Computational Complexity

Handout – Lecture 3

Definition 1. The decision problem TM-SAT is defined as follows:

$$\text{TM-SAT} = \{\ (\alpha, x, 1^n, 1^t)\ \mid\ \begin{array}{l}\text{there exists } u \in \{0,1\}^n \text{ such that} \\ \mathbb{M}_\alpha \text{ outputs 1 on input } (x,u) \text{ within } t \text{ steps}\ \}\end{array}$$

Or, described in a different format:

Input: A binary string α, a binary string x, a unary string 1^n, and a unary string 1^t.

Question: Does there exist a binary string $u \in \{0,1\}^n$ such that $\mathbb{M}_\alpha$ outputs 1 on input (x,u) within t steps?

Definition 2. The problem FORMULA-SAT is defined as follows:

Input: A propositional logic formula φ.

Question: Is φ satisfiable?

Definition 3. A propositional logic formula φ is in *conjunctive normal form (CNF)* if it is of the form $c_1 \wedge \cdots \wedge c_m$, where each c_j is a disjunction of literals (propositional variables or their negation). The conjuncts $c_1, \ldots, c_m$ are called *clauses* of φ.

Let $k \geq 2$. Then k-CNF denotes the set of all propositional formulas in CNF whose clauses contain (at most) k literals.

Definition 4. The problem k-SAT (or kSAT) is defined as follows:

Input: A propositional logic formula φ in k-CNF.

Question: Is φ satisfiable?