Errata to and remarks on the book: G. Friedlander and M. Joshi, *Introduction to the theory of distributions*, Cambridge University Press, 1999, second printing

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Warning In the first 1982 edition of the book there are many more errata, not listed below because they were corrected in the 1999 second printing.

Chapter 1

p.9, Proof of Theorem 1.3.2, l.5: Add $\phi \neq 0$.

p.9, Proof of Theorem 1.3.2, l.7: Add $\phi_N \neq 0$.

p.9, Proof of Theorem 1.3.2, l.10: Replace right-hand side by $\phi_N(x)/\Big(N \sum_{|\alpha|\le N} \sup|\partial^\alpha \phi_N|\Big)$.

p.9, Definition 1.3.2$'$: One may add:
"A sequence $(\phi_j)_{1\le j<\infty} \in C_c^m(X)$ is said to converge in $C_c^m(X)$ to a function $\phi \in C_c^m(X)$ if the sequence $\phi_j - \phi$ converges to zero in $C_c^m(X)$."

p.11, l.12 of Proof: Replace "tha" by "that".

p.12, Theorem 1.4.3, l.2: After "index set" insert "and $X_\lambda \subset X$".

p.12, Proof of Theorem 1.4.3, l.8: Replace ψ_n' by ψ_1'.

p.12, l.-1: Replace ψ_n' by ψ_1'

p.13, *Note on partitions of unity*, l.2: Replace 1.4.4 by 1.4.1.

p.16, Exercise 1.7: Replace last line by:
Show also that, if f_ε satisfies (a) and (c) and if $f_\varepsilon \to \delta$ in $\mathcal{D}'(\mathbf{R}^n)$ as $\varepsilon \downarrow 0$, then $\lim_{\varepsilon\downarrow 0} \int f_\varepsilon(x)\,dx = 1$.

p.16, Exercise 1.9, l.4: Replace C_k by c_k.

Chapter 2

p.18, first line after (2.2.2): Replace R by $\mathbf{R}$.

p.19, l.1: Insert minus sign after last equality sign.

p.19, second line after (2.2.8): Insert between minus sign and $\partial\phi(x)$: $\int_{-\infty}^{\infty}$.

p.21, l.2: Replace 8.2.1 by 8.1.2.

p.22, l.-9: Replace "chose" by "choose".

p.23, Proof of Theorem 2.4.1, lines 4 and 8: Twice replace "supp ϕ_0" by "hull of supp ϕ_0".

p.23, Proof of Theorem 2.4.1, l.5: Replace $\sup|\phi_0|$ by $\int_{-\infty}^{\infty} |\phi_0(t)|\,dt$.

p.24, l.5: Theorem 2.5.1 can be proved without using (2.5.1).

p.24, (2.5.4): Insert u after $(\partial_i f)$.

p.25, second line after (2.6.1): Replace $a_\alpha \equiv 0$ by $a_\alpha \not\equiv 0$.

p.25, (2.6.3): Replace $(-1)^\alpha$ by $(-1)^{|\alpha|}$.

p.26, (2.6.6): On the right-hand side move $\dfrac{\partial^\alpha u}{\alpha!}$ to the right of f.

p.26, l.−7: Insert “linear” before “differential”.

p.27, Theorem 2.7.1 (i): The converse implication also holds.

p.28, bottom: We need that the linear map μ is continuous in the sense of Definition 2.8.1.

p.31, last formula in Exercise 2.5: On the left-hand side replace v by v_i.

p.31, Exercise 2.6, l.4: Replace a_n by a_1.

p.31, Exercise 2.7, l.3: Replace “if $\beta > \alpha$” by “otherwise”.

p.31, Exercise 2.9: This Exercise can better be moved to Chapter 3.

p.31, Exercise 2.11: Replace $u \sin \pi x$ by $(\sin \pi x) u$.

p.32, l.2: Here $0(\varepsilon^{\operatorname{Re}\lambda+k-1})$ means $o(\varepsilon^{\operatorname{Re}\lambda+k-1})$ (small oh).

p.32, l.3: Replace $j = 1$ by $j = 0$.

p.32, Exercise 2.14, last formula: This should read: $|x|^{\lambda-1}\operatorname{sign} x = x_+^{\lambda-1} - x_-^{\lambda-1}$.

p.32, Exercise 2.15, l.3: Replace $\exp(\lambda - 1)$ by $\exp((\lambda - 1)$.

p.32, Exercise 2.15, l.5: Replace $x^{\lambda-1}$ by $x_-^{\lambda-1}$.

