Computational Social Choice and Complexity Theory

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ESSLLI 2018 – Day 4
Recap

- Voting theory
- (Parameterized) complexity theory
- Complexity of winner determination
- Strategic manipulation in voting
- Bribery
- (Nearly) single-peakedness
What we’ll do today

- Judgment Aggregation (JA)
- JA Procedures
- Complexity issues
- Modeling Voting in JA
Judgment Aggregation
Example: Discursive Dilemma

- Consider a court case with a jury
- Suppose that they use majority voting

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- $k$: the defendant killed the victim
- $i$: it was intentional
- $k \land i$: it was murder (*intentional killing*)
Example: Budget Spending

- Consider a city council meeting on budget spending
- Suppose that they use majority voting

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- $p_1, p_2, p_3$: projects to fund, each with a cost of 1000 leva
- A total budget of 2000 leva is available

$(\Gamma = \neg p_1 \lor \neg p_2 \lor \neg p_3)$
Judgment Aggregation

- **Agenda**: set of propositional formulas and their negations, \( \Phi = \{ \varphi_1, \ldots, \varphi_n, \neg \varphi_1, \ldots, \neg \varphi_n \} \) – set of issues
  - Pre-agenda: \([\Phi] = \{ \varphi_1, \ldots, \varphi_n \}\)

- **Integrity constraint**: propositional formula \( \Gamma \) – logical context

- **Judgment set**: \( J \subseteq \Phi \).
  - **Complete** if for each \( i \in [n] \), either \( \varphi_i \in J \) or \( \neg \varphi_i \in J \)
  - **Consistent** if \( J \) is logically consistent with \( \Gamma \)
  - **Feasible opinions**: complete and consistent judgment sets

- **Profile**: sequence \( J = (J_1, \ldots, J_m) \) of complete and consistent judgment sets – individual opinions

- **Judgment aggregation procedure**: a function \( F \) that assigns to each profile \( J \) a set \( F(J) \) of judgment sets – (possible) group opinions
The Majority JA Procedure

- **Majority** is the JA Procedure that outputs the judgment set $J$ containing all issues that are supported by a majority of judgment sets in the profile $J$
  - For an odd number of individuals, Majority($J$) is a complete judgment set
  - But it is not always consistent
Example

- **Pre-agenda:** \([\Phi] = \{x_1, x_2, x_3\}\)

- **Agenda:** \(\Phi = \{x_1, x_2, x_3, \neg x_1, \neg x_2, \neg x_3\}\)

- **Integrity constraint:**
  \[\Gamma = \neg (x_1 \land x_2 \land x_3)\]

- **Profile:** \(J = (J_1, J_2, J_3)\)
  - \(J_1 = \{x_1, x_2, \neg x_3\}\)
  - \(J_2 = \{x_1, \neg x_2, x_3\}\)
  - \(J_3 = \{\neg x_1, x_2, x_3\}\)

- **Majority**\((J)\) = \(\{x_1, x_2, x_3\}\)

- **Majority**\((J)\) is **not consistent**
Example

- **Pre-agenda**: $[\Phi] = \{x_1, x_2, x_3, y_1, y_2, y_3, z\}$

  **Agenda**: $\Phi = \{x_1, x_2, x_3, y_1, y_2, y_3, z, \neg x_1, \neg x_2, \neg x_3, \neg y_1, \neg y_2, \neg y_3, \neg z\}$

- **Integrity constraint**:
  \[ \Gamma = [(x_1 \land x_2 \land x_3) \rightarrow z] \land [(y_1 \land y_2 \land y_3) \rightarrow z] \]

- **Profile**: $J = (J_1, J_2, J_3)$
  - $J_1 = \{x_1, x_2, \neg x_3, y_1, y_2, \neg y_3, \neg z\}$
  - $J_2 = \{x_1, \neg x_2, x_3, y_1, \neg y_2, y_3, \neg z\}$
  - $J_3 = \{\neg x_1, x_2, x_3, \neg y_1, y_2, y_3, \neg z\}$

  **Majority**($J$) = $\{x_1, x_2, x_3, y_1, y_2, y_3, \neg z\}$

- **Majority**($J$) is not consistent
General Model

▶ Very general framework, where you can easily model a wide range of possible ‘elections’ on interrelated issues

