

# OP-SF NET – Volume 27, Number 6 – November 15, 2020

The Electronic News Net of the  
SIAM Activity Group on Orthogonal Polynomials and Special Functions

<http://math.nist.gov/opsf>

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Please send contributions to the OP-SF Net editors.

Editors:

Howard S. Cohl

[howard.cohl@nist.gov](mailto:howard.cohl@nist.gov)

Sarah Post

[spost@hawaii.edu](mailto:spost@hawaii.edu)

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**Calendar of Events:**

**January 6–9, 2021**

2021 Joint Mathematics Meetings, American Mathematical Society,  
Happening Virtually in 2021. *Join your colleagues online for the latest in mathematics.*

*AMS Special Session on The Legacy of Dick Askey,*

Organized by Howard Cohl, Mourad Ismail and George Andrews,

[https://jointmathematicsmeetings.org/meetings/national/jmm2021/2247\\_program\\_ss39.html](https://jointmathematicsmeetings.org/meetings/national/jmm2021/2247_program_ss39.html)

*AMS Special Session on Continued Fractions,*

Organized by James McLaughlin, Geremías Polanco Encarnación, Barry Smith, Nancy J. Wyshinski,

[https://jointmathematicsmeetings.org/meetings/national/jmm2021/2247\\_program\\_ss62.html](https://jointmathematicsmeetings.org/meetings/national/jmm2021/2247_program_ss62.html)

**June 7–11, 2021**

33<sup>rd</sup> International Colloquium on Group Theoretical Methods in Physics (Group33)

Cotonou, Benin

<http://www.cipma.net/group33-cotonou-benin>

### June 20–26, 2021

8<sup>th</sup> European Congress of Mathematics (8ECM)

Mini-symposium on Orthogonal Polynomials and Special Functions

Organized by Paco Marcellán, Juan J. Moreno-Balcázar and Galina Filipuk,  
Portorož, Slovenia

<https://www.8ecm.si/minisymposia>

### July 6–9, 2021

Functional Analysis, Approximation Theory and Numerical Analysis (FAATNA)

Matera, Italy

<http://web.unibas.it/faatna20/>

### July 12–16, 2021

9<sup>th</sup> International Conference on Computational Methods and Function Theory (CMFT 2021)

Federico Santa María Technical University, Valparaíso, Chile

<http://cmft2021.inf.utfsm.cl/>

### July 19–24, 2021

Mathematical Congress of the Americas (MCA 2021)

Special Session on *Special Functions and Orthogonal Polynomials*

Organized by Diego Dominici, Luis E. Garza, Jan Felipe van Diejen  
Buenos Aires, Argentina

<http://www.mca2021.org/en>

### August 9–13, 2021

OPSFA Summer School 2021

Radboud University, Nijmegen, The Netherlands

<https://www.ru.nl/radboudsummerschool/courses/2021/opsfa-summer-school/>

### July 11–15, 2022—tentative new dates due to COVID-19 outbreak.

OPSFA-16

Centre de Recherches Mathématiques, Montreal, Canada

Topic #1 ——— OP – SF Net 27.6 ——— November 15, 2020

From: Luc Vinet ([vinet@CRM.UMontreal.CA](mailto:vinet@CRM.UMontreal.CA))

Subject: Announcement: OPSFA-16 date changed to **2022**, dedicated to **Richard Askey**

The 16<sup>th</sup> International Symposium on Orthogonal Polynomials, Special Functions and Applications (OPSFA-16) will take place at the [Centre de Recherches Mathématiques](#) (CRM) in Montreal in **2022** (instead of 2021 as initially planned). The tentative dates are **July 11–15, 2022**.

The call for Mini Symposia proposals will be made in January 2021.

This conference will be dedicated to the memory of **Richard Askey**.

The Organizing Committee consists of:

- Hendrik De Bie (Ghent University)
- Jan Felipe van Diejen (University of Talca)
- Erik Koelink (Radboud University)
- Mourad H. E. Ismail (University of Central Florida)
- Ana Loureiro (University of Kent)
- Francesco Marcellán (University Carlos III de Madrid)
- Sarah Post (University of Hawaii)
- Margit Rösler (University of Paderborn)
- Luc Vinet (Chair) (CRM, University of Montréal)

From: Howard S.Cohl ([howard.cohl@nist.gov](mailto:howard.cohl@nist.gov))

and Mourad E. H. Ismail ([mourad.eh.ismail@gmail.com](mailto:mourad.eh.ismail@gmail.com))

Subject: Announcement: London Mathematical Society Lecture Note Series: OPSF–S6 2016

We would like to draw your attention to the following set of Lecture Notes published by Cambridge University Press: London Mathematical Society Lecture Note Series, **464**.