Chapter 3

p.34, (3.1.2): Put “$\sup |\partial^\alpha \phi| : x \in K$” in brackets.

p.35, l.3: The increasing sequence of compact subsets $K_1, K_2, \dots$ must also satisfy that $K_i \subset K_{i+1}^0$ for all i, where A^0 denotes the interior of $A \subset \mathbf{R}^n$.

p.35, Theorem 3.1.2: One may add:
If $u \in \mathcal{D}'(X)$ has compact support then (3.1.1) holds for any compact $K \subset X$ such that $\operatorname{supp}(u) \subset K^0$.

p.37, l.8: This formula should read:

$$|\partial^\beta \phi(x)| \le \varepsilon^{N-|\beta|+1} \sum_{|\gamma|=N+1-|\beta|} \sup\{|\partial^{\gamma+\beta}\phi(x)| : |x| \le \varepsilon\}/\gamma! \quad \text{if } |x| \le \varepsilon.$$

p.37, lines 14,15: This should read:

$$|\partial^\alpha(\phi(x)\psi(x/\varepsilon))| \le C_\alpha \sum_{\beta+\gamma=\alpha} \varepsilon^{N-|\beta|+1}\varepsilon^{-|\gamma|} = C'_\alpha \varepsilon^{N+1-|\alpha|},$$

where C_α, C'_α are constants independent of ε.

p.37, Proof of Theorem 3.2.1, l.4: Insert “$= 0$” after $\partial^\alpha \phi'(0)$.

p.38, l.2: Replace $\mathbf{R}^n$ by X.

p.38, l.6: Replace $\frac{1}{4}\varepsilon$ by 4ε.

p.38, l.7: One may insert after K_ε: "and $\operatorname{supp}\psi_\varepsilon \subset K_{3\varepsilon}$".

p.38, l.8: After the equality sign replace 1 by $|$.

p.38, l.14: Replace (3.2.1) by (3.1.1).

p.39, (3.2.8): Replace left-hand side by its absolute value.
On the right-hand side insert a factor N before the summation sign.

p.39, Exercise 3.1, l.2: Replace $C^\infty(x)$ by $C^\infty(X)$.

Chapter 4

p.40, Theorem 4.1.1, l.1: Replace $\mathbf{R}^n$ by $\mathbf{R}^m$.

p.40, Theorem 4.1.1, l.4: One may insert "$\subset X$" after "$K(y')$".

p.41, l.16, 21: Replace $\partial(\partial y_j$ by $\partial/\partial y_j$.

p.41, l.19: Replace χ by χ_ε.

p.41, l.24: Replace n by m.

p.41, Corollary 4.1.2, l.1: Replace ψ by $\psi(y)$.

p.43, two lines above Theorem 4.2.2: Replace "function" by "mapping".

p.45, Lemma 4.3.1: One may add:
"and such that for all α we have $\sum_{j=1}^\infty \sup|\partial^\alpha \psi_{j1} \otimes \cdots \otimes \psi_{jN}| < \infty$."

p.46, l.−9: Replace $\langle u(x), \phi(x,y)\rangle = g(y)$ by $g(y) = \langle u(x), \phi(x,y)\rangle$.

p.47, (4.3.8): Replace by: $\partial_x^\alpha \partial_y^\beta (u(x) \otimes v(y)) = \partial^\alpha u \otimes \partial^\beta v$.

p.47, Proof of Theorem 4.3.3, part (ii), l.5,6:
Replace $\operatorname{supp} y$ by $\operatorname{supp} v$.
Replace part of sentence between "can find" and "such that" by:
"for each open neighbourhood U of x in X and each neighbourhood V of y in Y functions $\phi \in C_c^\infty(U)$ and $\psi \in C_c^\infty(V)$"

p.48, l.10: Replace ",$\rangle$" by "$\rangle$, ".

p.49, Exercise 4.2, l.2: Replace u by $A^* u$.

p.49, Exercise 4.3 part (ii): One may extend this to:
"Show that Euler's equation $\sum_{i=1}^n x_i \partial_i u = \lambda u$ holds if and only if u is homogeneous of degree λ."

p.49, Exercise 4.4: Replace $\rangle\rangle$ by $\rangle$.

