Computational Complexity

Exercise Session 6

Exercise 1. Prove that $\mathsf{RP} \subseteq \mathsf{BPP}$ and that $\mathsf{coRP} \subseteq \mathsf{BPP}$.

Exercise 2.

(a) Show that $\mathsf{RP} \subseteq \mathsf{NP}$.

(b) Show that $\mathsf{ZPP} = \mathsf{RP} \cap \mathsf{coRP}$.

- *Hint:* use Markov's inequality for showing that $\mathsf{ZPP} \subseteq \mathsf{RP} \cap \mathsf{coRP}$. If X is a non-negative random variable and $a > 0$, then:
$$\mathbb{P}(X \geq a) \leq \frac{\mathbb{E}(X)}{a}.$$

Exercise 3. Prove that $\mathsf{BPP} \subseteq \mathsf{PSPACE}$.

Exercise 4. CLIQUE is the problem of deciding, given a graph $G = (V, E)$ and a natural number $k \in \mathbb{N}$, whether there exists a set $C \subseteq V$ such that $|C| = k$ and for all $c_1, c_2 \in C$ with $c_1 \neq c_2$ it holds that $\{c_1, c_2\} \in E$.

For every $\rho < 1$, an algorithm A is called a ρ-approximation algorithm for MAX-CLIQUE if for every graph $G = (V, E)$, the algorithms outputs a clique $C \subseteq V$ of G of size at least $\rho \cdot \mu_G$, where μ_G is the maximum size of any clique of G.

Show that for each $\rho < 1$, if there exists a polynomial-time ρ-approximation algorithm for MAX-CLIQUE, then $\mathsf{P} = \mathsf{NP}$.