Lectures on Orthogonal Polynomials and Special Functions,  
OPSF6 Sixth Summer School, Norbert Wiener Center, University of Maryland,  
College Park, Maryland, July 11–15, 2016.

Edited by **Howard S. Cohl**, National Institute of Standards and Technology, USA,  
and **Mourad E. H. Ismail**, University of Central Florida, Orlando, Florida, USA.

Publisher: Cambridge University Press

Online publication date: October 2020

publication year: 2020

Online ISBN: 9781108908993

DOI: <https://doi.org/10.1017/9781108908993>

Link to CUP: [LMS Lecture Note Series 464](#).

Description: Written by experts in their respective fields, this collection of pedagogic surveys provides detailed insight and background into five separate areas at the forefront of modern research in orthogonal polynomials and special functions at a level suited to graduate students. A broad range of topics are introduced including exceptional orthogonal polynomials,  $q$ -series, applications of spectral theory to special functions, elliptic hypergeometric functions, and combinatorics of orthogonal polynomials. Exercises, examples and some open problems are provided. The volume is derived from lectures presented at the OPSF–S6 Summer School in 2016 at the University of Maryland, and has been carefully edited to provide a coherent and consistent entry point for graduate students and newcomers.

Five lectures:

1. **Antonio Durán** (70 pages):  
Exceptional Orthogonal Polynomials via Krall Discrete Polynomials;
2. **Mourad E. H. Ismail** (53 pages):  
a Brief Review of  $q$ -Series;
3. **Erik Koelink** (76 pages):  
Applications of Spectral Theory to Special Functions;
4. **Hjalmar Rosengren** (65 pages):  
Elliptic hypergeometric functions; and
5. **Jiang Zeng** (52 pages):  
Combinatorics of Orthogonal Polynomials and their Moments

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From: Tom H. Koornwinder ([thkmath@xs4all.nl](mailto:thkmath@xs4all.nl))

Subject: Announcement: **Askey–Bateman** Project Vol. 1. Univariate Orthogonal Polynomials

We would like to draw your attention to the publication of the first volume of Encyclopedia of Special Functions, The **Askey–Bateman** Project.

Volume 1. Univariate Orthogonal Polynomials.

Edited by **Mourad E. H. Ismail**, University of Central Florida, Orlando, Florida, USA, assisted by **Walter Van Assche**, Katholieke Universiteit, Leuven, Belgium.

Publisher: Cambridge University Press

Publication date: September 2020

ISBN: 9780521197427

Link to CUP: [The Askey–Bateman Project. Volume 1. Univariate Orthogonal Polynomials.](#)

Description: This is the first of three volumes that form the Encyclopedia of Special Functions, an extensive update of the Bateman Manuscript Project. Volume 1 contains most of the material on orthogonal polynomials, from the classical orthogonal polynomials of Hermite, Laguerre and Jacobi to the Askey–Wilson polynomials, which are the most general basic hypergeometric orthogonal polynomials. Separate chapters cover orthogonal polynomials on the unit circle, zeros of orthogonal polynomials and matrix orthogonal polynomials, with detailed results about matrix-valued Jacobi polynomials. A chapter on moment problems provides many examples of indeterminate moment problems. A thorough bibliography rounds off what will be an essential reference.

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Topic #4 ——— OP – SF Net 27.6 ——— November 15, 2020

From: Tom H. Koornwinder ([thkmath@xs4all.nl](mailto:thkmath@xs4all.nl))

Subject: Announcement: **Askey-Bateman** Project Vol. 2. Multivariable Special Functions

We would like to draw your attention to the publication of the first volume of  
Encyclopedia of Special Functions, The **Askey-Bateman** Project.

Volume 2. Volume 2. Multivariable Special Functions.

Edited by **Tom H. Koornwinder**, Universiteit van Amsterdam, The Netherlands,  
and **Jasper V. Stokman**, Universiteit van Amsterdam, The Netherlands.