▶ E.g., pick your favorite combination of:
  ▶ budget constraints
    (“each issue costs $c_i$, and the total budget is $b$”)
  ▶ dependencies
    (“you can only choose $a$ if you also choose $b$ or $c$”)
  ▶ rankings
    (“rank the issues $a, b, c$ in order of importance”)
  ▶ etc.
The Premise-Based Procedure

- One approach to avoiding inconsistency is to use the Premise-Based Procedure (PBP)

- Split the agenda into premises and conclusions, such that:
  - the premises are logically independent of each other
  - any (complete) judgment set over the premises determines the truth value of the conclusions

- Then carry out the Majority rule on the premises, and take the entailed outcome for the conclusions
Example: PBP for the Discursive Dilemma

- Premises: $k$ and $i$
- Conclusions: $k \land i$

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A Judgment Aggregation procedure $F$ is **consistent** if for each agenda $\Phi$, each integrity constraint $\Gamma$ and each profile $J$, each $J \in F(J)$ is consistent with $\Gamma$.

The Premise-Based Procedure only work for agendas with a particular structure (i.e., agendas with ‘premises’ and ‘conclusions’).

For our example with the budget constraint, this is not the case.

Are there JA procedures that work for all agendas and that are consistent?
The Kemeny JA Procedure

- The Kemeny JA Procedure is defined as follows:

- For a profile $J = (J_1, \ldots, J_p)$ it returns those complete and consistent judgment sets $J^*$ that minimize:

$$\sum_{1 \leq i \leq p} d_H(J_i, J^*)$$

where $d_H(J, J')$ is the Hamming distance between two judgment sets:

$$d_H(J, J') = \frac{|J \setminus J'| + |J' \setminus J|}{2}$$

(the number of issues on which $J$ and $J'$ disagree)

- If the majority outcome is consistent, the Kemeny procedure outputs the majority outcome only.
Example: Kemeny for Budget Spending

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$$\Gamma = \neg p_1 \lor \neg p_2 \lor \neg p_3$$

- Kemeny($J$) = \{\{p_1, p_2, \neg p_3\}, \{p_1, \neg p_2, p_3\}, \{\neg p_1, p_2, p_3\}\}
We can model preference aggregation (voting) in judgment aggregation.

Let $A$ be a set of alternatives. Consider the agenda $\Phi_A$ and the integrity constraint $\Gamma_A$:

$$\Phi_A = \{ p_{a,b} : a, b \in A, a \neq b \}$$

$$\Gamma_A = \bigwedge_{a, b \in A \land a \neq b} (p_{a,b} \leftrightarrow \neg p_{b,a}) \land \bigwedge_{a, b, c \in A \land a \neq b, b \neq c, a \neq c} ((p_{a,b} \land p_{b,c}) \rightarrow p_{a,c})$$

The linear orders $\succ$ over $A$ are in one-to-one correspondence with the complete and consistent judgment sets $J_\succ$ for $\Phi_A$, $\Gamma_A$.

The Kendall-Tau distance between $\succ$ and $\succ'$ is exactly twice the Hamming distance between $J_\succ$ and $J_\succ'$. 
For example, take $A = \{a, b, c\}$

Then $[\Phi_A] = \{p_{a,b}, p_{a,c}, p_{b,c}\}$, and:

$$\Gamma_A = \left( p_{a,b} \leftrightarrow \neg p_{b,a} \right) \land \left( p_{a,c} \leftrightarrow \neg p_{c,a} \right) \land \left( p_{b,c} \leftrightarrow \neg p_{c,b} \right) \land \left( (p_{a,b} \land p_{b,c}) \rightarrow p_{a,c} \right) \land \left( (p_{a,c} \land p_{c,b}) \rightarrow p_{a,b} \right) \land \left( (p_{b,a} \land p_{a,c}) \rightarrow p_{b,c} \right) \land \left( (p_{b,c} \land p_{c,a}) \rightarrow p_{b,a} \right) \land \left( (p_{c,a} \land p_{a,b}) \rightarrow p_{c,b} \right) \land \left( (p_{c,b} \land p_{b,a}) \rightarrow p_{c,a} \right)$$
Modeling Preference Aggregation in JA

- **The Condorcet paradox**

- Take \( A = \{a, b, c\} \) and consider \( \Phi_A \) and \( \Gamma_A \)

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Hardness for the Kemeny JA Procedure

**JA-Outcome**$(F)$

**Input:** an agenda $\Phi$, an integrity constraint $\Gamma$ a profile $J$, and a partial judgment set $J_0$.

**Output:** Is there some $J^* \in F(J)$ that agrees with $J_0$?

**Theorem**

JA-Outcome(Kemeny) is $\Theta^p_2$-complete.

