Computational Complexity

Exercise Session 5

Note: These exercises are (likely) too much work to solve all during the exercise session.

Exercise 1. A decision problem $L \subseteq \{0,1\}^*$ is *sparse* if there exists a polynomial p such that for every $n \in \mathbb{N}$ it holds that $|L \cap \{0,1\}^n| \leq p(n)$. Show that every sparse decision problem is in $\mathsf{P/poly}$.

Definition 1. $\mathsf{P}^{\mathsf{NP}[\log]}$ is the class of all decision problems $L \subseteq \{0,1\}^*$ for which there exists a polynomial-time deterministic oracle TM $\mathbb{M}$ and an oracle language $O \in \mathsf{NP}$ such that $\mathbb{M}^O$ decides L, and a function $f(n) : \mathbb{N} \to \mathbb{N}$ that is $O(\log n)$ such that for each input $x \in \{0,1\}^*$, $\mathbb{M}^O(x)$ makes at most $f(|x|)$ queries to the oracle O.

Exercise 2. Show that the following problem is in $\mathsf{P}^{\mathsf{NP}[\log]}$:

$$\{\ \varphi \mid\ \varphi \text{ is a propositional logic formula, and the maximum number } m \text{ of variables among } \mathrm{var}(\varphi) \text{ that are set to true in any satisfying truth assignment of } \varphi \text{ is odd.}\ \}$$

Exercise 3. Prove that $\mathsf{RP} \subseteq \mathsf{BPP}$ and that $\mathsf{coRP} \subseteq \mathsf{BPP}$.

Exercise 4. Prove that $\mathsf{BPP} \subseteq \mathsf{PSPACE}$.