Chapter 5

p.50, formula (∗∗): Insert after the equality sign a second integral sign.

p.51, (5.11): Replace $=$ by $\subset$.

p.52, (5.1.5): Replace ∂_i by ∂_j.

p.52, 3 lines above Theorem 5.1.3: Replace $C_c^\infty(\mathbf{R}^n)$ by $\mathcal{D}'(\mathbf{R}^n)$.

p.53, Proof of Theorem 5.2.1, l.3: Replace $\phi(x-y)$ by $\rho(x-y)$.

p.54, l.4: Assume moreover about ψ that its support is convex and contains 0.

p.54, 5 lines above Theorem 5.2.3: Assume moreover that $K_j \subset (K_{j+1})^0$.

p.54, (5.2.4): Replace $K_j \subset K_{j+1}$ by $K_j \subset (K_{j+1})^0$.

p.54, (5.2.6): Replace $\psi(x(\epsilon_j)$ by $\psi(x/\epsilon_j)$.

p.55, l.−3: Replace $A_1^\varepsilon, \ldots, A_m^\varepsilon$ by $A_1^\varepsilon \times \cdots \times A_m^\varepsilon$.

p.56, l.2: Replace A by A^ε, B by B^ε.

p.56, l.7: Replace $|x-x'|$ by $|x-x'+x'|$.

p.56, l.8: Replace A by A^ε, B by B^ε.

p.56, l.13: Replace $C_c^\infty(\mathbf{R})$ by $C_c^\infty(\mathbf{R}^n)$.

p.56, l.14: Skip "is supported in $K_\varepsilon(\phi)$".

p.56, l.20: After $m=2$ insert "and when $u_2 \in \mathcal{E}'(\mathbf{R}^n)$"

p.56, l.22: Replace $u_1 * \ldots * u_m$ by $u_1 * \ldots * u_m$.

p.56, second line of Theorem 5.3.2(i): Here one has to use the definition of $\langle u, \phi\rangle$ for $u \in \mathcal{D}'(\mathbf{R}^n)$, $\phi \in C^\infty(\mathbf{R}^n)$ and $\mathrm{supp}(u) \cap \mathrm{supp}(\phi)$ compact, see Exercise 3.1.

p.56, Theorem 5.3.2(ii): Add that convolution is commutative.

p.56, third line of Theorem 5.3.2(ii): Replace $i \in J$ by $i \in I$.

p.57, first line after (5.3.3): Replace $\delta \geq 0$ by $\delta > 0$.

p.58, (5.3.9): Replace ∂E^+ by $\partial_n E^+$, ∂E^- by $\partial_n E^-$.

p.60, l.14: Insert ρ after ε^{-n}.

p.60, l.17: Insert at the end of the line: "(see Exercise 5.4)".

p.60, l.21: Insert after "$\varepsilon \to 0$." the sentence: "Here $\lim_{j\to\infty} \phi_j = \phi$ in $C^N(\mathbf{R}^n)$ means that for all compact $K \subset \mathbf{R}^n$ and for all α, $|\alpha| \leq N$, we have $\lim_{j\to\infty} \partial^\alpha \phi_j = \partial^\alpha \phi$, uniform on K."

p.61, l.1: Twice replace $N+1$ by $N+2$.

p.61, second line of Corollary 5.4.1: After "functions" insert "of compact support".

p.61, l.13: Replace h by n.

p.61, l.−8: Replace $\alpha \geq 0$ by $|\alpha| \geq 0$.

p.61, l.−4: Twice replace $\alpha \geq 0$ by $|\alpha| \geq 0$.

p.62, l.5: Replace $\pi^{1/2n}$ by $\pi^{(1/2)n}$.

p.62, (5.4.7): Replace by $1/\big((n-2)\omega_{n-1}|x|^{n-2}\big)$.

p.62, l.10: Replace $1/4\pi|x|$ by $1/(4\pi|x|)$.

