Recap

What we saw last time:

- The classes $\Sigma^p_i$ and $\Pi^p_i$
- The Polynomial Hierarchy
- $\Sigma^p_i$-complete and $\Pi^p_i$-complete QBF problems
- Characterizations using oracles and ATMs
What will we do today?

- Reflecting on what we’ve seen before
- Mostly using examples (of games)
One-/two-liner overview of complexity classes

- **L**: deterministic algorithm, logarithmic space (and polynomial time)
- **NL**: nondeterministic algorithm, logarithmic space (and polynomial time)
- **P**: solvable in (deterministic) polynomial time
- **NP**: solutions (for yes-answers) can be guessed/checked in polynomial time
- **coNP**: solutions (for no-answers) can be guessed/checked in polynomial time
- **Σ²_p**: solutions (for yes-answers) have “∃∀ structure”
- **Π²_p**: solutions (for yes-answers) have “∀∃ structure”
- **PSPACE**: (non)deterministic algorithm, polynomial space (and exponential time)
  OR: unbounded “∃∀∃∀∃··· structure”
- **EXP**: solvable in (deterministic) exponential time
Some oracle questions..

- Is it the case that $P^P = P$? Yes
- Is it the case that $NP^{NP} = NP$? We don’t know.
- Is it the case that $PSPACE^{PSPACE} = PSPACE$? Yes
- Is it the case that $EXP^{EXP} = EXP$? No
- Is it the case that $\text{DTIME}(n^2)^{\text{DTIME}(n^2)} = \text{DTIME}(n^2)$? No
- Is it the case that $\text{NTIME}(n)^{\text{NTIME}(n)} = \text{NTIME}(n)$? We don’t know.
Polls on $P \overset{?}{=} NP$ have been held among computational complexity researchers:

- In 2002, see: https://tiny.cc/pnp-poll1
- In 2012, see: https://tiny.cc/pnp-poll2
- In 2019, see: https://tiny.cc/pnp-poll3

In these papers, there are some very interesting opinions on the question (and some nerdy jokes)

Short answer: we have no clue (really), why $P = NP$ or $P \neq NP$ would be true, but most think that $P \neq NP$. 
What is the complexity of this problem?

Input: A propositional formula $\varphi$, and a satisfying truth assignment $\alpha$ for $\varphi$.

Question: Is $\alpha$ the only satisfying assignment for $\varphi$?
Quiz example #2: finding a minimal equivalent DNF formula

■ What is the complexity of this problem?

■ Input: A propositional formula $\varphi$, and $1^k$ for some $k \in \mathbb{N}$.

Question: Is there a DNF formula $\psi$ of size $\leq k$ such that $\varphi \equiv \psi$?
What is the complexity of this problem?

**Input:** Two propositional formulas $\varphi_1, \varphi_2$.

**Question:** $\varphi_1 \equiv \varphi_2$?
What is the complexity of this problem?

Input: A propositional 2CNF formula $\varphi$.

Question: Is $\varphi$ satisfiable?
What is the complexity of this problem?

Input: A basic modal logic formula $\varphi$.

Question: Is $\varphi$ satisfiable?
What is the complexity of this problem?

**Input:** A modal logic formula $\varphi$.

**Question:** Is there an S5 Kripke model where $\varphi$ is true?

**Theorem:** if there is an S5 Kripke model where $\varphi$ is true, then there exists an S5 Kripke model with at most $|\varphi|$ states where $\varphi$ is true.
Quiz example #7: Tiling I

What is the complexity of this problem?

**Input:** A set of 4-sided tile types, and $n, m \in \mathbb{N}$.

**Question:** Can we use these tile types to fill an $n \times m$ grid, so that
(1) the outsides of the grid all have side $s_0$, and
(2) neighboring tiles have matching sides?
Quiz example #8: Tiling II

What is the complexity of this problem?

Input: A set of 4-sided tile types, and $n \in \mathbb{N}$.

Question: Can we use these tile types to fill an $n \times m$ grid, for some $m \in \mathbb{N}$, so that

(1) the outsides of the grid all have side $s_0$, and
(2) neighboring tiles have matching sides?
Quiz example #9: Greedy Spiders

- What is the complexity of this problem?

- *Input:* An instance $I$ of the *greedy spiders* puzzle.
  
  *Question:* Is there a solution for $I$?
Quiz example #10: Generalized Geography

What is the complexity of this problem? (See: https://en.wikipedia.org/wiki/Generalized_geography)

Input: An instance $I$ of generalized geography.

Question: Does Player 1 have a winning strategy?
What is the complexity of this problem? (See: https://en.wikipedia.org/wiki/Generalized_geography)

**Input:** An instance $I$ of generalized geography.

**Question:** Does Player 1 have a winning strategy?
What is the complexity of this problem?
(See: https://en.wikipedia.org/wiki/Game_of_the_Amazons)

Input: A (2-player) Game of the Amazons position (on an $n \times n$ board).

Question: Does Player 1 have a winning strategy?
Quiz example #12: Game of the Amazons (1 player)

What is the complexity of this problem?
(See: https://en.wikipedia.org/wiki/Game_of_the_Amazons)

Input: A (1-player) Game of the Amazons position (on an $n \times n$ board), and $1^k$ for some $k \in \mathbb{N}$.

Question: Can Player 1 make at least $k$ consecutive moves?
What is the complexity of this problem?

**Input:** A propositional logic formula $\varphi(x_1, \ldots, x_n, x'_1, \ldots, x'_n)$, and two binary vectors $s, t \in \{0, 1\}^n$.

**Question:** Consider the directed graph $G = (V, E)$, where:

- $V = \{0, 1\}^n$,
- and for each $\vec{v}, \vec{w} \in V$, $(\vec{v}, \vec{w}) \in E$ if and only if $\varphi[\vec{u}, \vec{w}]$ is true.

Is $t$ reachable from $s$ in $G$?
Quiz example #14: 3-colorability for succinctly represented graphs

What is the complexity of this problem?

Input: A propositional logic formula $\varphi(x_1, \ldots, x_n, x'_1, \ldots, x'_n)$, and two binary vectors $s, t \in \{0, 1\}^n$.

Question: Consider the undirected graph $G = (V, E)$, where:
- $V = \{0, 1\}^n$, and for each $\overline{v}, \overline{w} \in V$,
- $\{\overline{v}, \overline{w}\} \in E$ if and only if $\varphi[\overline{u}, \overline{w}]$ is true.

Is $G$ 3-colorable?
Next time

- Non-uniform complexity
- Circuit complexity
- TMs that take advice
- The Karp-Lipton Theorem