Publisher: Cambridge University Press

Publication date: October 2020

publication year: 2020

ISBN: 9781107003736

Link to CUP: [The Askey-Bateman Project. Volume 2. Multivariable Special Functions.](#)

Description: This is the second of three volumes that form the Encyclopedia of Special Functions, an extensive update of the Bateman Manuscript Project. Volume 2 covers multivariable special functions. When the Bateman project appeared, study of these was in an early stage, but revolutionary developments began to be made in the 1980s and have continued ever since. World-renowned experts survey these over the course of 12 chapters, each containing an extensive bibliography. The reader encounters different perspectives on a wide range of topics, from Dunkl theory, to Macdonald theory, to the various deep generalizations of classical hypergeometric functions to the several variables case, including the elliptic level. Particular attention is paid to the close relation of the subject with Lie theory, geometry, mathematical physics and combinatorics.

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Topic #5 ——— OP – SF Net 27.6 ——— November 15, 2020

From: OP–SF Net Editors

Subject: Announcement: JMM 2021 AMS Special Session on The Legacy of **Dick Askey**

We would like to invite you to attend the virtual [JMM 2021  
AMS Special Session on The Legacy of Dick Askey](#).

Organizers: Howard S. Cohl, Mourad E.H. Ismail and George E. Andrews

All times below are **U.S. Mountain Standard Time** (the time zone of Denver, Colorado)

Thursday January 7, 2021, 8:00 a.m.–11:50 a.m.

AMS Special Session on The Legacy of Dick Askey, I

- 8:00 a.m. **Persi Diaconis\***, Chenyang Zhong  
Orthogonal Polynomials and the Burnside Process.
- 8:30 a.m. **Tom H. Koornwinder\***  
Charting the Askey and  $q$ -Askey schemes.
- 9:00 a.m. **Kathy A. Driver\***  
Zeros of Jacobi polynomials.
- 9:30 a.m. **Erik Koelink\***, Maarten van Puijssen, Pablo Román  
Multivariable matrix valued orthogonal polynomials from representation theory.
- 10:00 a.m. **Jacob Stordal Christiansen\***, Barry Simon, Maxim Zinchenko  
Residual Polynomials.
- 10:30 a.m. **Hjalmar Rosengren\***  
On the Kanade–Russell identities.
- 11:00 a.m. **Roger Howe\***  
Dick Askey and Mathematics Education.
- 11:30 a.m. **Al Cuoco\***  
Adventures with Dick in Mathematics Education.

Friday January 8, 2021, 1:00 p.m.–5:50 p.m.

AMS Special Session on The Legacy of Dick Askey, II

- 1:00 p.m. **George E. Andrews\***  
Chebyshev polynomials and Compositions.
- 1:30 p.m. Jennifer Balakrishnan, William Craig, **Ken Ono\***, Wei–Lun Tsai  
Variants of Lehmer’s Conjecture on Ramanujan’s tau–function.
- 2:00 p.m. **Luc Vinet\***, Alexei Zhedanov  
A unified algebraic underpinning for the Hahn polynomials and rational functions.

2:30 p.m. **Doron Zeilberger\***

How Richard Askey Inspired the Happy Marriage of Special Functions and Combinatorics.

3:00 p.m. **Dennis Stanton\***

Combinatorics of type  $R_I$  orthogonal polynomials.

3:30 p.m. **Warren P. Johnson\***

Functional Equations in the Research and Teaching of Dick Askey.

4:00 p.m. **Sarah Post\***, Ian Marquette, Lisa Ritter

Exceptional Orthogonal Polynomials and rational solutions of Painlevé Equations.

4:30 p.m. **Bruce C. Berndt\***

Ramanujan's Beautiful Integrals.

5:00 p.m. **Shaun Cooper\***

Some elliptic integrals in Ramanujan's lost notebook.

5:30 p.m. **Mourad E. H. Ismail\***

Mathematical Reminisce about Dick Askey.

