<http://arxiv.org/abs/2012.04165>

On an approximation of a  $J$ -Bessel integral and its applications (with an appendix by J.S. Friedman\*)

George Anton, Jessen A. Malathu, Shelby Stinson

<http://arxiv.org/abs/2012.04615>

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Andrew O'Desky, David Harry Richman

<http://arxiv.org/abs/2012.05121>

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Hao Pan, Zhi-Wei Sun

<http://arxiv.org/abs/2012.05450>

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Asma Ben Saber, Abderrazek Karoui

<http://arxiv.org/abs/2012.05469>

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Hamed Baghal Ghaffari, Jeffrey A. Hogan, Joseph D. Lakey

<http://arxiv.org/abs/2012.05639>

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V. E. Adler, V. V. Sokolov

<http://arxiv.org/abs/2012.06402>

New identities for Theta operators

Michele D'Adderio, Marino Romero

<http://arxiv.org/abs/2012.06581>

Analytical recurrence formulas for non-trivial zeros of the Riemann zeta function

Artur Kawalec

<http://arxiv.org/abs/2012.07026>

Motivic zeta function of the Hilbert schemes of points on a surface

Luigi Pagano

<http://arxiv.org/abs/2012.07067>

Supercongruences of multiple harmonic  $q$ -sums and generalized finite/symmetric multiple zeta values

Yoshihiro Takeyama, Koji Tasaka

<http://arxiv.org/abs/2012.07153>

Families of inverse functions: coefficient bodies and the Fekete–Szegő problem

Mark Elin, Fiana Jacobzon

<http://arxiv.org/abs/2012.07321>

Elliptic asymptotic representation of the fifth Painlevé transcendents

Shun Shimomura

<http://arxiv.org/abs/2012.07895>

An asymptotic structure of the bifurcation boundary of the perturbed Painlevé–2 equation

O. M. Kiselev

<http://arxiv.org/abs/2012.09045>

Distribution of Beurling primes and zeroes of the Beurling zeta function I. Distribution of the zeroes of the zeta function of Beurling

Szilárd Gy. Révész

<http://arxiv.org/abs/2012.09091>

Zero-free strips for the Riemann zeta-function derived from the Prime Number Theorem

Douglas Azevedo

<http://arxiv.org/abs/2012.09506>

Special zeta Mahler functions

Berend Ringeling

<http://arxiv.org/abs/2012.09840>

Functional equations of polygonal type for multiple polylogarithms in weights 5, 6 and 7

Steven Charlton, Herbert Gangl, Danylo Radchenko

<http://arxiv.org/abs/2012.09847>

Around spin Hurwitz numbers

A. D. Mironov, A. Yu Morozov, S. M. Natanzon, A. Yu Orlov

<http://arxiv.org/abs/2012.09969>

Gaussian Multiplicative Chaos for Gaussian Orthogonal and Symplectic Ensembles

Pax Kivimae

<http://arxiv.org/abs/2012.11005>

A non-hypergeometric  $E$ -function

Javier Fresán, Peter Jossen

<http://arxiv.org/abs/2012.11173>

Zeros of hypergeometric functions in the  $p$ -adic setting

Neelam Saikia

<http://arxiv.org/abs/2012.11231>

A smooth summation of Ramanujan expansions

Giovanni Coppola

<http://arxiv.org/abs/2012.11513>

A variant of van Hoeff's algorithm to compute hypergeometric term solutions of holonomic recurrence equations

Bertrand Teguia Tabuguia

<http://arxiv.org/abs/2012.12140>

Betti structures of hypergeometric equations

Davide Barco, Marco Hien, Andreas Hohl, Christian Sevenheck

<http://arxiv.org/abs/2012.12303>

Exact Christoffel–Darboux Expansions: A New, Multidimensional, Algebraic, Eigenenergy Bounding Method

Carlos R. Handy

<http://arxiv.org/abs/2012.12371>

On the influence of the discrete spectrum and resonances on the asymptotics of the Toda shock wave

Iryna Egorova, Johanna Michor

<http://arxiv.org/abs/2012.13213>

A cohomological interpretation of Archimedean zeta integrals for  $GL_3 \times GL_2$

Takashi Hara, Kenichi Namikawa

<http://arxiv.org/abs/2012.13274>

On the Area of the Fundamental Region of a Binary Form Associated with Algebraic Trigonometric Quantities

