Definition 1 (Probabilistic TMs). Probabilistic Turing machines (PTMs) are variants of (deterministic) Turing machines, where a few elements are modified.

- Instead of one halting state $q_{\text{halt}}$, there are two halting states $q_{\text{acc}}$ (the accept state) and $q_{\text{rej}}$ (the reject state).
- Instead of a single transition function $\delta$, there are two transition functions $\delta_1, \delta_2$.
- To execute a PTM on an input $x$, at each step, we use the transition function $\delta_1$ with probability $1/2$ and the transition function $\delta_2$ with probability $1/2$. At each step, this choice is made independently of all previous choices.
- The TM outputs only 0 (when halting in $q_{\text{acc}}$) or 1 (when halting in $q_{\text{rej}}$). We denote by $M(x)$ the random variable corresponding to the value that the machine $M$ outputs when executed on $x$.
- Let $T : \mathbb{N} \rightarrow \mathbb{N}$. The TM runs in time $T(n)$ if for every input $x$ the machine halts after at most $T(|x|)$ steps, regardless of its random choices that it makes.

Definition 2 (BPTIME, BPP). Let $T : \mathbb{N} \rightarrow \mathbb{N}$ be a function, and $L \subseteq \{0, 1\}^*$ be a language. We say that a PTM $M$ decides $L$ in time $T(n)$ if for every $x \in \{0, 1\}^*$, $M$ halts in $T(|x|)$ steps, regardless of its random choices, and $\Pr[M(x) = L(x)] \geq 2/3$.

The class BPTIME($O(T(n))$) is the set of all languages decided by PTMs in time $O(T(n))$.

The class BPP is defined as follows:

$$\text{BPP} = \bigcup_{c \geq 1} \text{BPTIME}(n^c)$$

Definition 3 (BPP, alternative definition). A language $L \subseteq \{0, 1\}^*$ is in BPP if there exists a polynomial-time TM $M$ and a polynomial $p : \mathbb{N} \rightarrow \mathbb{N}$ such that for every $x \in \{0, 1\}^*$:

$$\Pr_{r \in \mathbb{R}^{\{0, 1\}^{|x|}}}[M(x, r) = L(x)] \geq 2/3$$

Definition 4 (RTIME, RP, coRP). Let $T : \mathbb{N} \rightarrow \mathbb{N}$ be a function. The class RTIME($O(T(n))$) contains every language $L \subseteq \{0, 1\}^*$ for which there exists a PTM $M$ running in time $O(T(n))$ such that:

- if $x \in L$, then $\Pr[M(x) = 1] \geq 2/3$
- if $x \notin L$, then $\Pr[M(x) = 0] = 1$

The class RP is defined as follows:

$$\text{RP} = \bigcup_{c \geq 1} \text{RTIME}(n^c)$$
The class coRTIME($T(n)$) contains every language $L \subseteq \{0, 1\}^*$ for which there exists a PTM $M$ running in time $O(T(n))$ such that:

- if $x \in L$, then $\Pr[M(x) = 1] = 1$
- if $x \notin L$, then $\Pr[M(x) = 0] \geq 2/3$

The class coRP is defined as follows:

$$\text{coRP} = \bigcup_{c \geq 1} \text{coRTIME}(n^c)$$

Alternatively:

$$\text{coRP} = \{ L \mid L \in \text{RP} \}$$

**Definition 5 (ZTIME, ZPP).** Let $T : \mathbb{N} \to \mathbb{N}$ be a function. The class ZTIME($T(n)$) contains every language $L \subseteq \{0, 1\}^*$ for which there exists a PTM $M$ running in expected time $O(T(n))$ such that for every input $x \in \{0, 1\}^*$, whenever $M$ halts on $x$, the output of $M$ is exactly $L(x)$.

The class ZPP is defined as follows:

$$\text{ZPP} = \bigcup_{c \geq 1} \text{ZTIME}(n^c)$$