Topic #6 ——— OP – SF Net 27.6 ——— November 15, 2020

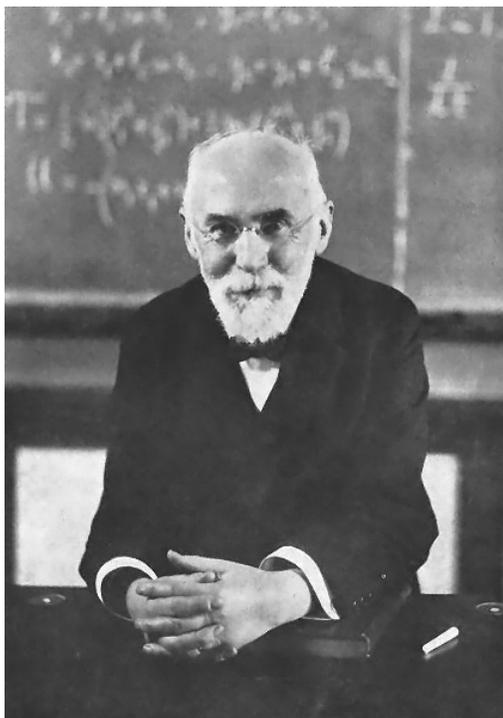
From: Tom Koornwinder ([thkmath@xs4all.nl](mailto:thkmath@xs4all.nl))

Subject: Essay by Tom Koornwinder: Hendrik Lorentz and Harry Bateman

## Lorentz and Bateman

by Tom Koornwinder

There are two ways by which the Dutch physicist Hendrik Antoon Lorentz (1853–1928) and the American, originally English, mathematician Harry Bateman (1882–1946) are scientifically related. Do you know which ones?



Hendrik Lorentz (1853–1928)



Harry Bateman (1882–1946)

1. Lorentz and Bateman both figure on a group photo (see [link](#) and below) taken at Caltech, Pasadena in 1922 (see [1] for a photo of less quality, but with Bateman closer to Lorentz.). Lorentz is central in the row of those sitting on chairs and Bateman is fourth to the right from him. The group consists of the people attending Lorentz' lecture course there. Bateman, who was a faculty



member at Caltech, was charged with taking notes and sending a draft to Lorentz so that it could be published as a book. Already in May 1922 Bateman had done his job. Lorentz was very slow in making corrections and writing additions. He was burdened by administrative duties. The book was finally published in 1927: Problems of Modern Physics, a course of lectures delivered in the California Institute of Technology [2], see a review in [3]. If we remember Bateman mainly as the grandfather of handbooks on special functions, we may be surprised that he was assigned to prepare this book for Lorentz. But according to Erdélyi's obituary [4] of Bateman, he had earlier done work on Maxwell equations which contained ideas of general relativity theory, independently of Einstein. So he may have been the best suitable person for taking notes.

2. After the flood of 1916, a law was passed in the Netherlands that a closure dam of the Zuiderzee (see old situation at [5] and new situation at [6]) should be built. In 1918, a committee of specialists was installed for preparing a final technical proposal for the construction of the dam. Lorentz became chairman. This turned out to be a major job since new theory had to be developed on how the tides would behave after the dam was built and how the dam should be in order to stand storm floods. Lorentz was the only person in this committee of hydraulic engineers who was able to develop such a theory. Even for him, the man who had completely understood Einstein's general relativity theory in a few months after its publication in 1915, it was a tour de force. It took until 1926 before the committee published its report. At a certain stage Bessel functions entered in Lorentz' research for this project. The hydraulic engineers were not familiar at all with these functions. Of course, Bateman knew these functions very well, but

probably he did not know that one of the reasons that he had to wait so long for the final version of the Caltech lecture notes, was that Lorentz was busy with Bessel functions. See the final report [7] (in Dutch); Bessel functions are introduced on p.232.

I found these facts from a recent biography of Lorentz (in Dutch). See a short description in English on [8].

## References

- [1] <https://www.aip.org/history-programs/niels-bohr-library/photos/caltech-e7>
- [2] Problems of Modern Physics: a Course of Lectures Delivered in the California Institute of Technology Hendrik Antoon Lorentz; Harry Bateman; California Institute of Technology, Dover, New York (N.Y.) 1967.
- [3] [https://projecteuclid.org/download/pdf\\_1/euclid.bams/1183492751](https://projecteuclid.org/download/pdf_1/euclid.bams/1183492751)
- [4] <https://www.jstor.org/stable/769103>
- [5] <http://www.ijsselmeervissen.nl/portals/32/Geschiedenis%20Zuiderzee/oude-zuiderzee.jpg>
- [6] <https://www.teylersmuseum.nl/nl/bezoek-het-museum/teylers-verhalen/knikwinklerprins.jpg>
- [7] <https://repository.tudelft.nl/islandora/object/uuid:f5a4fe20-b26a-4875-9f96-6be04ba16c59/datastream/OBJ1/download>
- [8] <https://www.universiteitleiden.nl/en/news/2019/10/lorentz-celebrated-physicist-born-mediator>

From: OP–SF Net Editors  
Subject: Preprints in arXiv.org

The following preprints related to the fields of orthogonal polynomials and special functions were posted or cross-listed to one of the subcategories of arXiv.org during September and October 2020. This list has been separated into two categories.