p.62, l.−2: In the middle part omit the integral sign.

p.62, l.−1: On the right-hand side insert the factor $\frac{1}{2}$ before the integral.

p.63, l.3: On the right-hand side insert a factor π.

p.63, l.10: Replace the exponent $-1/2n$ by $-\frac{1}{2}n$, and replace $-|x|^2/4t$ by $-|x|^2/(4t)$.

p.65, l.13: Replace $\phi(0,0)$ by $\phi(0)$.

p.65, Exercise 5.1(ii), l.2: Replace "to $A+B$" by "to $A \times B$".

p.65, Exercise 5.2, l.4: Replace $x = \operatorname{supp} u$ by $x \in \operatorname{supp} u$.

p.66, l.1: Replace $\mathcal{D}'(\mathbf{R})$ by $\mathcal{D}'^{+}(\mathbf{R})$.

p.66, Exercise 5.4, l.2: Replace $U * \psi$ by $u * \psi(x)$.

p.66, Exercise 5.4, l.3: Replace "if u is" by "if $u \in \mathcal{E}'(\mathbf{R}^n)$ is".

p.66, Exercise 5.5, l.5: Replace "$u_1 \dots u_m$ is" by "$u_1 * \dots * u_m$ is".

p.66, Exercise 5.5, l.7: Replace $u_1 \dots u_m * v$ by $u_1 * \dots * u_m * v$,

p.67, l.1: Replace 2^{k+1} by 2^{k-1}.

p.67, l.4: Replace "$\leq \phi_k$" by "$\leq \mu_k$".

Chapter 6

p.71, l.−5: Insert $(1+|h|)^N$ after $(1+|g|)^N$.

p.71, l.−2: Replace $\hat{\chi}$ by $\hat{\chi}_{g,h}$.

p.78, l.4: Replace "right" by "left".

p.78, (6.3.12): Replace $\langle E, \chi \rangle$ by $\langle {}^t E, \chi \rangle$.

p.78, Exercise 6.3: Here a differential operator is meant of the form in p.25, §2.6 with coefficients a_α in $\mathcal{D}'(X)$.
There is also an extension of Peetre's theorem stating that if $k\colon C_c^\infty(X) \to C^\infty(X)$ is a linear (a priori not necessarily continuous) map with $\operatorname{supp}(k(u)) \subset \operatorname{supp}(u)$ for all $u \in C_c^\infty(X)$ then k is a differential operator with C^∞ coefficients. See J. Peetre, *Rectification à l'article "Une caractérisation abstraite des opérateurs différentiels"* Math. Scand. 8 (1960), 116–120.

Chapter 8

p.91, Theorem 8.1.2, l.2: Insert "measurable" before "function".

p.91, Theorem 8.1.2, l.3: Insert "in t" after "function".

p.91, Proof of Theorem 8.1.2, l.1: Insert "(i) and" after "By".

p.92, l.−2: Omit the integral sign on the left-hand side.

p.93, l.2: Replace $e^{-\mathrm{i}x\cdot\xi}$ by $e^{-\mathrm{i}z\cdot\xi}$.

p.93, (8.1.8): Replace the last part by "($\mathrm{i} = \sqrt{-1}$)".

p.93, l.−3: Add: "for a linear map from a Fréchet space to a topological space".

p.95, l.3: Replace $D^\alpha\phi$ by $D^\alpha\hat{\phi}$.

p.95, l.6: Replace $\|(-1)^{|\beta|}(D^\alpha(x^\beta\phi))\hat{}\,\|$ by $\sup|(-1)^{|\beta|}(D^\alpha(x^\beta\phi))\hat{}\,|$

p.96, l.11: Replace $\hat{\phi}(e)$ by $\hat{\phi}(\xi)$.

p.98, l.7: Insert "and by Exercise 5.4" after "Theorem 5.4.1".

p.98, l.12: Take the sup of the absolute value of the given expression.

p.99, Corollary 8.3.1, l.2: Replace (8.3.1) by (8.1.1).

p.99, l.−6: Replace $\phi(\xi + h)$ by $\hat{\phi}(\xi + h)$.

p.101, l.−4: Replace 4.3.6 by 4.3.3.