**Idea:** we can use the modeling of voting in judgment aggregation as a reduction from WinDet(Kemeny) to JA-Outcome(Kemeny)

- with $J_0 = \{ p_{a^*,b} : b \in A, b \neq a^* \}$
A Poly-time Consistent JA Procedure

- Does there exist a **polynomial-time computable** judgment aggregation procedure that is **consistent**?

- Yes: the **Plurality JA procedure**
  - Selects as outcomes those judgment sets that appear most often in the profile

- **Note**: this rule can give arguably undesirable outcomes:

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The Axiomatic Method

- Just like with voting, one can use the axiomatic method to investigate the existence of JA procedures with certain normatively appealing properties.

- A JA procedure is:
  - **anonymous** if the outcome is always preserved under permuting the individuals in the profile.
  - **majority preserving** if the procedure outputs the majority outcome whenever it is consistent.
  - **unanimous** if whenever all judgment sets in the profile agree on some issue \( \varphi \), then \( \varphi \) is in all outcomes.
  - etc.
An Impossibility Result

Theorem (List, Pettit, 2002)

There is no JA procedure, for agendas $\Phi \supseteq \{p, q, p \land q\}$, that is resolute, anonymous, neutral, independent, complete and consistent.

- **Neutral**: all formulas are treated the same (i.e., if $\varphi$ and $\varphi'$ are in exactly the same judgment sets in the profile, then either both or none should be in the outcome)

- **Independent**: whether or not $\varphi$ is in the outcome only depends on the pattern of individual acceptances of $\varphi$

The Axiomatic Method

- **Takeaway:** there is no unique best JA procedure

- Determine what axioms / normative properties are most desirable in the domain where you want to apply judgment aggregation, and try to find a JA procedure that satisfies these axioms

Agenda Safety for the Majority Rule

- For some agendas, the Majority JA procedure will never lead to inconsistent outcomes
- E.g., $\Phi = \{p, q, r, \neg p, \neg q, \neg r\}$, $\Gamma = (p \lor q)$

**Theorem (Nehring, Puppe, 2007)**

The majority rule is consistent for an agenda $\Phi$ and an integrity constraint $\Gamma$ if and only if $\Phi$ has the median property w.r.t. $\Gamma$.

- Median property: every inconsistent subset of $\Phi$ (w.r.t., $\Gamma$) does itself have an inconsistent subset of size at most 2

Agenda Safety for the Majority Rule

- Suppose some agenda $\Phi$ has the median property w.r.t. some integrity constraint $\Gamma$

- Why is the majority rule consistent for $\Phi$, for an odd number of individuals?

  - Suppose the contrary, i.e., that there is some profile $J$ such that $\text{Majority}(J)$ is inconsistent with $\Gamma$
  
  - Take a minimally inconsistent subset $J^*$ of $\text{Majority}(J)$
  
  - Then $|J^*| \leq 2$
  
  - Since $J^*$ is part of $\text{Majority}(J)$, each $\varphi \in J^*$ is supported by a strict majority
  
  - Then there must be some individual whose judgment set agrees with $J^*$
  
  - **Contradiction:** then $J^*$ is consistent
Complexity of Agenda Safety

**Agenda-Safety**

**Input:** An agenda \( \Phi \), and an integrity constraint \( \Gamma \).

**Output:** Does \( \Phi \) have the median property (w.r.t. \( \Gamma \))—that is, is every minimally inconsistent subset of \( \Phi \) of size at most 2?

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**Theorem (Endriss, Grandi, Porello, 2012)**

Agenda-Safety is \( \Pi_2^p \)-complete.