Anton Mosunov

<http://arxiv.org/abs/2012.14081>

Objective Bayesian Analysis for the Differential Entropy of the Gamma Distribution

Eduardo Ramos, Osafu A. Egbon, Pedro L. Ramos, Francisco A. Rodrigues, Francisco Louzada

<http://arxiv.org/abs/2012.14189>

The two-dimensional fractional orthogonal derivative  
Enno Diekema

<http://arxiv.org/abs/2012.14492>

Hypergeometric Functions and Feynman Diagrams  
Mikhail Kalmykov, Vladimir Bytev, Bernd Kniehl, Sven-Olaf Moch, Bennie Ward, Scott Yost

<http://arxiv.org/abs/2012.14723>

Topological recursion for Kadomtsev–Petviashvili tau functions of hypergeometric type  
Boris Bychkov, Petr Dunin–Barkowski, Maxim Kazarian, Sergey Shadrin

<http://arxiv.org/abs/2012.14924>

Cutoff profile of ASEP on a segment  
Alexey Bufetov, Peter Nejjar

<http://arxiv.org/abs/2012.15108>

Multiple Series Representations of  $N$ -fold Mellin–Barnes Integrals  
B. Ananthanarayan, Sumit Banik, Samuel Friot, Shayan Ghosh

<http://arxiv.org/abs/2012.15611>

Semi-Parametric Estimation of Incubation and Generation Times by Means of Laguerre Polynomials  
Alexander Kreiss, Ingrid Van Keilegom

<http://arxiv.org/abs/2012.15646>

Massive One-loop Conformal Feynman Integrals and Quadratic Transformations of Multiple Hypergeometric Series  
B. Ananthanarayan, Sumit Banik, Samuel Friot, Shayan Ghosh

Topic #10 ——— OP – SF Net 28.1 ——— January 15, 2021

From: OP–SF Net Editors

Subject: Submitting contributions to OP–SF NET and SIAM–OPSF (OP–SF Talk)

To contribute a news item to OP–SF NET, send e-mail to one of the OP–SF Editors  
[howard.cohl@nist.gov](mailto:howard.cohl@nist.gov), or [spost@hawaii.edu](mailto:spost@hawaii.edu).

Contributions to OP–SF NET 28.1 should be sent by March 1, 2021.

OP–SF NET is an electronic newsletter of the SIAM Activity Group on Special Functions and Orthogonal Polynomials. We disseminate your contributions on anything of interest to the special functions and orthogonal polynomials community. This includes announcements of conferences, forthcoming books, new software, electronic archives, research questions, and job openings as well as news about new appointments, promotions, research visitors, awards and prizes. OP–SF Net is transmitted periodically through a post to SIAM–OPSF (OP–SF Talk).

SIAM–OPSF (OP–SF Talk) is a listserv of the SIAM Activity Group on Special Functions and Orthogonal Polynomials, which facilitates communication among members, and friends of the Activity Group. See the previous Topic. To post an item to the listserv, send e-mail to [siam-opsf@siam.org](mailto:siam-opsf@siam.org).

WWW home page of this Activity Group:

<http://math.nist.gov/opsf>

Information on joining SIAM and this activity group: [service@siam.org](mailto:service@siam.org)

The elected Officers of the Activity Group (2020–2022) are:

Peter Alan Clarkson, Chair

Luc Vinet, Vice Chair

Andrei Martínez–Finkelshtein, Program Director

Teresa E. Pérez, Secretary and OP–SF Talk moderator

The appointed officers are:

Howard Cohl, OP–SF NET co–editor

Sarah Post, OP–SF NET co–editor

Diego Dominici, OP–SF Talk moderator

Bonita Saunders, Webmaster and OP–SF Talk moderator

Topic #11      OP – SF Net 28.1      January 15, 2021

From: OP–SF Net Editors

Subject: Thought of the Month by **George Pólya**

“The elegance of a mathematical theorem is directly proportional to the number of independent ideas one can see in the theorem and inversely proportional to the efforts it takes to see them.”

**George Pólya** (1887–1985), *Mathematical Discovery. On Understanding, Learning, and Teaching Problem Solving*, Volume I, John Wiley & Sons, New York–London, MA, 1962.