**Comparison of Macdonald's and Koornwinder's relations for the double affine Hecke algebra of type** $(C_1^\vee, C_1)$

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Macdonald [2] gives relations (6.4.2), (6.4.6), (6.4.8) for the double affine Hecke algebra of type $(C_1^\vee, C_1)$ with generators $T_1, T_0, T_1', T_0'$ and parameters $\tau_1, \tau_0, \tau_1', \tau_0'$:

$$\begin{aligned}
&(T_1-\tau_1)(T_1+\tau_1^{-1})=0, \qquad (T_0-\tau_0)(T_0+\tau_0^{-1})=0,\\
&(T_1'-\tau_1')(T_1'+\tau_1'^{-1})=0, \qquad (T_0'-\tau_0')(T_0'+\tau_0'^{-1})=0,\\
&T_0'\,T_0\,T_1'\,T_1=q^{-\frac12}.
\end{aligned}$$

Koornwinder [1] equivalently gives relations (3.1)–(3.4) for the double affine Hecke algebra of type $(C_1^\vee, C_1)$ with generators $\widetilde T_1, \widetilde T_0, \widetilde Z, \widetilde Z^{-1}$ and parameters $a, b, c, d$:

$$\begin{aligned}
&(\widetilde T_1+ab)(\widetilde T_1+1)=0, \qquad (\widetilde T_0+q^{-1}cd)(\widetilde T_0+1)=0,\\
&(\widetilde T_1\widetilde Z+a)(\widetilde T_1\widetilde Z+b)=0, \qquad (q\widetilde T_0\widetilde Z^{-1}+c)(q\widetilde T_0\widetilde Z^{-1}+d)=0.
\end{aligned}$$

We can go from Macdonald's relations to Koornwinder's relations by putting

$$\begin{aligned}
&T_1=\tau_1^{-1}\widetilde T_1, \quad T_0=\tau_0^{-1}\widetilde T_0, \quad T_1'=\tau_1\widetilde Z^{-1}\widetilde T_1^{-1}, \quad T_0'=q^{-\frac12}\tau_0\widetilde Z\widetilde T_0^{-1},\\
&\tau_1=i(ab)^{\frac12}, \quad \tau_0=i(cd/q)^{\frac12}, \quad \tau_1'=-i(b/a)^{\frac12}, \quad \tau_0'=-i(d/c)^{\frac12}.
\end{aligned}$$

Conversely we can go from Koornwinder's relations to Macdonald's relations by putting

$$\begin{aligned}
&a=-\tau_1/\tau_1', \quad b=\tau_1\tau_1', \quad c=-q^{\frac12}\tau_0/\tau_0', \quad d=q^{\frac12}\tau_0\tau_0',\\
&\widetilde T_1=\tau_1T_1, \quad \widetilde T_0=\tau_0T_0, \quad \widetilde Z=q^{\frac12}T_0'\,T_0=T_1^{-1}T_1'^{-1},
\end{aligned}$$

## References

[1] T. H. Koornwinder, *The relationship between Zhedanov's algebra $AW(3)$ and the double affine Hecke algebra in the rank one case*, SIGMA 3 (2007), 063, 15 pp.; arXiv:math/0612730v4 [math.QA].

[2] I. G. Macdonald, *Affine Hecke algebra and orthogonal polynomials*, Cambridge University Press, 2003.