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Complexity Issues in the Framework

- Suppose $\Phi = \{y, y \to \varphi\}$, where:
  $$\varphi = (\neg x_5 \lor \neg x_1 \lor x_7) \land (x_4 \lor \neg x_2 \lor x_1) \land (\neg x_1 \lor x_6 \lor \neg x_7) \land (x_1 \lor \neg x_4 \lor \neg x_2) \land (x_6 \lor \neg x_1 \lor x_5) \land (x_3 \lor x_2 \lor x_1) \land (x_2 \lor x_1 \lor \neg x_3) \land (\neg x_6 \lor \neg x_7 \lor \neg x_1)$$

- Suppose that an individual reports the judgment set $J = \{y, y \to \varphi\}$

- Is it easy to check whether this is a valid ballot? *No*: this boils down to solving the SAT problem *(NP-hard)*

- It is *polynomial-time solvable* to come up with *some* ballot but it could be hard to find a valid ballot that includes some given issue $\varphi \in \Phi$

- This is a *computational hurdle* for using JA

- Also: could open up a way to *control* the election
A Different Framework  
(Binary Aggregation with Integrity Constraints – BAIC)

- **Issues**: a set $\mathcal{I} = \{x_1, \ldots, x_n\}$ of propositional variables

- **Integrity constraint**: propositional formula $\Gamma$ over the variables $x_1, \ldots, x_n$ – logical context

- **Ballot**: $(b_1, \ldots, b_n) \in \{0, 1\}^n$
  - corresponds to a truth assignment $\alpha$ to the variables $x_1, \ldots, x_n$ – namely, $\alpha(x_i) = b_i$
  - consistent if $\alpha \models \Gamma$
  - feasible opinions: consistent ballots

- **Profile**: sequence $r = (r_1, \ldots, r_m)$ of consistent ballots – individual opinions

- **Judgment aggregation procedure**: a function $F$ that assigns to each profile $r$ a set $F(r)$ of ballots – (possible) group opinions
Example

- Issues: $\mathcal{I} = \{x_1, x_2, x_3\}$

- Integrity constraint:
  \[
  \Gamma = \neg (x_1 \land x_2 \land x_3)
  \]

- Profile: $r = (r_1, r_2, r_3)$
  - $r_1 = \{1, 1, 0\}$
  - $r_2 = \{1, 0, 1\}$
  - $r_3 = \{0, 1, 1\}$

  - Majority($r$) = $\{1, 1, 1\}$

- Majority($r$) is not consistent
Complexity Issues in the Different Framework

- Suppose:
  \[ \Gamma = (\neg x_5 \lor \neg x_1 \lor x_7) \land (x_4 \lor \neg x_2 \lor x_1) \land (\neg x_1 \lor x_6 \lor \neg x_7) \land (x_1 \lor \neg x_4 \lor \neg x_2) \land (x_6 \lor \neg x_1 \lor x_5) \land (x_3 \lor x_2 \lor x_1) \land (x_2 \lor x_1 \lor \neg x_3) \land (\neg x_6 \lor \neg x_7 \lor \neg x_1) \]

- It is polynomial-time solvable to check if a ballot is valid.

- Is it easy to come up with a valid ballot?
  
  No: this boils down to solving the SAT problem (NP-hard)

- This is an unreasonable computational burden to put on participants in the election.

- Also: could open up a way to control the election.
Using Logic Fragments (for the BAIC framework)

- We would like to use a **logic fragment** (a subset $C$ of propositional formulas) for the integrity constraint $\Gamma$, with the property that:
  
  - Given a formula $\varphi \in C$ and a partial truth assignment $\alpha : \text{Var}(\varphi) \to \{0, 1\}$,
    
    it is **polynomial-time solvable** to find a truth assignment $\alpha' : \text{Var}(\varphi) \to \{0, 1\}$ that extends $\alpha$ and that satisfies $\varphi$ (if it exists)

- This way, the basic operations for participating in and administering the election are efficient
The 2CNF Fragment

- A 2CNF formula is a propositional formula $\varphi$ in conjunctive normal form (CNF), where each clause has at most 2 literals
  - In other words, conjunctions of clauses of the following form:
    
    $$
    (x_1 \lor x_2) \text{ or } (\neg x_1 \lor x_2) \text{ or } (x_1 \lor \neg x_2) \text{ or } (\neg x_1 \lor \neg x_2)
    $$

- Given a 2CNF formula $\varphi$ and a partial truth assignment $\alpha : \text{Var}(\varphi) \rightarrow \{0, 1\}$, it is polynomial-time solvable to find a truth assignment $\alpha' : \text{Var}(\varphi) \rightarrow \{0, 1\}$ that extends $\alpha$ and that satisfies $\varphi$ (if it exists)

- If we take the second judgment aggregation framework (BAIC) and restrict the constraints to be 2CNF formulas, none of the complexity issues for basic operations arise
Theorem

JA-Outcome(Kemeny) for the BAIC framework is polynomial-time solvable, when the constraint is restricted to be a 2CNF formula.

