

Nico Temme, the Askey scheme and me, 1968–2005 *

Tom H. Koornwinder

version of 28 May 2005

In December 1968 I started working at the Mathematical Centre (MC) in Amsterdam, having freshly obtained my *doctorandus* degree in mathematics at the University of Leiden. I became the room mate of Nico Temme, at the third (upper) floor of the building at Tweede Boerhaavestraat. This was a former school building. The upper floor, with narrow corridor and tiny offices, had been put on top of the original school building. We were *aio's* (as we would now say in the Netherlands), i.e., PhD students, but in those days one was immediately appointed in such a position for indefinite time, without the strict requirement to finish the PhD thesis within, say, four years. Nico had studied mathematics at the University of Amsterdam (UvA). In the final stage of his study he had become a student assistant at the MC of prof. Hans Lauwerier, who was both professor at the UvA and head of the department of Applied Mathematics at the MC. After Nico had obtained his doctorandus degree, he had been appointed at the MC in a similar position as me.

Thus Nico knew a lot about people and habits at the MC. He was also quite experienced already in applied mathematics, special functions, asymptotics and numerical mathematics. For me everything at the MC was new, and I was also rather blank in the fields of mathematics just mentioned, since education in Leiden was very much biased to pure mathematics. Nico was very helpful in introducing me to all this.

Nico owned the original 1964 edition of the *Handbook of Mathematical Functions (with Formulas, Graphs, and Mathematical Tables)*, edited by M. Abramowitz and I. A. Stegun. This impressive volume was published by the Government Printing Office in the USA. Later I bought the (much less prestigious) softcover Dover edition of this handbook. However, for special functions I preferred the three volumes of *Higher Transcendental Functions*, edited by A. Erdélyi, because it was less aiming at a readership of physicists and engineers, and closer to pure mathematics.

In the academic year 1969–1970 a young professor from Madison (Wisconsin, USA) spent his sabbatical at the MC. His name was Dick Askey. He taught a marvellous introductory course on orthogonal polynomials. Quite a few of the young people attending his lectures were infected by his enthusiasm and started working on problems suggested by him. Dick Askey's lectures focused on the *classical orthogonal polynomials*, which we would now call *very classical*: the three families in the small *Askey scheme avant la lettre*, see Figure 1.

The MC had a very good library. In the tall library rooms at a lower floor of the school building the walls were packed with books and back volumes of journals, see Figure 2.

In 1975 I defended my PhD thesis at the UvA. Nico was one of my paranymphs, see Figure 3. In 1978 Nico defended his thesis at the UvA. I was asked to be an opponent *uit de zaal*, i.e.

*This paper appeared in *Liber Amicorum voor Nico Temme*, CWI, Amsterdam, 2005, pp. 125–131.

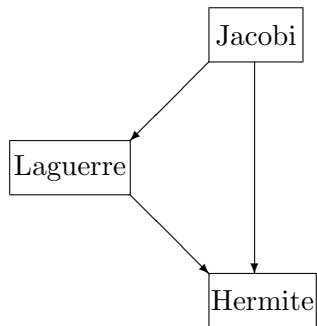


Figure 1: small Askey scheme avant la lettre



Figure 2: Nico and Tom sitting in the journals room of the MC library

from the floor (a custom allowed at UvA; this opposition from the floor precedes the regular opposition by the committee), see Figure 4.

Although employees of the MC could not be fired after they got their PhD, it could happen that the board of the MC advised them to look for a job elsewhere. But some of the young PhD's got a perspective for making further scientific career at the MC. Nico and I belonged to this group. Our ways then diverged somewhat: Nico stayed at the Department of Applied Mathematics and became *souschef* of this department, for surveying the daily matters, while the final responsibility remained with the department head Hans Lauwerier, who was not present so often. I moved within the MC to the Department of Pure Mathematics, headed by prof. Cor Baayen.