p.102, Lemma 8.4.1: After "then" replace v by $\hat{v}$.

p.102, Proof of Lema 8.4.1, l.3: The fact that $x^\alpha v$ is in $\mathcal{E}'(\mathbf{R}^n)$ is true, but it is not used.

p.103, Lemma 8.4.2: This is essentially the Remark after Definition 8.3.2.

p.103, l.−5: Replace $C_c(\mathbf{R}^n)$ by $C_c^\infty(\mathbf{R}^n)$.

p.104, Lemma 8.5.1, l.4: Replace "$(\tau_g\psi)_g \in \mathbb{Z}^n$" by "$(\tau_g\psi)_{g\in\mathbb{Z}^n}$".

p.105, l.−10: Replace h by n.

p.107, l.1: Replace "$u \in \mathcal{E}'(\mathbf{R}^n)$" by "$\psi u \in \mathcal{E}'(\mathbf{R}^n)$".

p.107, (8.5.12): On the right-hand side insert a factor $(2\pi)^n$.

p.108, Definition 8.6.1, l.1: Replace $\mathcal{S}'$ by $\mathcal{D}'$.

p.108, Lemma 8.6.1, l.1: Replace $\mathcal{D}'$ by $\mathcal{E}'$.

p.108, Proof of Lemma 8.6.1, l.1: Replace $\rho \in C^\infty(\mathbf{R}^n)$ by $\rho \in C_c^\infty(\mathbf{R}^n)$. Also replace $\psi \in C_c^\infty(\mathbf{R}^n)$ by $\psi \in C^\infty(\mathbf{R}^n)$.

p.108, (8.6.3): Replace $|\alpha| \le m$ by $|\alpha| = m$,

p.109, Proof of Theorem 8.6.1, l.6: Insert "and m is the order of P" after "$c > 0$".

p.109, (8.6.10): Replace "$= \{0\}$" by "$\subset \{0\}$".

p.110, (8.6.11): Replace DE by PE.

p.110, l.3: Insert "Let $u \in \mathcal{D}'(X)$." at beginning of line.

p.110, l.5, 8, 10, 12: At five places replace $P\psi u$ by $P(\psi u)$.

p.110, last line before Exercises: Theorem 8.6.1 (also the generaliztion with C^∞ coefficients) was first proved by K. O. Friedrichs, *On the differentiability of the solutions of linear elliptic differential equations*, Comm. Pure Appl. Math. 6 (1953), 299–326.

p.110, Exercise 8.6, l.2: Replace $(u\psi)\hat{}$ by $(\psi u)\hat{}$.

p.110, Exercise 8.7, l.3: Replace $\mathbb{C}\backslash 0, -1, \ldots$ by $\mathbb{C}\backslash\{0, -1, \ldots\}$.

p.111, l.1: On the right-hand side replace $2\pi\mathrm{i}$ by $-2\pi\mathrm{i}$.

p.111, l.4: Replace $\mathbf{R}$ by $\mathbf{R}\backslash\{0\}$.

Chapter 9

p.117, l.−10: If indeed the authors prefer to take the statement that $C_c^0(\mathbf{R}^n)$ is dense in $L_2(\mathbf{R}^n)$

from the literature instead of proving it here, then the density of $C_c^\infty(\mathbf{R}^n)$ can be proved quicker from this statement together with Theorem 1.2.1.

p.120, l.−1: Replace the exponent $\frac{1}{2}$ by $\frac{1}{2}s$.

p.121, (9.3.2): Replace exponent $\frac{1}{2}s$ by s.

p.121, Proof of Theorem 9.3.1, l.14: Replace $(1+|\xi|^2)^{\frac{1}{2}}$ by $(1+|\xi|^2)^{\frac{1}{2}s}$.

p.122, Proof of Theorem 9.3.2, l.3: Replace $\hat{u}$ by $\overline{\hat{u}(\xi)}$.

p.123, l.6: Replace $\xi^\alpha u$ by $\xi^\alpha \hat{u}$. Also insert "for $|\alpha| \le m$" after "$L_2(\mathbf{R}^n)$".

p.123, l.17: Replace u_α by u.

p.124, Proof of Corollary 9.3.3, l.1: Replace 9.3.1 by 9.3.2.