▶ Idea:

▶ If \( \Gamma \in 2\text{CNF} \), then the agenda satisfies the median property
▶ So for every profile \( J \), Majority\( (J) \) is consistent, and the Kemeny procedure selects the majority outcome

▶ Downside: the fragment of 2CNF has limited expressivity

▶ E.g., the example of the discursive dilemma and the example of budget spending that we say cannot be expressed using a 2CNF constraint \( \Gamma \)
The Horn Fragment

- A **Horn formula** is a propositional formula $\varphi$ in conjunctive normal form (CNF), where each clause has at most 1 positive literal.
  - In other words, conjunctions of clauses of the following form:
    
    $$(\neg x_1 \lor \cdots \lor \neg x_u) \text{ or } (y \lor \neg x_1 \lor \cdots \lor \neg x_u)$$

- Given a Horn formula $\varphi$ and a partial truth assignment $\alpha : \text{Var}(\varphi) \rightarrow \{0, 1\}$, it is polynomial-time solvable to find a truth assignment $\alpha' : \text{Var}(\varphi) \rightarrow \{0, 1\}$ that extends $\alpha$ and that satisfies $\varphi$ (if it exists).

- If we take the second judgment aggregation framework (BAIC) and restrict the constraints to be Horn formulas, none of the complexity issues for basic operations arise.
Theorem (De Haan, 2018)

JA-Outcome(Kemeny) for the BAIC framework is $\Theta^P_2$-hard, even when the constraint is restricted to be a Horn formula.

- So restricting the setting to avoid complexity issues in the basic operations of the election does not automatically buy us efficient elections.

Further Fragments

- There are fragments that strike a certain balance between
  (1) expressivity and compactness, and
  (2) efficiency of computing outcomes for various judgment
  aggregation procedures.

- Boolean circuits in Decomposable Negation Normal Form
  (DNNF)

- These offer an efficient encoding of budget constraints, for
  example

**Theorem (De Haan, 2018)**

JA-Outcome(Kemeny) for the BAIC framework is polynomial-time
solvable, even when the constraint is restricted to be a DNNF
Circuit.
Relation Between the Frameworks

For every agenda $\Phi$ and constraint $\Gamma$ in the original judgment aggregation framework one can find an equivalent set $\mathcal{I}$ of issues and constraint $\Gamma'$ in the BAIC framework,

and vice versa

Equivalent translations might be hard to find and be of exponential size

Another variant on the judgment aggregation framework:

- have two integrity constraints $\Gamma_1, \Gamma_2$
  - $\Gamma_1$ used as constraint for the individual opinions
  - $\Gamma_2$ used as constraint for the collective opinion

Modeling Borda Voting

- With two integrity constraints, we can model Borda voting:
  
  - Constraint $\Gamma_1$ for the individual opinions:
    $$\Gamma_1 = \bigwedge_{a,b \in A, a \neq b} (p_{a,b} \iff \neg p_{b,a}) \land \bigwedge_{a,b,c \in A, a \neq b, b \neq c, a \neq c} ((p_{a,b} \land p_{b,c}) \rightarrow p_{a,c})$$

  - Constraint $\Gamma_2$ for the collective opinion:
    $$\Gamma_2 = \bigvee_{a \in A} \left( \bigwedge_{b \in A, a \neq b} (p_{a,b} \land \neg p_{b,a}) \land \bigwedge_{b,c \in A, a \neq b, a \neq c, b \neq c} (p_{b,c} \land p_{c,b}) \right)$$

**Theorem (Endriss, 2018)**

When encoding a preference profile $P$ using $\Gamma_1$, the outcomes of the Kemeny judgment procedure w.r.t. $\Gamma_2$ correspond exactly to the Borda winners of $P$. 
Recap

- Judgment Aggregation (JA)
- Insufficiency of the majority procedure
- Other JA Procedures
- Complexity issues
- Different JA frameworks
- Modeling Voting in JA