

Computational Complexity

Exercise Session 6

Exercise 1. Prove that $\text{RP} \subseteq \text{BPP}$ and that $\text{coRP} \subseteq \text{BPP}$.

Exercise 2. Prove that $\text{BPP} \subseteq \text{PSPACE}$.

Exercise 3. CLIQUE is the problem of deciding, given a graph $G = (V, E)$ and a natural number $k \in \mathbb{N}$, whether there exists a set $C \subseteq V$ such that $|C| = k$ and for all $c_1, c_2 \in C$ with $c_1 \neq c_2$ it holds that $\{c_1, c_2\} \in E$.

For every $\rho < 1$, an algorithm A is called a ρ -approximation algorithm for MAX-CLIQUE if for every graph $G = (V, E)$, the algorithm outputs a clique $C \subseteq V$ of G of size at least $\rho \cdot \mu_G$, where μ_G is the maximum size of any clique of G .

Show that for each $\rho < 1$, if there exists a polynomial-time ρ -approximation algorithm for MAX-CLIQUE, then $\text{P} = \text{NP}$.