Around 1980 many things changed. The new name of the institute became CWI. It got a new building further out from the city centre at Kruislaan. Cor Baayen became director of the CWI. The departments of Pure and Applied Mathematics merged into the department of Analysis, Algebra and Geometry (AAG), and got one new department head: Michiel Hazewinkel. Senior researchers as Nico and me got offices of their own. During the first years in the new building we



Figure 3: Nico as a paranymp at Tom's thesis defense

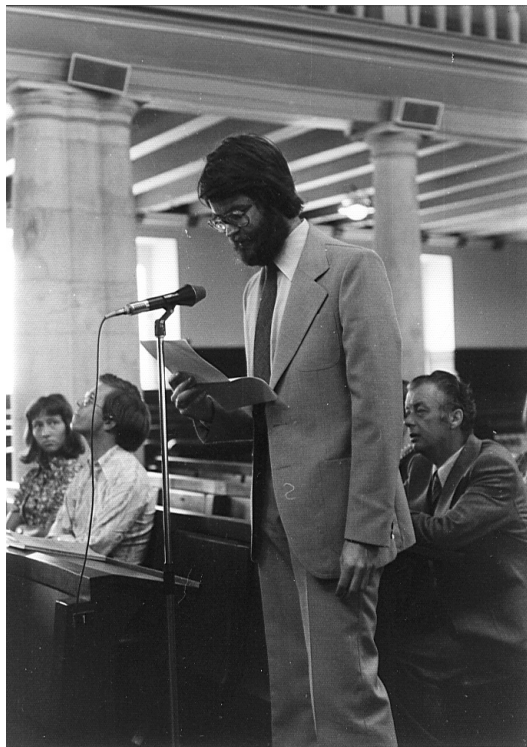


Figure 4: Tom as an opponent at Nico's thesis defense

had splendid offices near the director's office, from where we could often hear homeric laughter. Later we moved to smaller offices, and next we went into exile in portacabins (see Figure 5).

Our offices were spacious, but altogether it felt and sounded like staying on a ship.



Figure 5: AAG department members in front of the portacabins

In the eighties the Askey scheme (see Figure 8 for the scheme, and see Askey lecturing on some families from the scheme in Figure 6) became widely known as a convenient graphical way to see the hierarchy of hypergeometric orthogonal polynomials. The arrows denote limit transitions. The families on top have 4 parameters. When you go down one level then you lose one parameter. In the bottom level (Hermite polynomials), no parameters are left.

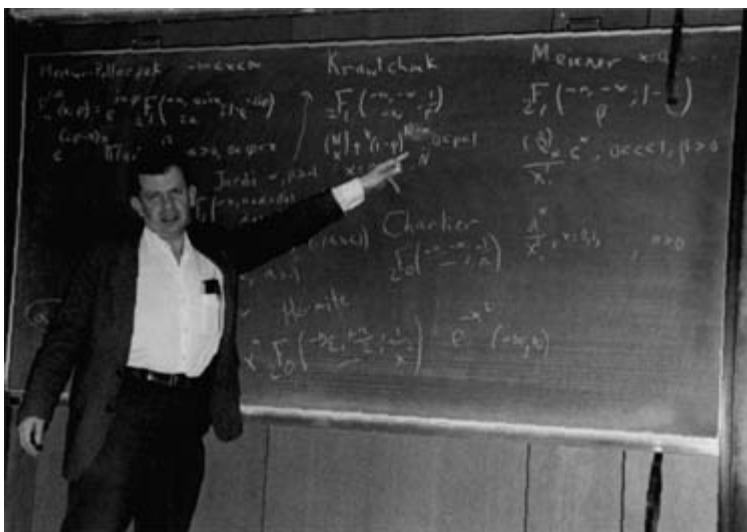


Figure 6: Dick Askey lecturing in Oberwolfach, 1983

In the second half of the eighties, both Nico and I bought an Apple Macintosh computer for usage at home, and we have both remained Apple fans since then. When Apple came out with the *HyperCard* easy programming environment, I made there an implementation of the Askey scheme, such that one could look locally which arrows start or end at a certain family. This



Figure 7: Our intensive PhD course at Twente University

implementation still runs under Apple system OS 9, see <http://www.science.uva.nl/~thk/art/software.html> for download.

The Askey scheme soon got a q -analogue, which was made possible by the discovery of the *Askey-Wilson polynomials*. The q -Askey scheme has the same qualitative features, but it contains many more families; it takes two pages to print it, see R. Koekoek & R. F. Swarttouw, *The Askey scheme of hypergeometric orthogonal polynomials and its q -analogue*. The q -scheme is lying in a complicated way above the classical scheme. There are $q \uparrow 1$ limit arrows from q -families to classical families, but often one q -family has limits to several classical families, and one classical family can be seen as limit family of several q -families.