### OP–SF Net Subscriber E-Prints

<http://arxiv.org/abs/1301.3556> [major update in September 2020]  
Multi-integral representations for associated Legendre and Ferrers functions  
Howard S. Cohl, Roberto S. Costas-Santos

<http://arxiv.org/abs/2009.00230>  
Generalized Bessel functions of dihedral-type: expression as a series of confluent Horn functions and Laplace-type integral representation  
Luc Deleaval, Nizar Demni

<http://arxiv.org/abs/2009.00446>  
A Series Representation for Riemann's Zeta Function and some Interesting Identities that Follow  
Michael Milgram

<http://arxiv.org/abs/2009.01962>  
Uniformization and Constructive Analytic Continuation of Taylor Series  
Ovidiu Costin, Gerald V. Dunne

<http://arxiv.org/abs/2009.02087>  
Dunkl intertwining operator for symmetric groups  
Hendrik De Bie, Pan Lian

<http://arxiv.org/abs/2009.02360>  
Self-similar potentials in quantum mechanics and coherent states  
V. P. Spiridonov

<http://arxiv.org/abs/2009.02871>  
Resolving singularities and monodromy reduction of Fuchsian connections  
Yik-Man Chiang, Avery Ching, Chiu-Yin Tsang

<http://arxiv.org/abs/2009.04269>  
Refined Wilf-equivalences by Comtet statistics  
Shishuo Fu, Zhicong Lin, Yaling Wang

<http://arxiv.org/abs/2009.04666>  
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<http://arxiv.org/abs/2010.09702>

Appell and Sheffer sequences: on their characterizations through functionals and examples  
Sergio A. Carrillo, Miguel Hurtado

<http://arxiv.org/abs/2010.09860>

The Polylogarithm Function in Julia  
Matthew Roughan

<http://arxiv.org/abs/2010.09862>

Higher Derivatives of the Tangent and Inverse Tangent Functions and Chebyshev Polynomials  
M.J. Kronenburg

<http://arxiv.org/abs/2010.10160>

An exact solution to asymptotic Bethe equation  
Yuan Miao

<http://arxiv.org/abs/2010.10290>

Where did the examples of Abel’s continuity theorem go?  
Sergio A. Carrillo

<http://arxiv.org/abs/2010.10358>

Non-real zeros of polynomials in a polynomial sequence satisfying a three-term recurrence relation

Innocent Ndikubwayo

<http://arxiv.org/abs/2010.10376>

On derivatives, Riesz transforms and Sobolev spaces for Fourier-Bessel expansions

Bartosz Langowski, Adam Nowak

<http://arxiv.org/abs/2010.10490>

The number of zeros of linear combinations of  $L$ -functions near the critical line

Youness Lamzouri, Yoonbok Lee

<http://arxiv.org/abs/2010.10576>

A Sharp Isoperimetric Inequality for the Second Eigenvalue of the Robin Plate

L. Mercredi Chasman, Jeffrey J. Langford

<http://arxiv.org/abs/2010.10675>

Some explicit and unconditional results on gaps between zeroes of the Riemann zeta-function

A. Simonič, T. Trudgian, C. L. Turnage-Butterbaugh

<http://arxiv.org/abs/2010.10689>

On Ramanujan Sums of a Real Variable and a New Ramanujan Expansion for the Divisor Function

Matthew S. Fox, Chaitanya Karamchedu

<http://arxiv.org/abs/2010.11216>

Null Kähler geometry and isomonodromic deformations

Maciej Dunajski

<http://arxiv.org/abs/2010.11235>

Trans-Series Asymptotics of Solutions to the Degenerate Painlevé III Equation: A Case Study

A. Vartanian

<http://arxiv.org/abs/2010.11360>

Normal zeta functions of small  $\mathfrak{S}_2$ -groups and their behaviour on residue classes

Seungjai Lee

<http://arxiv.org/abs/2010.13079>

Dwork hypersurfaces of degree six and Greene's hypergeometric function

Satoshi Kumabe

<http://arxiv.org/abs/2010.13258>

Gaussian Asymptotics of Jack Measures on Partitions from Weighted Enumeration of Ribbon Paths

