Errata and comments on the book *Basic hypergeometric series*, Second edition, by G. Gasper and M. Rahman

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These are errata and comments on the book

G. Gasper and M. Rahman, *Basic hypergeometric series*, Cambridge University Press, Second ed., 2004, ISBN 9780521833578.

p. 101, Exercise 3.2(iii):
We can combine the two equalities in (i) and (ii) as

$${}_3\phi_2\left(\begin{matrix}a,b,-b\\b^2,-az\end{matrix};q,z\right)=\frac{(-z;q)_\infty}{(-az;q)_\infty}\,{}_2\phi_1\left(\begin{matrix}a,aq\\qb^2\end{matrix};q^2,z^2\right)=\frac{(az^2;q^2)_\infty}{(z,-az;q)_\infty}\,{}_2\phi_2\left(\begin{matrix}a,a^{-1}b^2\\qb^2,az^2\end{matrix};q^2,az^2q\right).$$

Then the two equalities in (iii) are the limit case $a\to 0$ of the above two equalities. In the two equalities in (iii), with b replaced by q^b and z by $(1-q)z$, we obtain for $q\to 1$ that

$${}_1F_1\left(\begin{matrix}b\\2b\end{matrix};2z\right)=\mathrm{e}^z\,{}_0F_1\left(\begin{matrix}-\\b+\frac12\end{matrix};\tfrac14 z^2\right)=\mathrm{e}^z\,{}_0F_1\left(\begin{matrix}-\\b+\frac12\end{matrix};\tfrac14 z^2\right).$$

Equivalently, see Erdélyi [1953, Vol. 2, 7.2(3)],

$$J_\nu(z):=\frac{(\frac12 z)^\nu}{\Gamma(\nu+1)}\,{}_0F_1\left(\begin{matrix}-\\\nu+1\end{matrix};-\tfrac14 z^2\right)=\frac{(\frac12 z)^\nu\mathrm{e}^{-\mathrm{i}z}}{\Gamma(\nu+1)}\,{}_1F_1\left(\begin{matrix}\nu+\frac12\\2\nu+1\end{matrix};2\mathrm{i}z\right).$$

On the q-level, with notation as in Exercise 1.24, the equalities in Exercise 3.2(iii) can be equivalently written as

$$J_\nu^{(1)}(z;q^2)=\frac{1}{(-\frac14 z^2;q^2)_\infty}\,J_\nu^{(2)}(z;q^2)=\frac{(q^{2\nu+2};q^2)_\infty}{(q^2;q^2)_\infty}\,\frac{(\frac12 z)^\nu}{(-\frac12\mathrm{i}z;q)_\infty}\,{}_2\phi_1\left(\begin{matrix}q^{\nu+\frac12},-q^{\nu+\frac12}\\q^{2\nu+1}\end{matrix};q,\tfrac12\mathrm{i}z\right).$$

The first equality in this last formula is also given in Exercise 33.2(iii), and it is attributed there to Hahn [1949c].

Note that by (i) respectively (iii) the functions $(az,-z;q)_\infty\,{}_3\phi_2\left(\begin{smallmatrix}a,b,-b\\b^2,az\end{smallmatrix};q,-z\right)$ and $(z;q)_\infty\,{}_2\phi_1\left(\begin{smallmatrix}b,-b\\b^2\end{smallmatrix};q,z\right)$ are even in z.

By the expression given above for $J_\nu^{(1)}(z;q^2)$ the product formula

$${}_2\phi_1\left(\begin{matrix}a,-a\\a^2\end{matrix};q,z\right)\,{}_2\phi_1\left(\begin{matrix}b,-b\\b^2\end{matrix};q,-z\right)={}_4\phi_3\left(\begin{matrix}ab,-ab,abq,-abq\\a^2q,b^2q,a^2b^2\end{matrix};q^2,z^2\right)$$

(see formula (4.9) in H. M. Srivastava & V. K. Jain, *q-Series identities and reducibility of basic double hypergeometric functions*, Canad. J. Math. 38 (1986), 215–231, and formula

(2.1) in M. J. Schlosser, *q-Analogues of two product formulas of hypergeometric functions by Bailey*, in *Frontiers in orthogonal polynomials and q-series*, World Scientific, 2018, pp. 445–449) can be rewritten as

$$J_\mu^{(1)}(z;q^2)J_\nu^{(1)}(z;q^2) = \frac{(q^{2\mu+2},q^{2\nu+2};q^2)_\infty}{(q^2,q^2;q^2)_\infty}\,\frac{(\frac12 z)^{\mu+\nu}}{(-\frac14 z^2;q^2)_\infty}$$
$$\times\ {}_4\phi_3\left(\begin{matrix} q^{\mu+\nu+1},-q^{\mu+\nu+1},q^{\mu+\nu+2},-q^{\mu+\nu+2} \\ q^{2\mu+2},q^{2\nu+2},q^{2\mu+2\nu+2}\end{matrix};q^2,-\tfrac14 z^2\right).$$

For the $q=1$ limits of these product formulas see formulas (16.12.1) and (10.8.3) in DLMF, `https://dlmf.nist.gov/`.

p. 147, Exercise 5.10:
In the numerator on the left-hand side replace e/ab and q^2f/e by c/qf and q^2f/c
(error observed in p. 841 of W. Groenevelt & E. Koelink,
J. Approx. Theory 163 (2011), 836–863).
The formula with the same error occurs in (7.2.6) in the book
L. J. Slater, *Generalized hypergeometric functions*, Cambridge University Press, 1966.
A reference for Exercise 5.10 with the correct formula is formula (5) in
L. J. Slater, *General transformations of bilateral series*,
Quart. J. Math., Oxford Ser. (2) 3 (1952), 73–80.

p. 189, (7.5.7): On the second line the comma after $d\,e^{-i\theta}$ should be deleted.

p. 189, (7.5.8): Insert "q," after the second semicolon of the first ${}_8W_7$.

p. 212, l.5: (communicated by Slobodan Lj. Damjanovic)
In the ${}_5\phi_4$ insert a numerator parameter $\sqrt{q}$ and replace the denominator parameter $eq^{-2i\theta}$ by $qe^{-2i\theta}$.

p. 236, (8.8.19), l.4: (communicated by Slobodan Lj. Damjanovic)
In the ${}_6\phi_5$ replace the numerator parameter abz by az/b.
The same correction should be made in formula (4.11) of the paper

G. Gasper and M. Rahman, *A non-terminating q-Clausen formula and some related product formulas*, SIAM J. Math. Anal. 20 (1989), 1270–1282.

p. 420, list of Jackson, F. H.: Move the number 138 to the list of Jackson, M.