Most (but not all) objects and formulas associated with a certain family in the scheme also admit limits following the arrows in the scheme. These limits may be rigorous or only formal.

Many families in the classical and q -scheme have also limits to transcendental functions, which generalized orthogonal systems outside the polynomial world. See for instance the Bessel functions as limits of Jacobi polynomials. Here the generalized orthogonal system is given by the Hankel transform pair.

Very spectacular were the developments starting in the second half of the eighties with Heckman and Opdam's introduction of *Jacobi polynomials associated with root systems* and Ian Macdonald's introduction of *Macdonald polynomials*, also associated with root systems. In subsequent work by many people, for root system BC_n a full analogue of the Askey scheme and the q -Askey scheme took gradually shape.

In 1992 I got a full professor's position at the UvA, while Nico remained at CWI. However, we remained in touch. Nico became co-promotor at UvA of two of his PhD students, with me as promotor: these were Adri Olde Daalhuis (on the left in Figure 7) and Patrick Oonincx. In an NWO sponsored project we together supervised postdoc Raimundas Vidunas. We also taught together an intensive course on special functions for Dutch PhD students at Twente University, see Figure 7.

All families of orthogonal polynomials satisfy a three-term recurrence relation, and conversely each three-term recurrence relation, with the coefficients satisfying a certain positivity condition, uniquely generates a system of orthogonal polynomials, although its orthogonality measure is

not always unique. For all families in the Askey scheme the explicit recurrence relations of course have been written down. There is the converse problem: given an explicit three-term recurrence relation, does it generate a family in the Askey scheme, and, if so, with which parameters? For that, René Swarttouw and I together wrote a Maple procedure, `rec2ortho`, which could do this up to the two-parameter level of the Askey scheme. Already up to that level, the procedure was far from trivial. See again <http://www.science.uva.nl/~thk/art/software.html> for download.

A major project, in which we got both involved, started around 2001: the Digital Library of Mathematical Functions (DLMF), a full rewriting of the Handbook of Mathematical Functions by M. Abramowitz and I. A. Stegun. This project was coordinated by the institute NIST (Maryland, USA), in particular by Frank Olver and Dan Lozier. Nico became an associate editor of the project, and he wrote a number of chapters. I wrote, together with Roelof Koekoek, René Swarttouw and Roderick Wong, the chapter on Orthogonal Polynomials. For a while, I got hospitality at CWI during one day a week for working on this project, while I took refuge on that day from my administrative tasks as institute director at UvA. The book is not yet available, but work on it is near conclusion.

The Askey scheme, with its many limit transitions, of course invites to further asymptotic analysis. Nico also felt challenged by this, see for instance N. M. Temme & J. L. López, *The Askey scheme for hypergeometric orthogonal polynomials viewed from asymptotic analysis*, J. Comput. Appl. Math. 133 (2001), 623–633. I worked on this from another point of view, by considering the generic 4-parameter situation in the Askey scheme as a four-dimensional manifold, and considering families in lower levels of the scheme as being on the boundary of this manifold. More about this in my lecture on 27 May, 2005 at the symposium *From Here to Infinity* at CWI on occasion of the official retirement of Nico Temme.

Sources of pictures

The pictures in Figures 2–7 are from the author’s archive. The pictures in Figure 8 were obtained as follows (I thank Prof. Dr. Paul Butzer for help in obtaining the pictures of Hahn and Meixner):

Askey: <http://www.math.wisc.edu/~askey/>

Wilson: <http://orion.math.iastate.edu/wilson/>

Racah: <http://www-personal.umich.edu/~szwetch/Stamps.of.Israel/62.html>

Hahn: Prof. Dr. Ludwig Reich

Meixner: Dr. Ing. Michael Meixner and Prof. Dr. Hubert Geller

Pollaczek: <http://www2.uwindsor.ca/~hlynka/qhist.html>

Jacobi, Laguerre and Hermite: <http://www-groups.dcs.st-and.ac.uk/~history/BiogIndex.html>