Alexander Moll

<http://arxiv.org/abs/2010.13307>

On explicit estimates for  $S(t)$ ,  $S_1(t)$ , and  $\zeta(\frac{1}{2} + it)$  under the Riemann hypothesis

Aleksander Simonič

<http://arxiv.org/abs/2010.13335>

Convergence Acceleration via Chebyshev Step: Plausible Interpretation of Deep-Unfolded Gradient Descent

Satoshi Takabe, Tadashi Wadayama

<http://arxiv.org/abs/2010.13638>

Proof of some supercongruences concerning truncated hypergeometric series

Chen Wang, Dian-Wang Hu

<http://arxiv.org/abs/2010.14468>

The  $p$ -Airy distribution

Sergio Caracciolo, Vittorio Erba, Andrea Sportiello

<http://arxiv.org/abs/2010.15275>

A direct method for solving inverse Sturm-Liouville problems

Vladislav V. Kravchenko, Sergii M. Torba

<http://arxiv.org/abs/2010.15473>

Combinatorics of multisequant Fay identities

V.E. Vekslerchik

<http://arxiv.org/abs/2010.15529>

Discrete and continuous Muttalib-Borodin processes I: the hard edge

Dan Betea, Alessandra Occelli

<http://arxiv.org/abs/2010.16093>

Bailey-type factorizations for Horn functions

Carlo Verschoor

<http://arxiv.org/abs/2010.16371>

A Kronecker limit formula for indefinite zeta functions

Gene S. Kopp

<http://arxiv.org/abs/2010.16406>

$(p, q)$ -complex Itô-Hermite polynomials

Abdelhadi Benahmadi, Allal Ghanmi

Topic #8 ——— OP – SF Net 27.6 ——— November 15, 2020

From: OP-SF Net Editors

Subject: Submitting contributions to OP-SF NET and SIAM-OPSF (OP-SF Talk)

To contribute a news item to OP-SF NET, send e-mail to one of the OP-SF Editors

[howard.cohl@nist.gov](mailto:howard.cohl@nist.gov), or [spost@hawaii.edu](mailto:spost@hawaii.edu).

Contributions to OP-SF NET 28.1 should be sent by January 1, 2021.

OP-SF NET is an electronic newsletter of the SIAM Activity Group on Special Functions and Orthogonal Polynomials. We disseminate your contributions on anything of interest to the special functions and orthogonal polynomials community. This includes announcements of conferences, forthcoming books, new software, electronic archives, research questions, and job openings as well as news about new appointments, promotions, research visitors, awards and prizes. OP-SF Net is transmitted periodically through a post to SIAM-OPSF (OP-SF Talk).

SIAM-OPSF (OP–SF Talk) is a listserv of the SIAM Activity Group on Special Functions and Orthogonal Polynomials, which facilitates communication among members, and friends of the Activity Group. See the previous Topic. To post an item to the listserv, send e-mail to [siam-opsf@siam.org](mailto:siam-opsf@siam.org).

WWW home page of this Activity Group:

<http://math.nist.gov/opsf>

Information on joining SIAM and this activity group: [service@siam.org](mailto:service@siam.org)

The elected Officers of the Activity Group (2020–2022) are:

Peter Alan Clarkson, Chair

Luc Vinet, Vice Chair

Andrei Martínez–Finkelshtein, Program Director

Teresa E. Pérez, Secretary and OP–SF Talk moderator

The appointed officers are:

Howard Cohl, OP–SF NET co–editor

Sarah Post, OP–SF NET co–editor

Diego Dominici, OP–SF Talk moderator

Bonita Saunders, Webmaster and OP–SF Talk moderator

## Topic #9 ——— OP – SF Net 27.6 ——— November 15, 2020

From: OP–SF Net Editors

Subject: Thoughts of the Month by **Bertrand Russell**

“It seems to me now that mathematics is capable of an artistic excellence as great as that of any music, perhaps greater ... because it gives in absolute perfection that combination, characteristic of great art, of godlike freedom, with the sense of inevitable destiny; because, in fact, it constructs an ideal world where everything is perfect and yet true.”

**Bertrand Russell** (1872–1970), In Letter to Gilbert Murray, April 3, 1902.

”One of the painful things about our time is that those who feel certainty are stupid, and those with any imagination and understanding are filled with doubt and indecision.”

**Bertrand Russell**, In *New Hopes for a Changing World*, Simon and Schuster, New York, pp. 4–5 (1951).