Krawtchouk: <http://www.gulag.hu/links.htm>

Charlier: <http://www.phys-astro.sonoma.edu/BruceMedalists/Charlier/>

T. H. Koornwinder, Korteweg-de Vries Institute, University of Amsterdam,
Plantage Muidergracht 24, 1018 TV Amsterdam, The Netherlands

email: thk@science.uva.nl

Askey



scheme

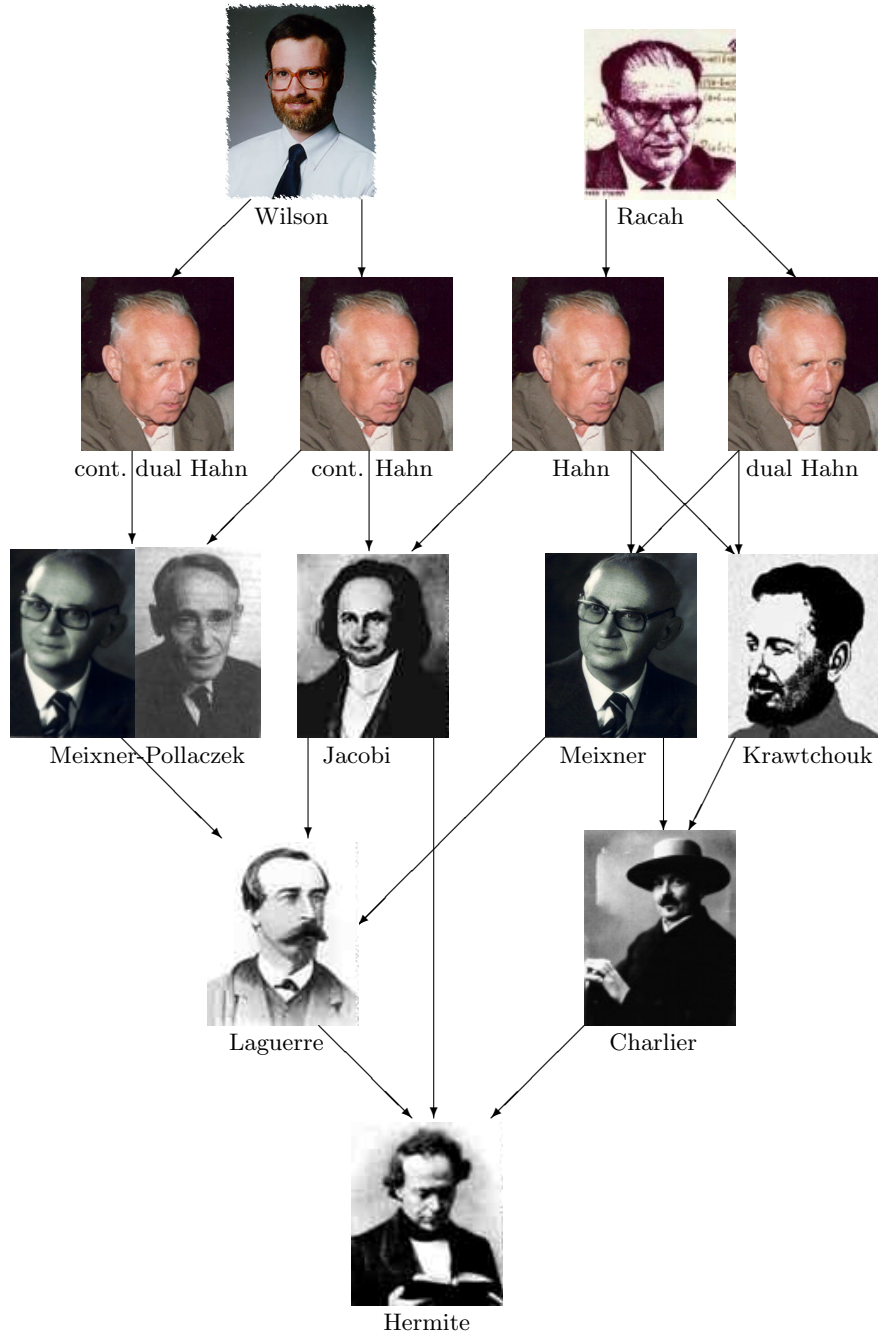


Figure 8: Askey scheme